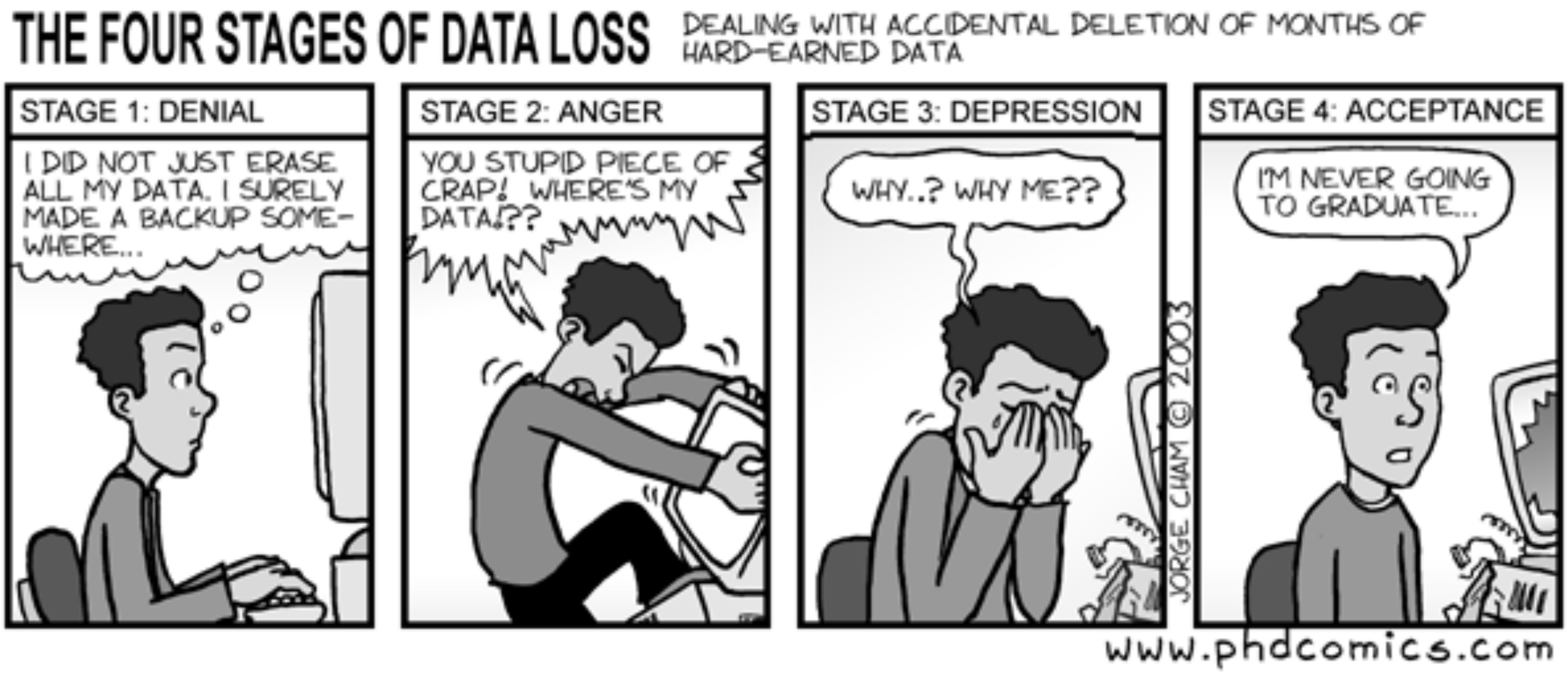
07 - balance principles

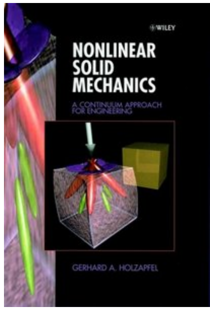


holzapfel ‘nonlinear solid mechanics ‘ [2000], chapter 4, pages 131-179

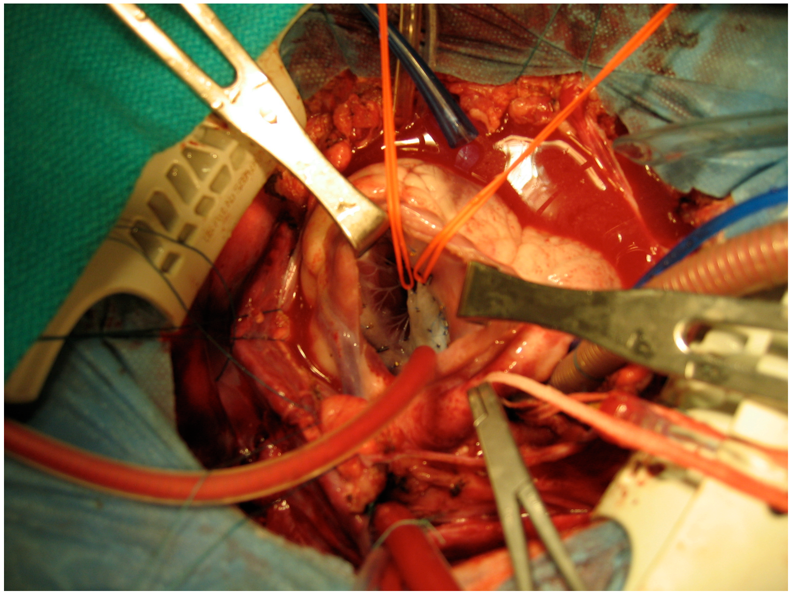
07 - balance principles

me338 - syllabus

day	date		topic	chapters	pages
tue	sep	25	why continuum mechanics?		
thu	sep	27	introduction to vectors and tensors	1.1-1.5	1-32
tue	oct	02	introduction to vectors and tensors	1.6-1.9	32-55
thu	oct	04	kinematics	2.1-2.4	55-76
tue	oct	09	kinematics	2.5-2.8	76-109
thu	oct	11	concept of stress	3.1-3.4	109-131
tue	oct	16	balance principles	4.1-4.4	131-161
thu	oct	18	balance principles	4.5-4.7	161-179
tue	oct	23	aspects of objectivity	5.1-5.4	179-205
thu	oct	25	hyperelastic materials	6.1-6.2	205-222
tue	oct	30	hyperelastic materials	6.3-6.5	222-252
thu	nov	01	hyperelastic materials	6.6-6.8	252-278
tue	nov	06	hyperelastic materials	6.9-6.11	278-305
thu	nov	08	thermodynamics of materials	7.1-7.6	305-337
tue	nov	13	midterm prep		
thu	nov	15	midterm		
tue	nov	27	thermodynamics of materials	7.7-7.9	337-371
thu	nov	29	variational principles	8.1-8.3	371-392
tue	dec	04	variational principles	8.4-8.6	392-414
thu	dec	06	final project discussion		



videofluoroscopic markers

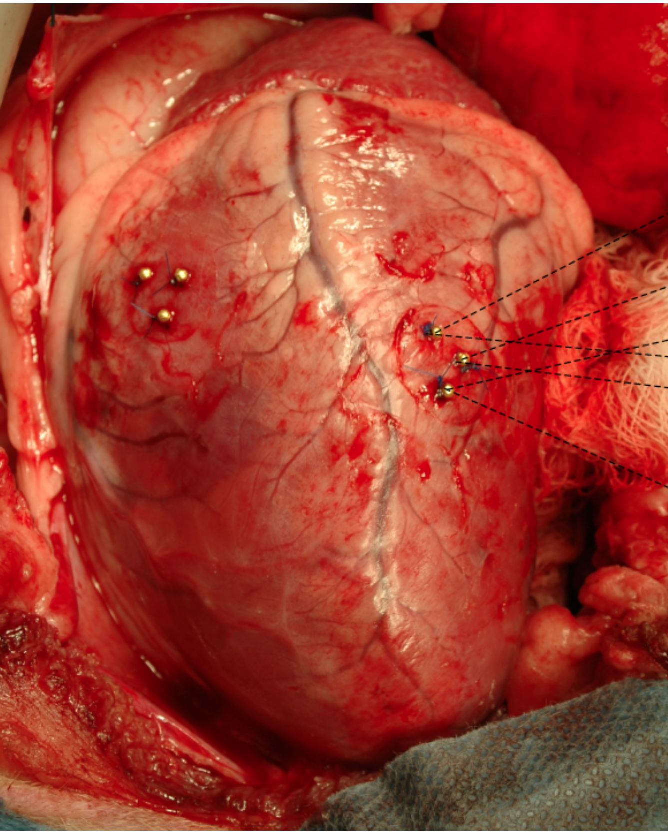


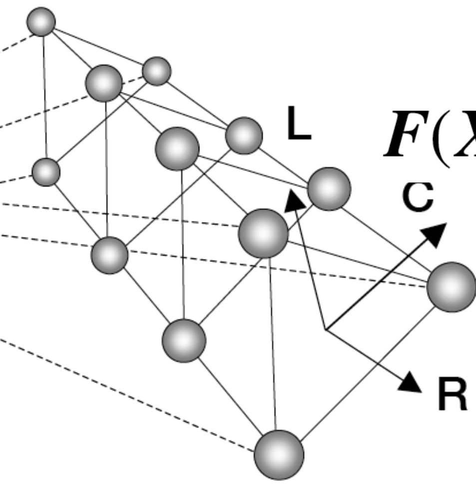
surgically implanted epicardial markers and transmural bead set

4d coordiantes from in vivo biplane videofluoroscopic marker images

itoh, bothe, swanson birchill, escobar kvitting, nguyen, langer,, rodriguez, criscione, ingels, miller

how much does the heart contract?





deformation

$$\boldsymbol{\varphi}(\boldsymbol{X},t) = \sum_{I=1}^{n_{\text{apx}}} \boldsymbol{c}_I(t)\; N_I(\boldsymbol{X})$$

deformation gradient

$$\boldsymbol{F}(\boldsymbol{X},t) = \sum_{I=1}^{n_{\text{apx}}} \boldsymbol{c}_I(t) \otimes \nabla N_I(\boldsymbol{X})$$

green lagrange strains

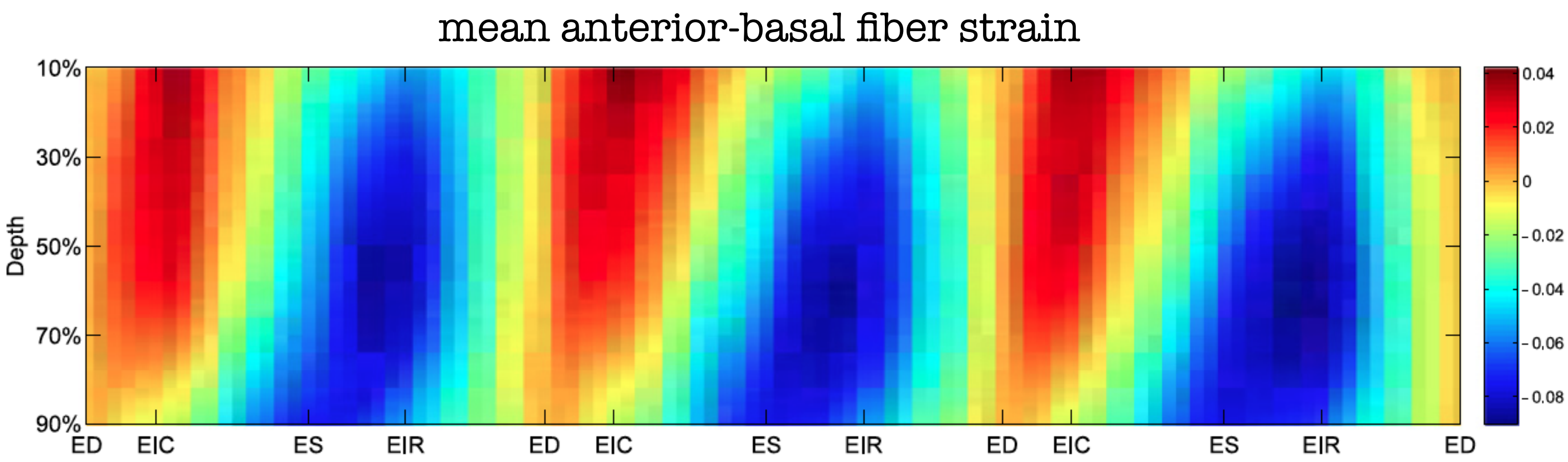
$$\boldsymbol{E}(\boldsymbol{X},t) = \frac{1}{2}\,[\,\boldsymbol{F}^{\text{t}} \cdot \boldsymbol{F} - \boldsymbol{I}\,]$$

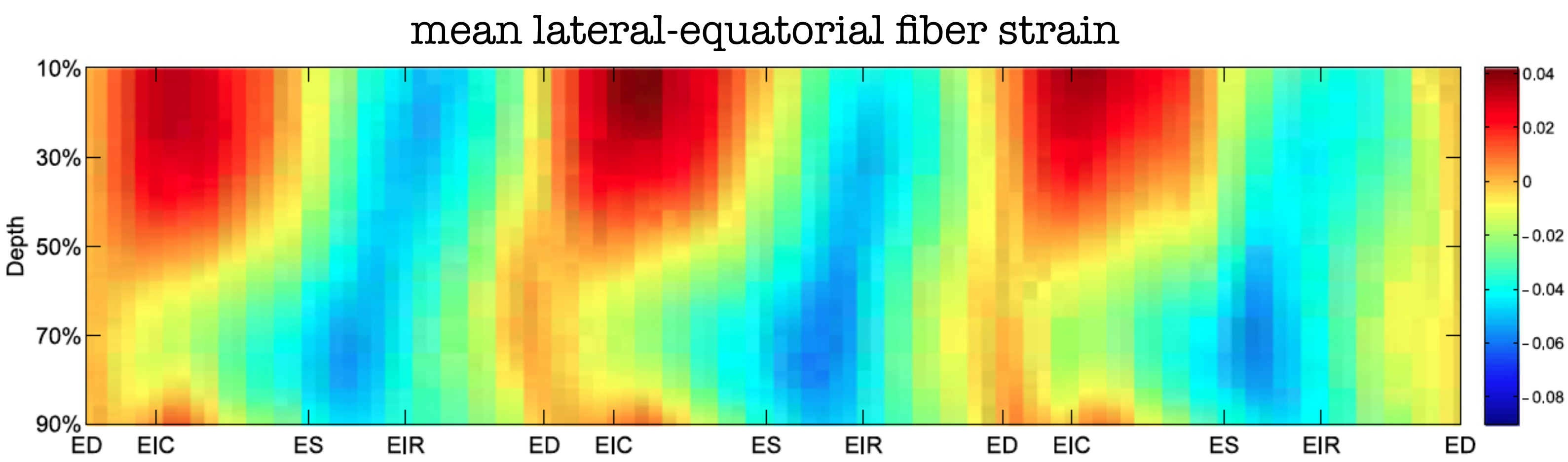
fiber strain

$$\mathsf{E}_{\mathsf{FF}}(\boldsymbol{X},t) = \boldsymbol{f}(\boldsymbol{X}) \cdot \boldsymbol{E}(\boldsymbol{X},t) \cdot \boldsymbol{f}(\boldsymbol{X})$$

tsamis, bothe, kvitting, swanson, miller, kuhl [2011]

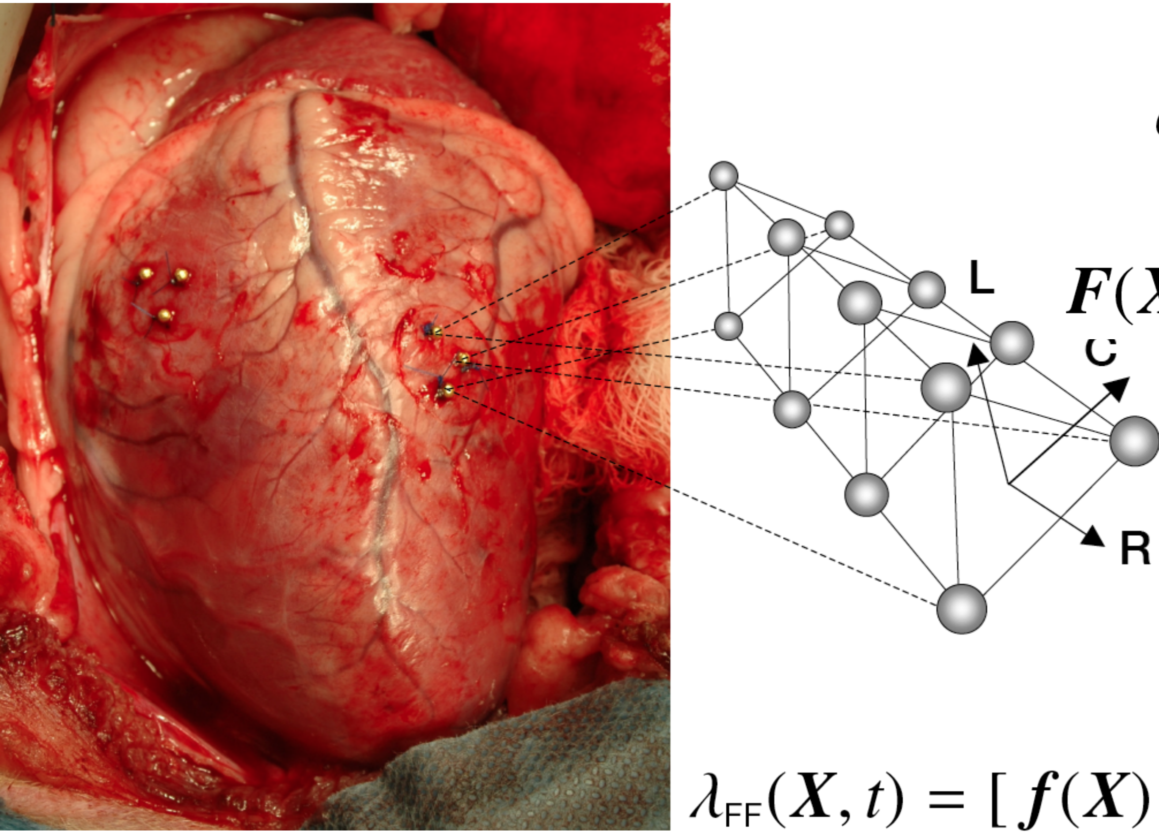
how much does the heart contract?





tsamis, bothe, kvitting, swanson, miller, kuhl [2011]

how much does the heart grow?



deformation

$$\boldsymbol{\varphi}(\boldsymbol{X},t) = \sum_{I=1}^{n_{\text{apx}}} \boldsymbol{c}_I(t)\, N_I(\boldsymbol{X})$$

deformation gradient

$$\boldsymbol{F}(\boldsymbol{X},t) = \sum_{I=1}^{n_{\text{apx}}} \boldsymbol{c}_I(t) \otimes \nabla N_I(\boldsymbol{X})$$

volume changes

$$J(\boldsymbol{X},t) = \det\left(\boldsymbol{F}(\boldsymbol{X},t)\right)$$

fiber stretch

$$\lambda_{\text{FF}}(\boldsymbol{X},t) = [\, \boldsymbol{f}(\boldsymbol{X}) \cdot \boldsymbol{F}^{\text{t}}(\boldsymbol{X},t) \cdot \boldsymbol{F}(\boldsymbol{X},t) \cdot \boldsymbol{f}(\boldsymbol{X}) \,]^{1/2}$$

tsamis, cheng, nguyen, langer, miller, kuhl [2012]

homework 02

how much does the heart grow?

	epi 20% depth	p	mid 50% depth	p	endo 80% depth	p
F^g_{CC}	1.00±0.12	0.96	1.03±0.14	0.46	1.02±0.10	0.44
F^g_{CL}	0.04±0.14	0.42	0.01±0.10	0.77	0.01±0.09	0.61
F^g_{CR}	−0.07±0.29	0.46	−0.03±0.16	0.61	0.05±0.14	0.29
F^g_{LC}	−0.02±0.17	0.75	−0.04±0.13	0.33	−0.04±0.11	0.24
F^g_{LL}	1.10±0.15	0.06	1.10±0.13	0.03	1.11±0.11	0.01
F^g_{LR}	0.02±0.16	0.71	0.10±0.20	0.11	0.18±0.34	0.12
F^g_{RC}	−0.01±0.09	0.64	−0.03±0.17	0.54	−0.05±0.19	0.41
F^g_{RL}	0.00±0.05	0.86	−0.00±0.09	0.96	−0.01±0.11	0.67
F^g_{RR}	0.68±0.15	0.00	0.73±0.15	0.00	0.77±0.22	0.01
J^g	0.74±0.19	0.00	0.82±0.19	0.01	0.89±0.21	0.10
λ^g_{FF}	1.03±0.12	0.49	1.04±0.16	0.36	1.08±0.11	0.04

tsamis, cheng, nguyen, langer, miller, kuhl [2012]

how much does the heart grow?

	epi 20% depth	p	mid 50% depth	p	endo 80% depth	p
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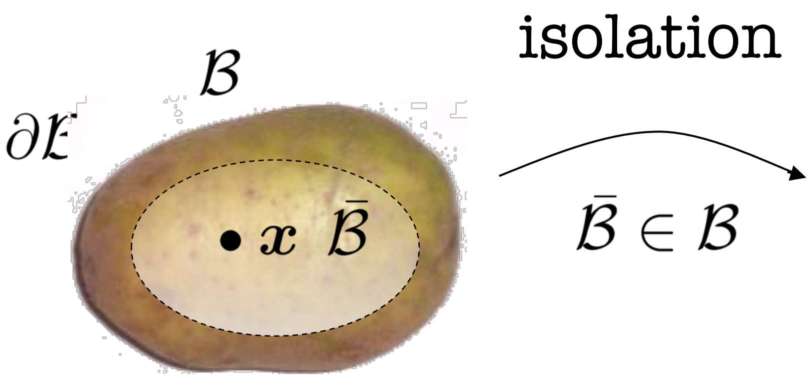
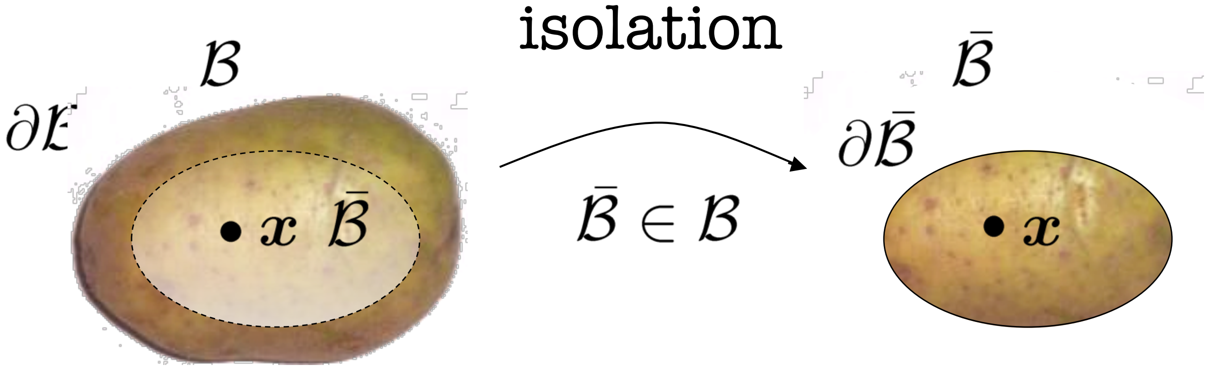
- longitudinal growth by more than 10%
- radial thinning by more than 20%
- volume decrease by more than 15%
- fiber lengthening by more than 5%

tsamis, cheng, nguyen, langer, miller, kuhl [2012]



[1] isolate subset $\bar{\mathcal{B}}$ from $\mathcal{B}$

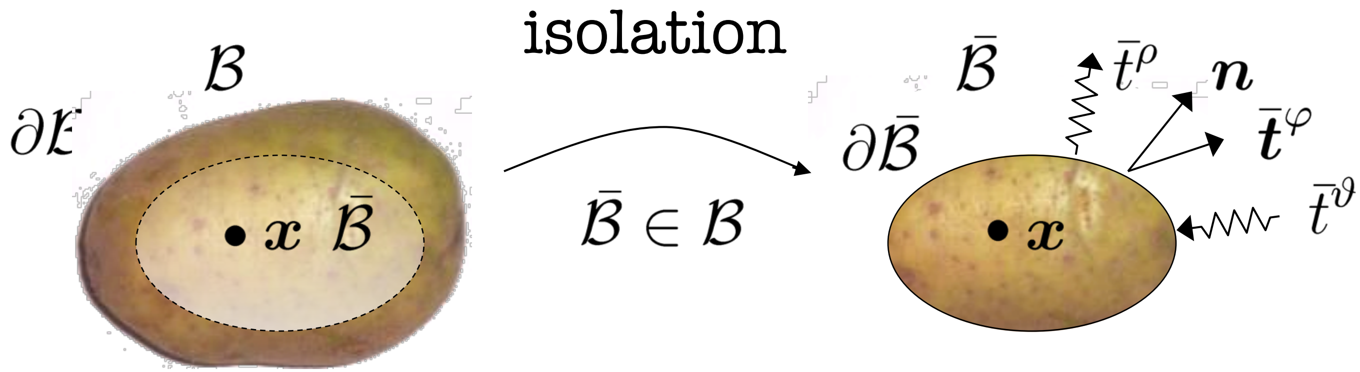
potato  balance equations



[1] isolate subset $\bar{\mathcal{B}}$ from $\mathcal{B}$

[2] **characterize** influence of remaining body through phenomenological quantities - contact fluxes $\bar{t}^{\rho}$, $\bar{\boldsymbol{t}}^{\varphi}$& $\bar{t}^{\vartheta}$

potato  balance equations



[1] isolate subset $\bar{\mathcal{B}}$ from $\mathcal{B}$

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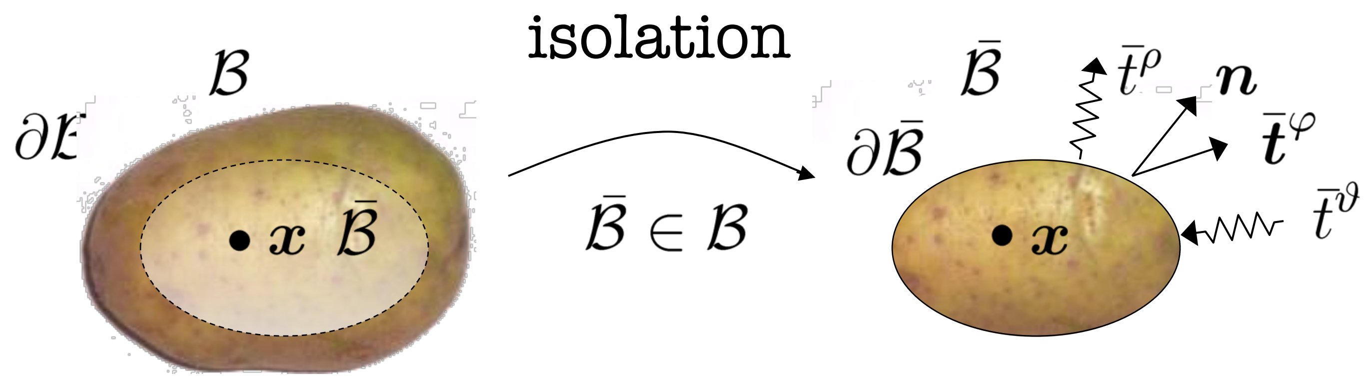
[3] **define** basic physical quantities - mass, linear and angular momentum, energy

potato  balance equations



[1] isolate subset $\bar{\mathcal{B}}$ from $\mathcal{B}$

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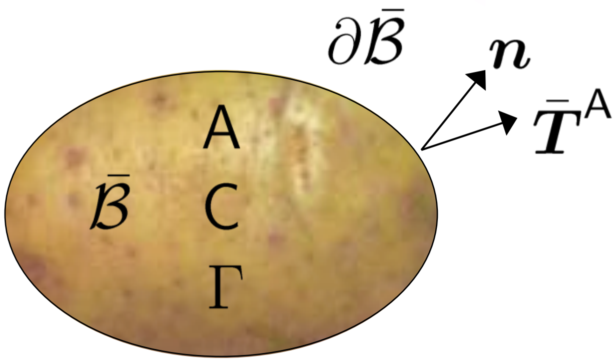
[3] define basic physical quantities - mass, linear and angular momentum, energy

[4] **postulate** balance of these quantities

generic balance equation



general format

A ... balance quantity

$\mathbf{B}$ ... flux $\quad \mathbf{B}\cdot\boldsymbol{n} = \bar{\boldsymbol{T}}^{\mathsf{A}}$

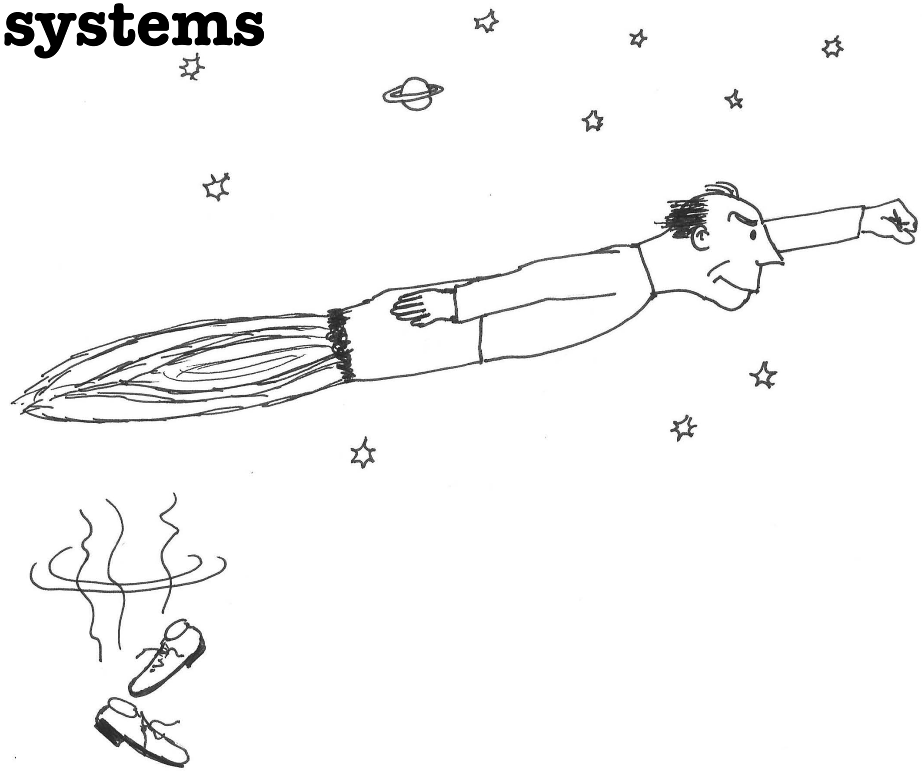
C ... source

Γ ... production

$$\mathrm{D}_t\mathsf{A} = \mathrm{Div}(\mathbf{B}) + \mathsf{C} + \Gamma$$

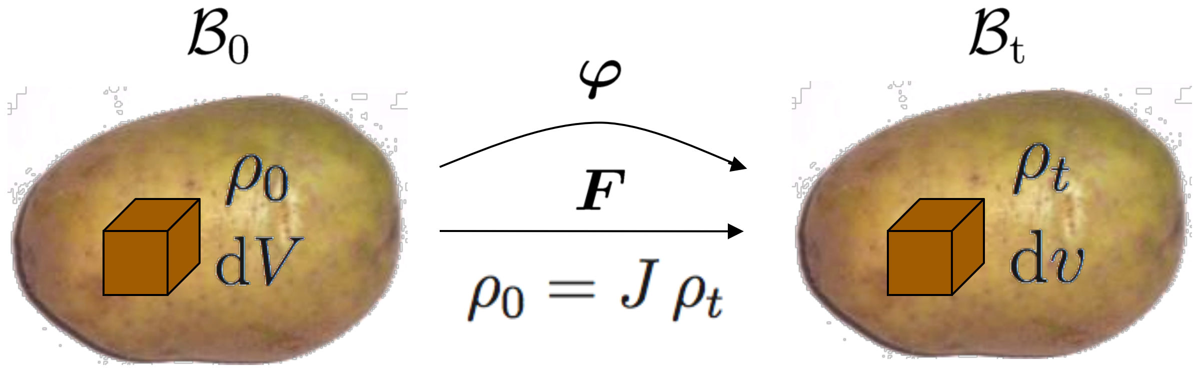
balance of mass

- here: **closed systems**



- unlike open systems **closed systems** have a **constant mass**
- examples of open systems: **rocket propulsion** and **biological growth** (me337)

balance of mass



$$\mathrm{d}m = \rho_0\,\mathrm{d}V \qquad\qquad \mathrm{d}m = \rho_t\,\mathrm{d}v$$

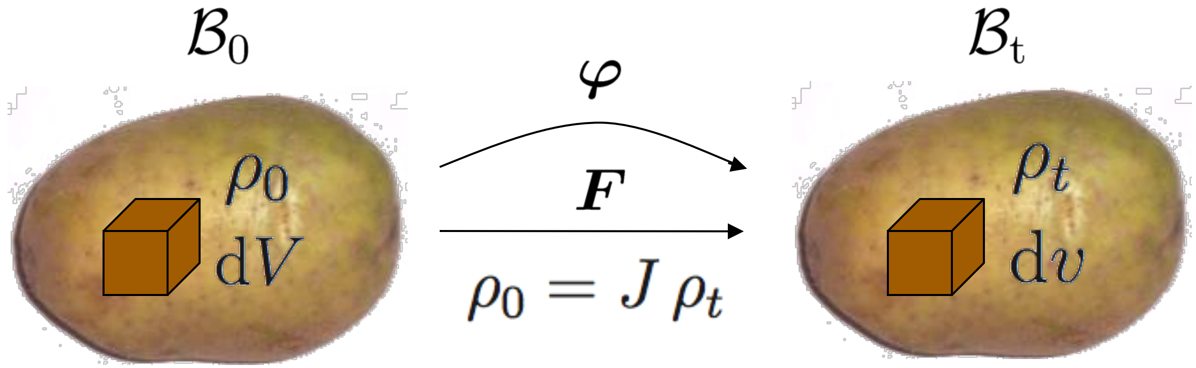
- mass is always constant and positive

$$m = \int_{\mathcal{B}_0} \rho_0\,\mathrm{d}V = \int_{\mathcal{B}_t} \rho_t\,\mathrm{d}v = \mathrm{const} > 0$$

- changes in volume and density are related through J

$$\mathrm{d}v = J\,\mathrm{d}V \quad \text{and} \quad \rho_0 = J\,\rho_t$$

balance of mass



mass is constant

$$\dot{m} = \mathrm{D}_t\, m = \mathrm{D}_t \int_{\mathcal{B}_0} \rho_0 \,\mathrm{d}V \doteq 0 \qquad \dot{m} = \mathrm{D}_t\, m = \mathrm{D}_t \int_{\mathcal{B}_t} \rho_t \,\mathrm{d}v \doteq 0$$

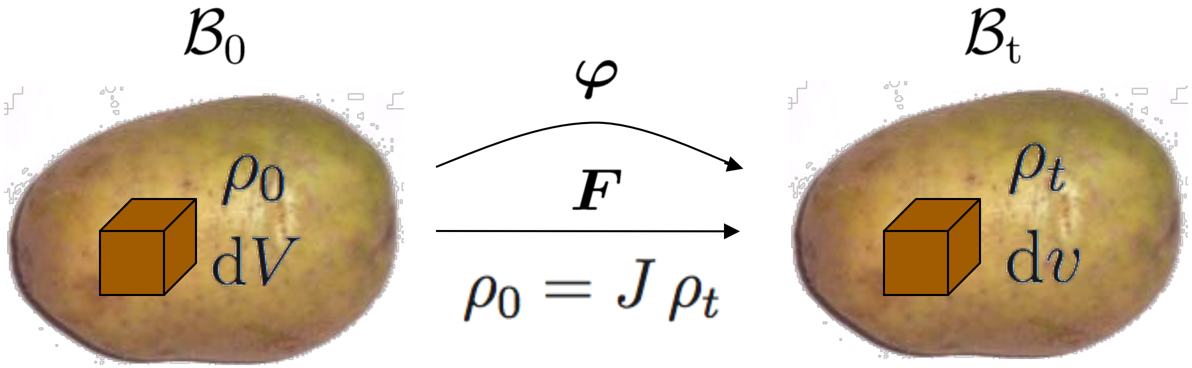
$$= \int_{\mathcal{B}_0} \mathrm{D}_t\, \rho_0 \,\mathrm{d}V \doteq 0 \qquad = \int_{\mathcal{B}_0} \mathrm{D}_t (J\rho_t) \,\mathrm{d}V \doteq 0$$

material density is constant

$$\dot{\rho}_0 \doteq 0 \rightarrow \rho_0 = \text{const} \qquad \dot{\rho}_t + \rho_t \,\mathrm{div}(\boldsymbol{v}) \doteq 0$$

$$\mathrm{D}_t(J\rho_t) = J\,\mathrm{D}_t\rho_t + \rho_t\,\mathrm{D}_t J = J\,\mathrm{D}_t\rho_t + \rho_t\,J\,\mathrm{div}(\boldsymbol{v}) = J\,[\,\dot{\rho}_t + \rho_t\,\mathrm{div}(\boldsymbol{v})\,]$$

balance of mass

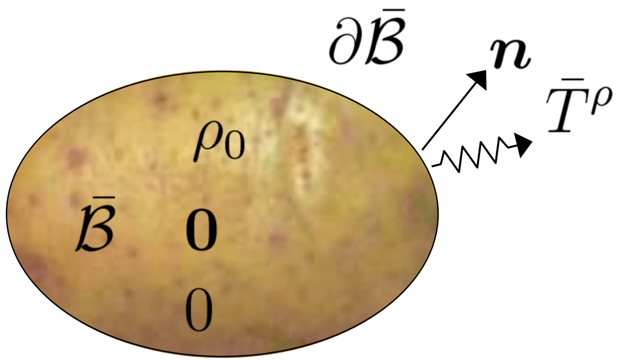


global balance of mass

$$\dot{m} = \int_{\mathcal{B}_0} \dot{\rho}_0 \,\mathrm{d}V \doteq 0 \qquad \dot{m} = \int_{\mathcal{B}_0} \dot{\rho}_t + \rho_t \,\mathrm{div}(\boldsymbol{v})\,\mathrm{d}V \doteq 0$$

local balance of mass

$$\dot{\rho}_0 \doteq 0 \qquad \dot{\rho}_t + \rho_t \,\mathrm{div}(\boldsymbol{v}) \doteq 0$$



ρ_0 ... density

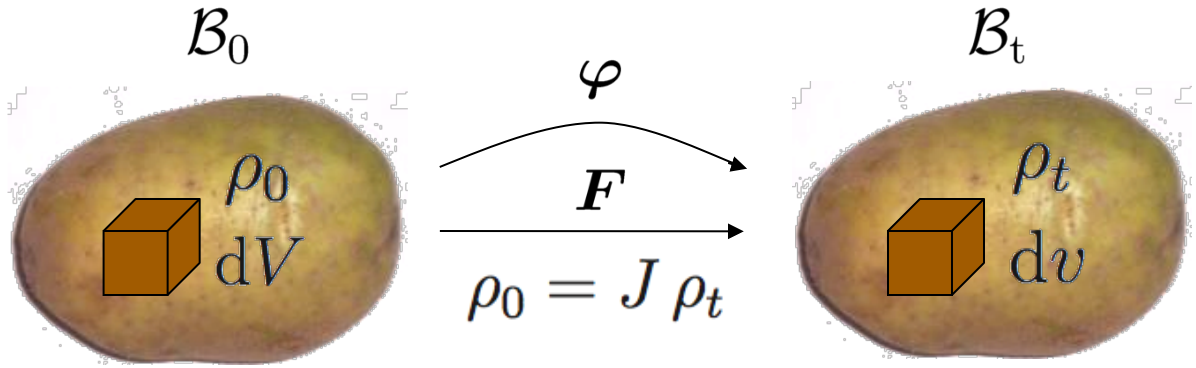
$\mathbf{0}$ ... no mass flux $\qquad \bar{T}^\rho = 0$

0 ... no mass source

0 ... no mass production

continuity equation $\qquad \mathrm{D}_t \rho_0 = 0$

balance of (linear) momentum



linear momentum density

$$\dot{\boldsymbol{L}} = \mathrm{D}_t \int_{\mathcal{B}_0} \rho_0 \boldsymbol{v}\,\mathrm{d}V = \int_{\mathcal{B}_0} \rho_0 \dot{\boldsymbol{v}}\,\mathrm{d}V = \boldsymbol{F}$$

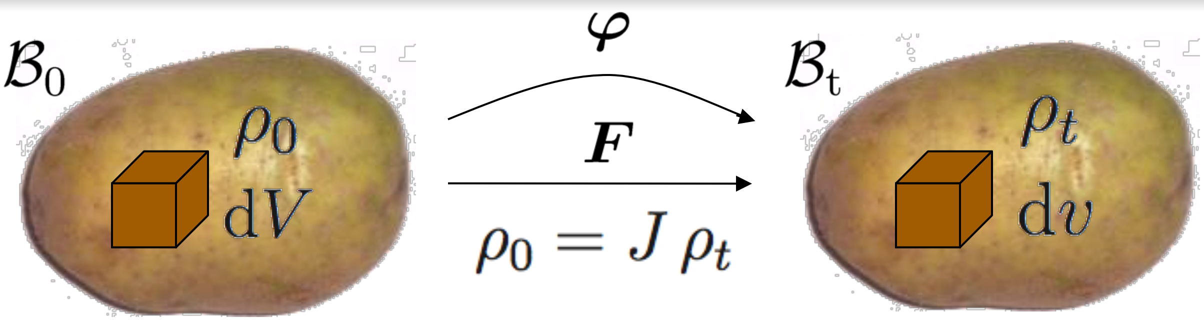
$$\dot{\boldsymbol{L}} = \mathrm{D}_t \int_{\mathcal{B}_t} \rho_t \boldsymbol{v}\,\mathrm{d}v = \int_{\mathcal{B}_t} \rho_t \dot{\boldsymbol{v}}\,\mathrm{d}v = \boldsymbol{F}$$

surface force (flux) and volume force (source)

$$\boldsymbol{F} = \int_{\partial\mathcal{B}_0} \boldsymbol{T}\,\mathrm{d}A + \int_{\mathcal{B}_0} \boldsymbol{b}_0\,\mathrm{d}A$$

$$\boldsymbol{F} = \int_{\partial\mathcal{B}_t} \boldsymbol{t}\,\mathrm{d}a + \int_{\mathcal{B}_t} \boldsymbol{b}_t\,\mathrm{d}a$$

balance of (linear) momentum



global balance of linear momentum

$$\int_{\mathcal{B}_0} \rho_0 \dot{\boldsymbol{v}}\,\mathrm{d}V = \int_{\mathcal{B}_0} \mathrm{Div}(\boldsymbol{P})\,\mathrm{d}V + \int_{\mathcal{B}_0} \boldsymbol{b}_0\,\mathrm{d}V$$

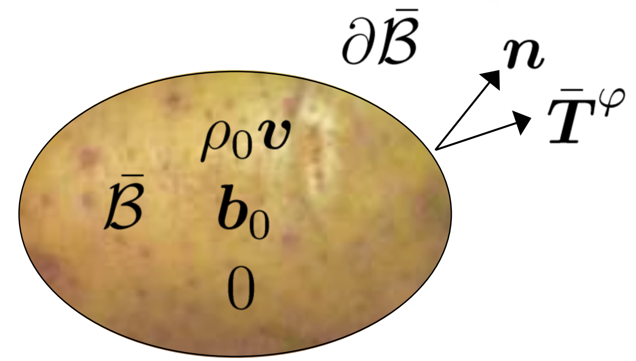
$$\int_{\mathcal{B}_t} \rho_t \dot{\boldsymbol{v}}\,\mathrm{d}v = \int_{\mathcal{B}_t} \mathrm{div}(\boldsymbol{\sigma})\,\mathrm{d}v + \int_{\mathcal{B}_t} \boldsymbol{b}_t\,\mathrm{d}v$$

local balance of linear momentum

$$\rho_0 \dot{\boldsymbol{v}} = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0 \qquad \rho_t \dot{\boldsymbol{v}} = \mathrm{div}(\boldsymbol{\sigma}) + \boldsymbol{b}_t$$

$$\rho_0 = J\,\rho_t \qquad \boldsymbol{P} = J\,\boldsymbol{\sigma}\cdot\boldsymbol{F}^{-\mathrm{t}} \qquad \boldsymbol{b}_0 = J\boldsymbol{b}_0$$

balance of (linear) momentum



$\rho_0\,\boldsymbol{v}$ ... linear momentum density

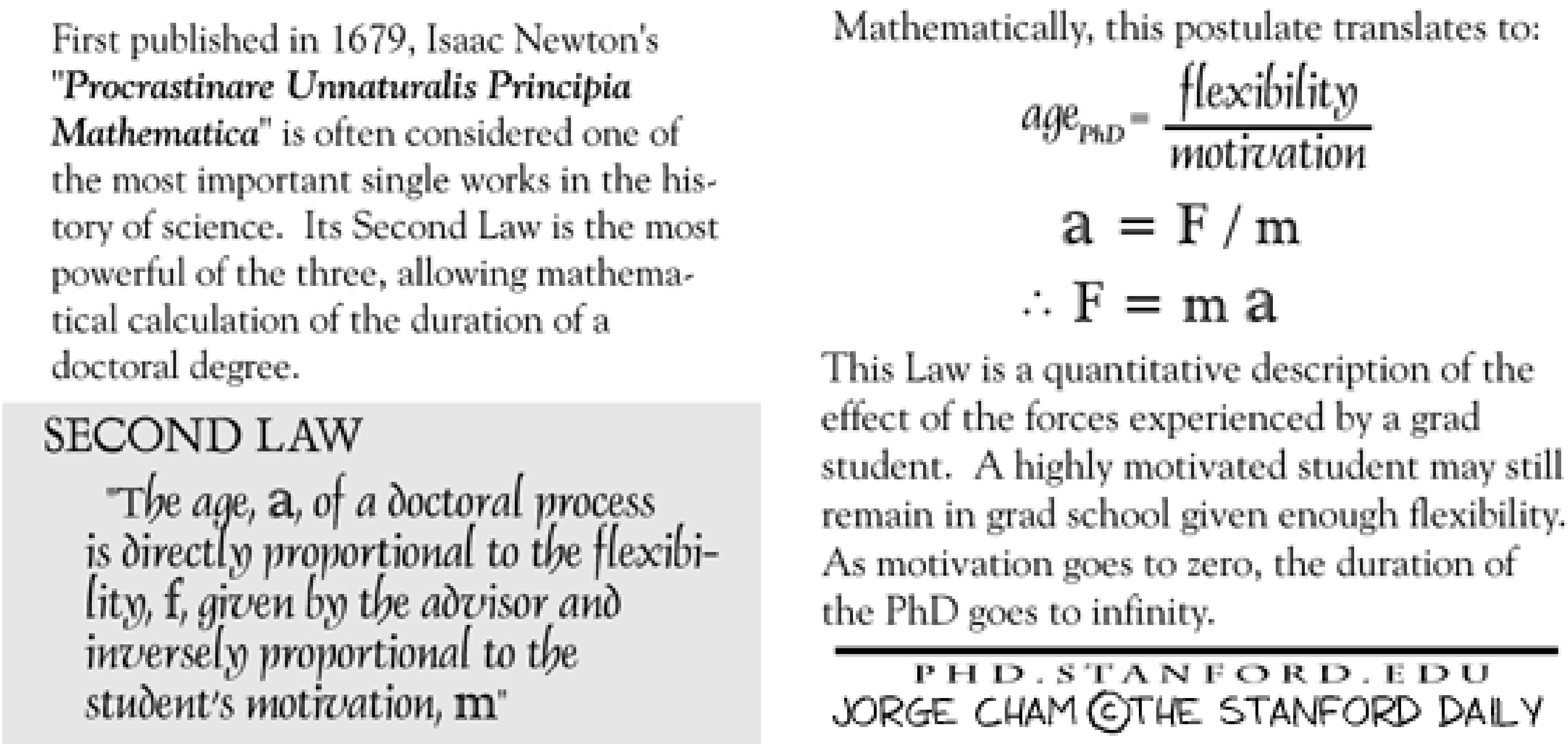
$\boldsymbol{P}$ ... momentum flux - stress $\quad \boldsymbol{P}\cdot\boldsymbol{n} = \bar{\boldsymbol{T}}^{\varphi}$

$\boldsymbol{b}_0$ ... momentum source - force

0 ... no momentum production

equilibrium equation $\rho_0 \dot{\boldsymbol{v}} = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0$

compare


First published in 1679, Isaac Newton's "*Procrastinare Unnaturalis Principia Mathematica*" is often considered one of the most important single works in the history of science. Its Second Law is the most powerful of the three, allowing mathematical calculation of the duration of a doctoral degree.

SECOND LAW

"*The age,* **a**, *of a doctoral process is directly proportional to the flexibility,* **f**, *given by the advisor and inversely proportional to the student's motivation,* **m**"

Mathematically, this postulate translates to:

$$age_{PhD} = \frac{flexibility}{motivation}$$

$$a = F / m$$

$$\therefore F = m\,a$$

This Law is a quantitative description of the effect of the forces experienced by a grad student. A highly motivated student may still remain in grad school given enough flexibility. As motivation goes to zero, the duration of the PhD goes to infinity.

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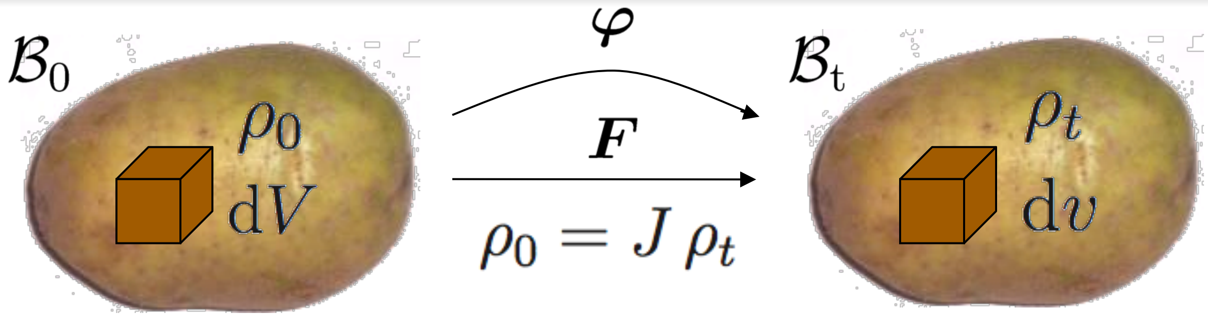
$$\rho_0 \dot{\boldsymbol{v}} = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0 \qquad \textbf{mass point} \qquad m\,\mathrm{D}_t \boldsymbol{v} = m\,\boldsymbol{a} = \boldsymbol{F}$$

07 - balance principles

balance of (internal) energy



global balance of internal energy

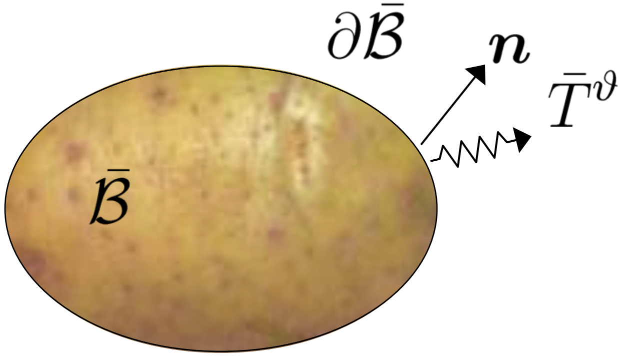
$$\mathrm{D}_t \int_{\mathcal{B}_0} E_0 \,\mathrm{d}V = \int_{\mathcal{B}_0} \boldsymbol{P} : \dot{\boldsymbol{F}} \,\mathrm{d}V - \int_{\mathcal{B}_0} \mathrm{Div}(\boldsymbol{Q}) \,\mathrm{d}V + \int_{\mathcal{B}_0} R_0 \,\mathrm{d}V$$

$$\mathrm{D}_t \int_{\mathcal{B}_t} E_t \,\mathrm{d}v = \int_{\mathcal{B}_t} \boldsymbol{\sigma} : \boldsymbol{d} \,\mathrm{d}v - \int_{\mathcal{B}_t} \mathrm{div}(\boldsymbol{q}) \,\mathrm{d}v + \int_{\mathcal{B}_t} R_t \,\mathrm{d}v$$

local balance of internal energy

$$\dot{E}_0 = \boldsymbol{P} : \dot{\boldsymbol{F}} - \mathrm{Div}(\boldsymbol{Q}) + R_0 \qquad \dot{E}_t = \boldsymbol{\sigma} : \boldsymbol{d} - \mathrm{div}(\boldsymbol{q}) + R_t$$

$$E_0 = J\,E_t \qquad \boldsymbol{Q} = J\,\boldsymbol{q} \cdot \boldsymbol{F}^{-\mathrm{t}} \qquad R_0 = J\,R_0$$



E_0 ... internal energy density

$\boldsymbol{Q}$ ... heat flux $\quad -\boldsymbol{Q}\cdot\boldsymbol{n} = \bar{T}^{\vartheta}$

R_0 ... heat source

0 ... no heat production

energy equation $$\dot{E}_0 = \boldsymbol{P} : \dot{\boldsymbol{F}} - \mathrm{Div}(\boldsymbol{Q}) + R_0$$

internal mechanical power external thermal power