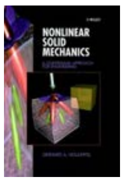
05 - kinematics

holzapfel 'nonlinear solid mechanics' [2000], chapter 2.5-2.8, pages 76-109

me338 - syllabus

day	date		topic	chapters	pages
tue	sep	24	why continuum mechanics?		
thu	sep	26	introduction to vectors and tensors	1.1-1.5	1-32
tue	oct	01	introduction to vectors and tensors	1.6-1.9	32-55
thu	oct	03	kinematics	2.1-2.4	55-76
tue	oct	08	kinematics	2.5-2.8	76-109
thu	oct	10	concept of stress	3.1-3.4	109-131
tue	oct	15	balance principles	4.1-4.4	131-161
thu	oct	17	balance principles	4.5-4.7	161-179
tue	oct	22	aspects of objectivity	5.1-5.4	179-205
thu	oct	24	hyperelastic materials	6.1-6.2	205-222
tue	oct	29	hyperelastic materials	6.3-6.5	222-252
thu	oct	31	hyperelastic materials	6.6-6.8	252-278
tue	nov	05	concept of internal variables	6.9-6.10	285-295
thu	nov	07	isotropic damage	6.11	295-304
tue	nov	12	thermodynamics of materials	7.1-7.6	305-337
thu	nov	14	thermodynamics of materials	7.7-7.9	337-371
tue	nov	19	final prep		
thu	nov	21	final		
tue	dec	03	selected topics		
thu	dec	05	selected topics		
thu	dec	05	written part of final projects due		



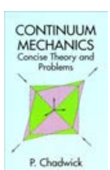
kinematic equations

kinematic equations [kɪnəˈmætɪk ɪˈkweɪ.ʒəns] describe the motion of objects without the consideration of the masses or forces that bring about the motion. the basis of kinematics is the choice of coordinates. the 1st and 2nd time derivatives of the position coordinates give the velocities and accelerations. the difference in placement between the beginning and the final state of two points in a body expresses the numerical value of strain. strain expresses itself as a change in size and/or shape.

Wikipedia The Free Encyclopedia

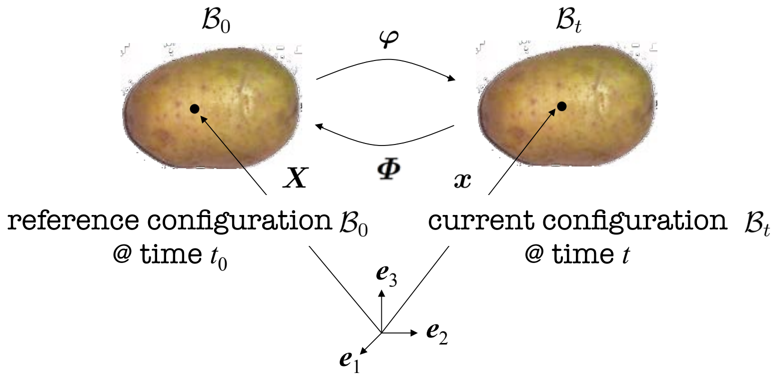
kinematic equations

kinematics [kɪnəˈmætɪks] is the study of motion per se, regardless of the forces causing it. the primitive concepts concerned are position, time and body, the latter abstracting into mathematical terms intuitive ideas about aggregations of matter capable of motion and deformation.

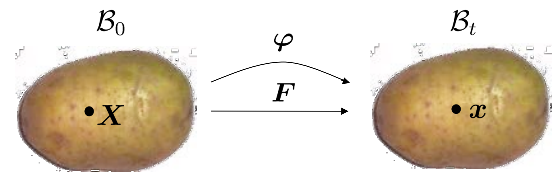


chadwick 'continuum mechanics' [1976]

motion



motion



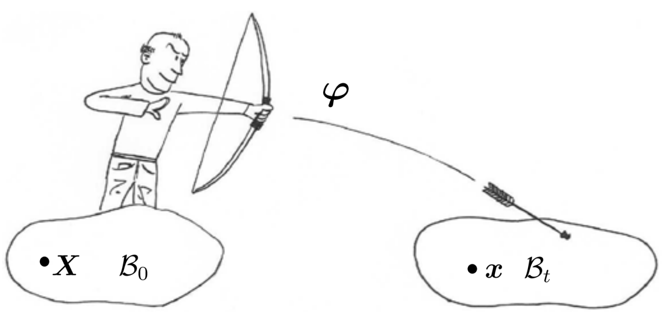
- spatial deformation map

$$\boldsymbol{x} = \boldsymbol{\varphi}(\boldsymbol{X}, t) \quad \text{with} \quad \boldsymbol{\varphi} : \mathcal{B}_0 \times \mathbb{R} \rightarrow \mathcal{B}_t$$

- material deformation map

$$\boldsymbol{X} = \boldsymbol{\Phi}(\boldsymbol{x}, t) \quad \text{with} \quad \boldsymbol{\Phi} : \mathcal{B}_t \times \mathbb{R} \rightarrow \mathcal{B}_0$$

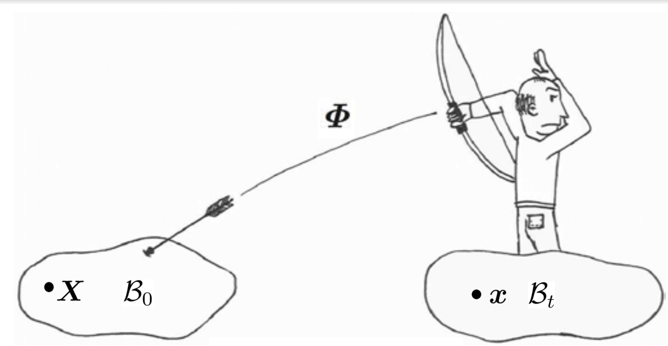
spatial deformation map



- spatial deformation map

$$\boldsymbol{x} = \boldsymbol{\varphi}(\boldsymbol{X}, t) \quad \text{with} \quad \boldsymbol{\varphi} : \mathcal{B}_0 \times \mathbb{R} \rightarrow \mathcal{B}_t$$

material deformation map



- material deformation map

$$\boldsymbol{X} = \boldsymbol{\Phi}(\boldsymbol{x}, t) \quad \text{with} \quad \boldsymbol{\Phi} : \mathcal{B}_t \times \mathbb{R} \rightarrow \mathcal{B}_0$$

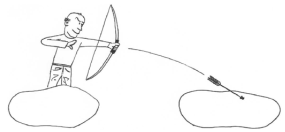
derivatives

- material time derivative of a material field

$$\dot{\mathcal{F}}(\mathbf{X},t) = \frac{\mathrm{D}\mathcal{F}(\mathbf{X},t)}{\mathrm{D}t} = \left(\frac{\partial \mathcal{F}(\mathbf{X},t)}{\partial t}\right)\bigg|_{\mathbf{X}}$$

- material gradient of a material field

$$\mathrm{Grad}\mathcal{F}(\mathbf{X},t) = \frac{\partial \mathcal{F}(\mathbf{X},t)}{\partial \mathbf{X}}\bigg|_{t}$$

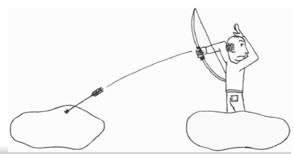


- spatial time derivative of a spatial field

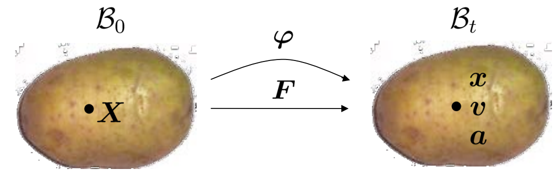
$$\frac{\partial f(\mathbf{x},t)}{\partial t}\bigg|_{\mathbf{x}}$$

- spatial gradient of a spatial field

$$\mathrm{grad} f(\mathbf{x},t) = \frac{\partial f(\mathbf{x},t)}{\partial \mathbf{x}}\bigg|_{t}$$



temporal derivatives



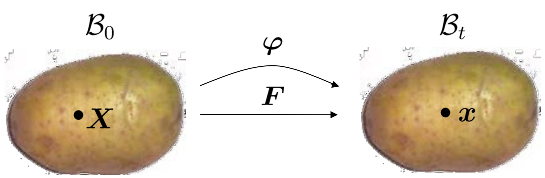
- temporal derivative of φ - velocity (material time deriv.)

$$\boldsymbol{v} = \mathrm{D}_t \boldsymbol{\varphi} = \frac{\partial \varphi}{\partial t}\bigg|_{X \text{ fixed}} \qquad \text{with} \qquad \boldsymbol{v} : \mathcal{B}_0 \times \mathbb{R} \to \mathbb{R}^3$$

- temporal derivative of $\boldsymbol{v}$ - acceleration (material time der.)

$$\boldsymbol{a} = \mathrm{D}_t \boldsymbol{v} = \frac{\partial \boldsymbol{v}}{\partial t}\bigg|_{X \text{ fixed}} = \frac{\partial^2 \varphi}{\partial t^2}\bigg|_{X \text{ fixed}} \qquad \text{with} \qquad \boldsymbol{a} : \mathcal{B}_0 \times \mathbb{R} \to \mathbb{R}^3$$

spatial derivatives



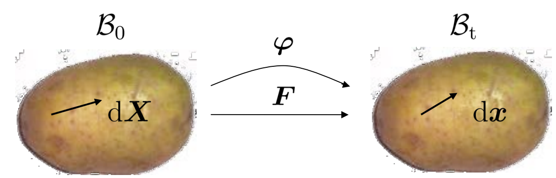
- nonlinear deformation map φ

$$\boldsymbol{x} = \boldsymbol{\varphi}(\boldsymbol{X},t) \quad \text{with} \quad \boldsymbol{\varphi} : \mathcal{B}_0 \times \mathbb{R} \to \mathcal{B}_t$$

- spatial derivative of φ - deformation gradient $\boldsymbol{F}$

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{X} \quad \text{with} \quad \boldsymbol{F} : T\mathcal{B}_0 \to T\mathcal{B}_t \qquad \boldsymbol{F} = \frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{X}}\bigg|_{t \text{ fixed}}$$

deformation gradient



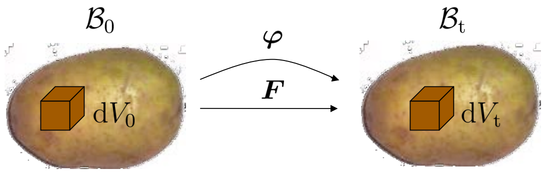
- transformation of line elements deformation gradient F_{ij}

$$\mathrm{d}x_i = F_{ij}\,\mathrm{d}X_j \quad \text{with} \quad F_{ij} : T\mathcal{B}_0 \to T\mathcal{B}_t \qquad F_{ij} = \frac{\partial \varphi_i}{\partial X_j}\bigg|_{t \text{ fixed}}$$

- uniaxial tension (incompr), simple shear, rotation

$$F_{ij}^{\mathrm{uni}} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha^{-\frac{1}{2}} & 0 \\ 0 & 0 & \alpha^{-\frac{1}{2}} \end{bmatrix} \quad F_{ij}^{\mathrm{shr}} = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad F_{ij}^{\mathrm{rot}} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

jacobian & volume change



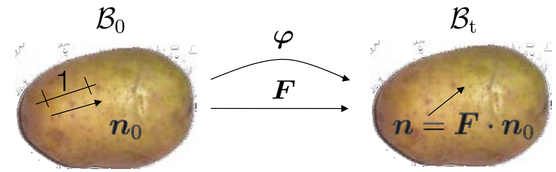
- transformation of volume elements - determinant of $\boldsymbol{F}$

$$\mathrm{d}V_0 = \mathrm{d}\boldsymbol{X}_1 \cdot [\mathrm{d}\boldsymbol{X}_2 \times \mathrm{d}\boldsymbol{X}_3] \qquad \mathrm{d}V_t = \mathrm{d}\boldsymbol{x}_1 \cdot [\mathrm{d}\boldsymbol{x}_2 \times \mathrm{d}\boldsymbol{x}_3]$$
$$= \det([\mathrm{d}\boldsymbol{x}_1, \mathrm{d}\boldsymbol{x}_2, \mathrm{d}\boldsymbol{x}_3])$$
$$= \det([\mathrm{d}\boldsymbol{X}_1, \mathrm{d}\boldsymbol{X}_2, \mathrm{d}\boldsymbol{X}_3]) \qquad = \det(\boldsymbol{F})\ \det([\mathrm{d}\boldsymbol{X}_1, \mathrm{d}\boldsymbol{X}_2, \mathrm{d}\boldsymbol{X}_3])$$

- changes in volume - determinant of deformation gradient J

$$\mathrm{d}V_t = J\,\mathrm{d}V_0 \qquad J = \det(\boldsymbol{F})$$

stretch & deformation tensor



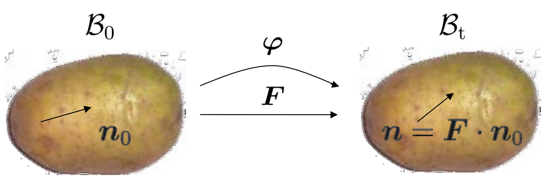
- stretch $\quad \lambda = |\boldsymbol{n}|$ with $\boldsymbol{n} = \boldsymbol{F} \cdot \boldsymbol{n}_0$ and $|\boldsymbol{n}_0| = 1$

$$\lambda^2 = \boldsymbol{n} \cdot \boldsymbol{n} = [\boldsymbol{F} \cdot \boldsymbol{n}_0] \cdot [\boldsymbol{F} \cdot \boldsymbol{n}_0]$$
$$= \boldsymbol{n}_0 \cdot \boldsymbol{F}^t \cdot \boldsymbol{F} \cdot \boldsymbol{n}_0 = \boldsymbol{n}_0 \cdot \boldsymbol{C} \cdot \boldsymbol{n}_0$$

- right Cauchy-Green deformation tensor

$$\boldsymbol{C} = \boldsymbol{F}^t \cdot \boldsymbol{F} \quad \text{with} \quad C_{AB} = F_{aA} F_{aB}$$
$$\det(\boldsymbol{C}) = \det{}^2(\boldsymbol{F}) = J^2 > 0$$

green lagrange strain tensor



- stretch $\quad \frac{1}{2}[\lambda^2 - \lambda_0^2] = \frac{1}{2}[\boldsymbol{n} \cdot \boldsymbol{n} - \boldsymbol{n}_0 \cdot \boldsymbol{n}_0]$

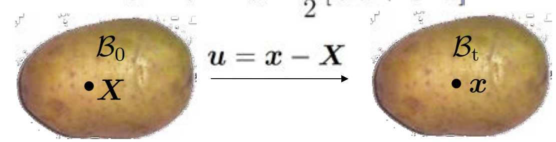
$$= \tfrac{1}{2}[\boldsymbol{n}_0 \cdot \boldsymbol{F}^t \cdot \boldsymbol{F} \cdot \boldsymbol{n}_0 - \boldsymbol{n}_0 \cdot \boldsymbol{I} \cdot \boldsymbol{n}_0]$$
$$= \boldsymbol{n}_0 \cdot \tfrac{1}{2}[\boldsymbol{F}^t \cdot \boldsymbol{F} - \boldsymbol{I}] \cdot \boldsymbol{n}_0 = \boldsymbol{n}_0 \cdot \boldsymbol{E} \cdot \boldsymbol{n}_0$$

- green-lagrange strain tensor

$$\boldsymbol{E} = \tfrac{1}{2}[\boldsymbol{F}^t \cdot \boldsymbol{F} - \boldsymbol{I}] \quad \text{with} \quad E_{AB} = \tfrac{1}{2}[F_{aA} F_{aB} - \delta_{AB}]$$

green lagrange strain tensor

- small strain $\quad \boldsymbol{\varepsilon} = \nabla^{\mathrm{sym}}\boldsymbol{u} = \dfrac{1}{2}[\nabla\boldsymbol{u} + \nabla^t\boldsymbol{u}]$



- deformation gradient

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\boldsymbol{X}} = \frac{\mathrm{d}[\boldsymbol{X} + \boldsymbol{u}]}{\mathrm{d}\boldsymbol{X}} = \frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}\boldsymbol{X}} + \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}\boldsymbol{X}} = \boldsymbol{I} + \nabla_{\boldsymbol{X}}\boldsymbol{u}$$

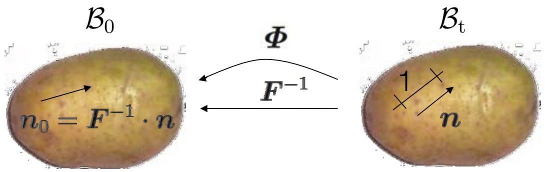
- green-lagrange strain tensor

$$\boldsymbol{E} = \tfrac{1}{2}[\boldsymbol{F}^t \cdot \boldsymbol{F} - \boldsymbol{I}] = \tfrac{1}{2}[[\boldsymbol{I}^t + \nabla^t\boldsymbol{u}] \cdot [\boldsymbol{I} + \nabla\boldsymbol{u}] - \boldsymbol{I}]$$

nonlinear term

$$\boldsymbol{E} = \tfrac{1}{2}[\boldsymbol{I} + \nabla^t\boldsymbol{u} + \nabla\boldsymbol{u} + \nabla^t\boldsymbol{u} \cdot \nabla\boldsymbol{u} - \boldsymbol{I}] = \boldsymbol{\varepsilon} + \tfrac{1}{2}[\nabla^t\boldsymbol{u} \cdot \nabla\boldsymbol{u}]$$

stretch & deformation tensor



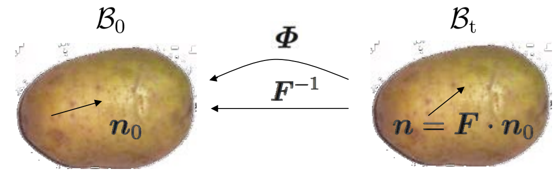
- stretch $\quad \Lambda = |\boldsymbol{n}_0|$ with $\boldsymbol{n}_0 = \boldsymbol{F}^{-1}\cdot\boldsymbol{n}$ and $|\boldsymbol{n}| = 1$

$$\Lambda^2 = \boldsymbol{n}_0\cdot\boldsymbol{n}_0 = [\boldsymbol{F}^{-1}\cdot\boldsymbol{n}]\cdot[\boldsymbol{F}^{-1}\cdot\boldsymbol{n}]$$
$$= \boldsymbol{n}\cdot\boldsymbol{F}^{-\mathrm{t}}\cdot\boldsymbol{F}^{-1}\cdot\boldsymbol{n} = \boldsymbol{n}\cdot\boldsymbol{b}^{-1}\cdot\boldsymbol{n}$$

- left Cauchy-Green deformation tensor / finger tensor

$$\boldsymbol{b} = \boldsymbol{F}\cdot\boldsymbol{F}^{\mathrm{t}} \quad \text{with} \quad b_{ab} = F_{aA}F_{bA}$$
$$\det(\boldsymbol{b}) = \det{}^2(\boldsymbol{F}) = J^2 > 0$$

euler almansi strain tensor

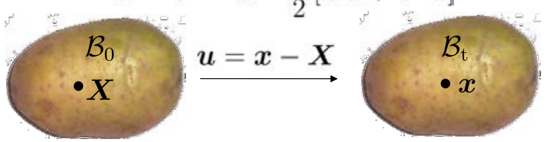


- euler-almansi strain tensor and co-variant push forward

$$\boldsymbol{e} = \tfrac{1}{2}\,[\boldsymbol{I} - \boldsymbol{F}^{-\mathrm{t}}\cdot\boldsymbol{F}^{-1}] \quad \text{with} \quad e_{ab} = \tfrac{1}{2}\,[\delta_{ab} - F^{-1}_{Aa}F^{-1}_{Ab}]$$
$$= \boldsymbol{F}^{-\mathrm{t}}\cdot\tfrac{1}{2}\,[\boldsymbol{F}^{\mathrm{t}}\cdot[\boldsymbol{I} - \boldsymbol{F}^{-\mathrm{t}}\cdot\boldsymbol{F}^{-1}]\cdot\boldsymbol{F}]\cdot\boldsymbol{F}^{-1}$$
$$= \boldsymbol{F}^{-\mathrm{t}}\cdot\tfrac{1}{2}\,[\boldsymbol{F}^{\mathrm{t}}\cdot\boldsymbol{F} - \boldsymbol{I}]\cdot\boldsymbol{F}^{-1} = \boldsymbol{F}^{-\mathrm{t}}\cdot\boldsymbol{E}\cdot\boldsymbol{F}^{-1}$$
$$\boldsymbol{E} = \tfrac{1}{2}\,[\boldsymbol{F}^{\mathrm{t}}\cdot\boldsymbol{F} - \boldsymbol{I}\,] \quad \text{with } E_{AB} = \tfrac{1}{2}\,[F_{aA}F_{aB} - \delta_{AB}\,]$$

euler almansi strain tensor

- small strain $\quad \boldsymbol{\varepsilon} = \nabla^{\mathrm{sym}}\boldsymbol{u} = \dfrac{1}{2}\,[\nabla\boldsymbol{u} + \nabla^{\mathrm{t}}\boldsymbol{u}]$



- inverse deformation gradient

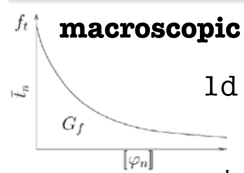
$$\boldsymbol{F}^{-1} = \frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}\boldsymbol{x}} = \frac{\mathrm{d}[\boldsymbol{x} - \boldsymbol{u}]}{\mathrm{d}\boldsymbol{x}} = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\boldsymbol{x}} - \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}\boldsymbol{x}} = \boldsymbol{I} - \nabla_{\boldsymbol{x}}\boldsymbol{u}$$

- euler-almansi strain tensor

$$\boldsymbol{e} = \tfrac{1}{2}\,[\boldsymbol{I} - \boldsymbol{F}^{-\mathrm{t}}\cdot\boldsymbol{F}^{-1}] = \tfrac{1}{2}\,[\boldsymbol{I} - [\boldsymbol{I}^{\mathrm{t}} - \nabla^{\mathrm{t}}\boldsymbol{u}]\cdot[\boldsymbol{I} - \nabla\boldsymbol{u}]]$$
$$\boldsymbol{e} = \tfrac{1}{2}\,[\boldsymbol{I} - \boldsymbol{I} + \nabla^{\mathrm{t}}\boldsymbol{u} + \nabla\boldsymbol{u} - \nabla^{\mathrm{t}}\boldsymbol{u}\cdot\nabla\boldsymbol{u}\,] = \boldsymbol{\varepsilon} - \tfrac{1}{2}\,[\nabla^{\mathrm{t}}\boldsymbol{u}\cdot\nabla\boldsymbol{u}]$$

nonlinear term

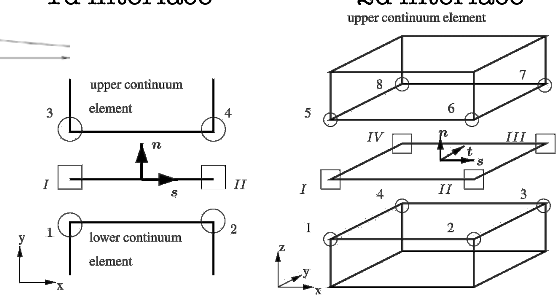
example 01: interface elements



macroscopic approach: lump failure between elements

1d interface

2d interface

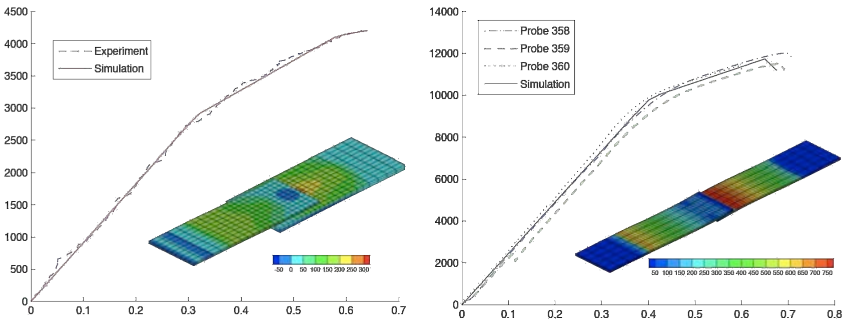


limitation: failure zone must be known a priori

utzinger, bos, floeck, menzel, kuhl, renz, friedrich, schlarb, steinmann [2008]

example 01: interface elements

thermal impact welded single lap tensile specimen



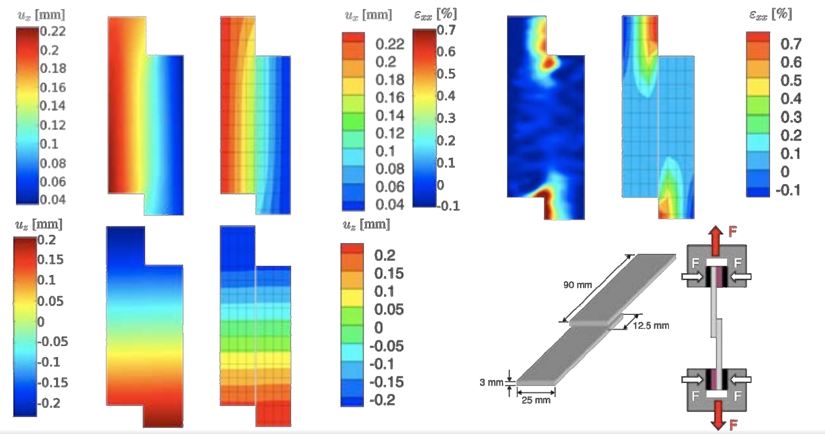
simulation with interfaces predicts characteristic behavior

utzinger, bos, floeck, menzel, kuhl, renz, friedrich, schlarb, steinmann [2008]

discontinuous kinematics

example 01: interface elements

simulation vs electronic speckle pattern interferometry



utzinger, bos, floeck, menzel, kuhl, renz, friedrich, schlarb, steinmann [2008]

discontinuous kinematics

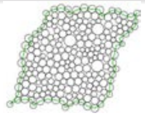
example 02: multiscale elements

slope stability problem

discontinuous kinematics

example 02: multiscale elements

san francisco landslide



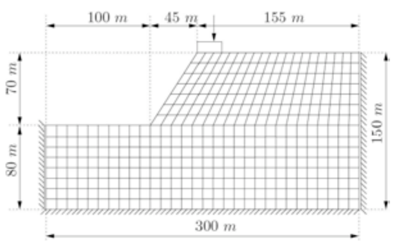
NBC, Feb 27, 2007. A landslide overnight in San Francisco damaged an apartment building and left other structures in precarious situations. A wide swath of hillside came thundering down on a strip club and several apartment buildings in the city's North Beach district Tuesday. At least 120 residents were displaced and several buildings declared off-limits as engineers tried to figure out how to stabilize the cliff to prevent further damage.

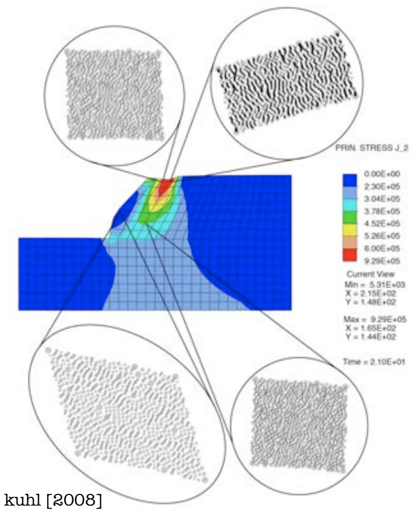
discontinuous kinematics

example 02: multiscale elements

slope stability problem

discretization of granular microstructure to visualize failure mechanism





meier, steinmann, kuhl [2008]

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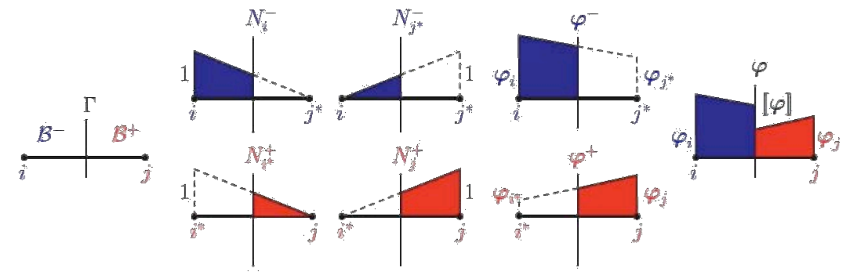
example 03: discontinuous elements

1d crack element



element nodes are doubled upon cracking

$$[\![\varphi]\!]|_\Gamma = \sum_{i=1}^{n_{en}^+} N^i|_\Gamma \, \varphi_i^+ - \sum_{i=1}^{n_{en}^-} N^i|_\Gamma \, \varphi_i^-$$

mergheim, kuhl, steinmann, kuhl [2005]

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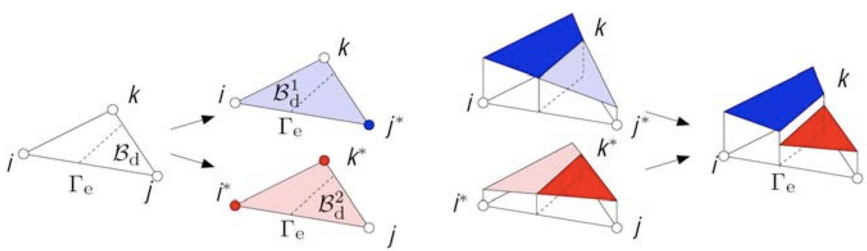
example 03: discontinuous elements

2d crack element

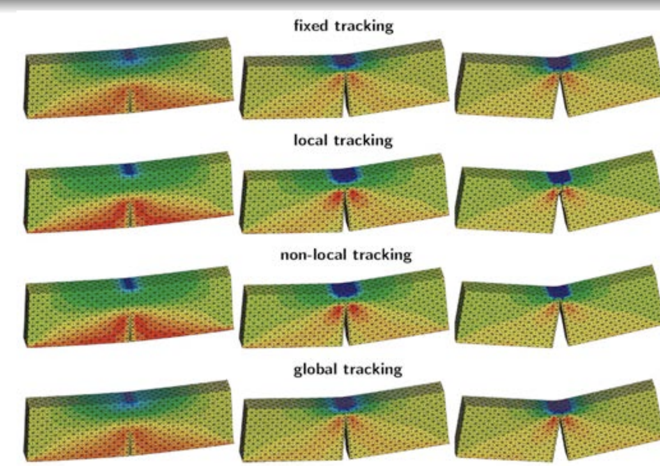


crack elements evolve dynamically

mergheim, kuhl, steinmann, kuhl [2005]

discontinuous kinematics

example 03: discontinuous elements

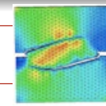


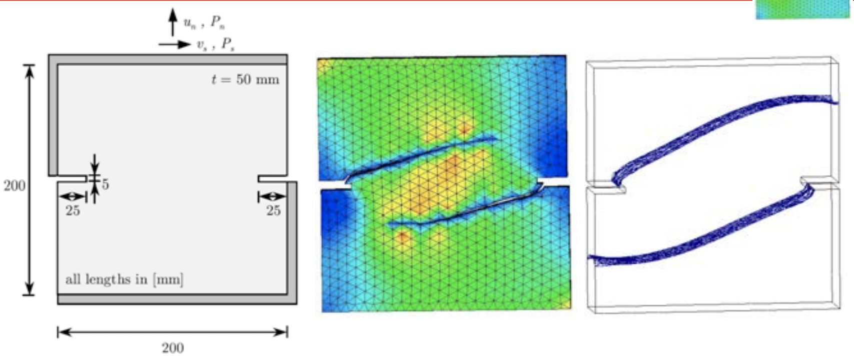
jager, steinmann, kuhl [2008]

discontinuous kinematics

example 03: discontinuous elements

nooru-mohamed test



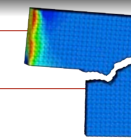


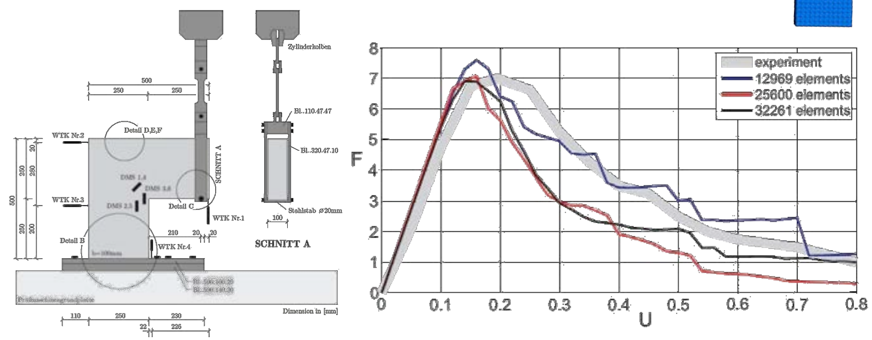
jager, steinmann, kuhl [2008]

discontinuous kinematics

example 03: discontinuous elements

L-shaped specimen test





jager, steinmann, kuhl [2008]

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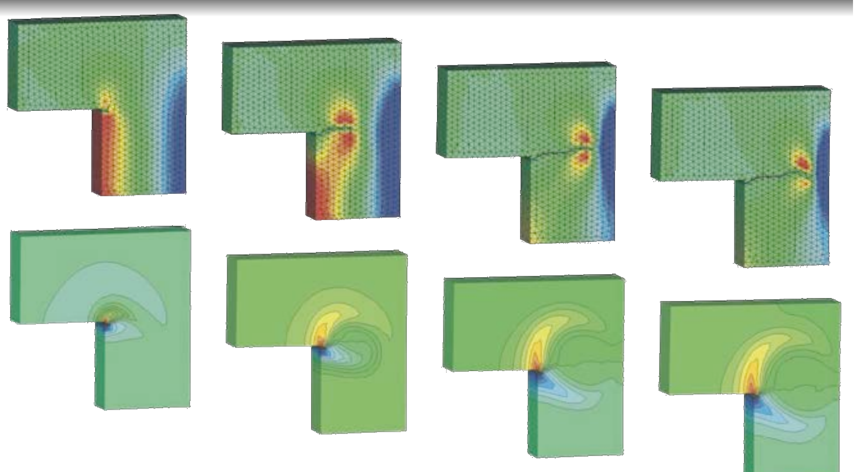
example 03: discontinuous elements



jager, steinmann, kuhl [2008]

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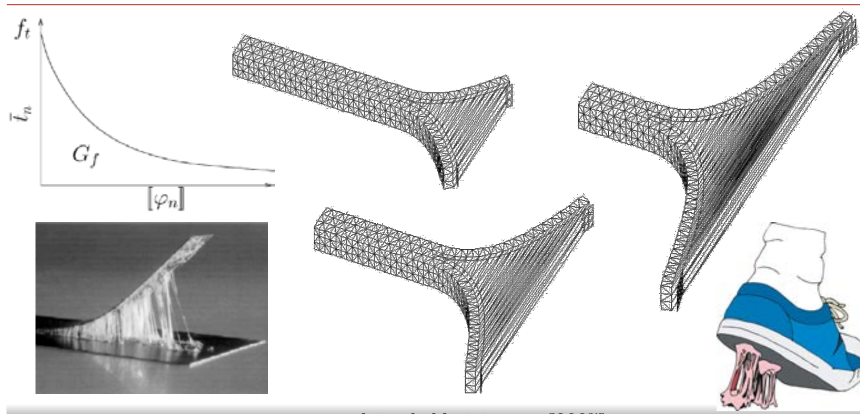
example 03: discontinuous elements

peel test & the chewing gum model

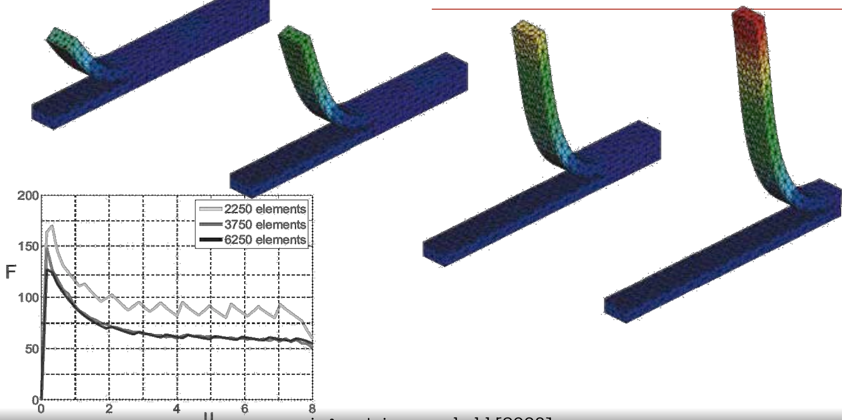


mergheim, kuhl, steinmann [2007]

discontinuous kinematics

example 03: discontinuous elements

non-symmetric peel test

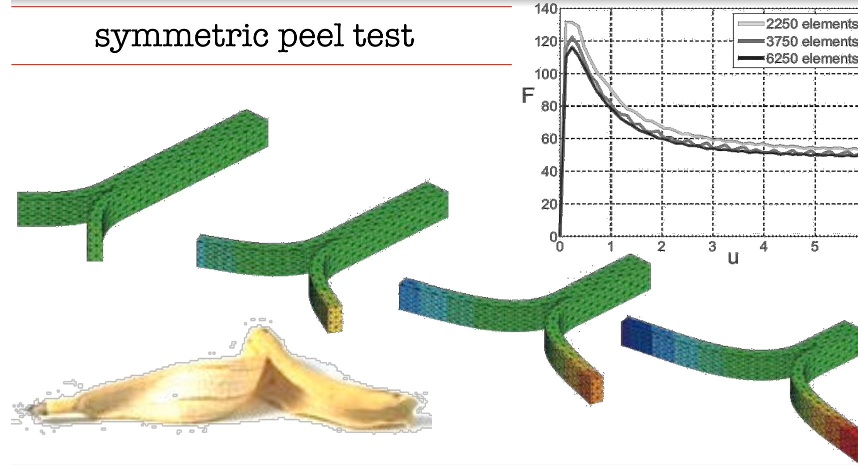


jager, steinmann, kuhl [2008]

discontinuous kinematics

example 03: discontinuous elements

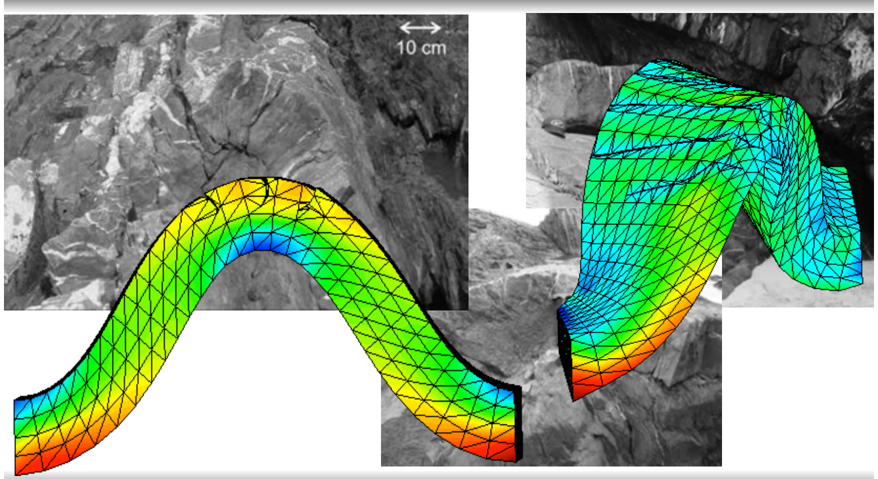
symmetric peel test



jager, steinmann, kuhl [2008]

discontinuous kinematics

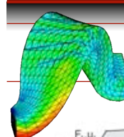
example 03: discontinuous elements



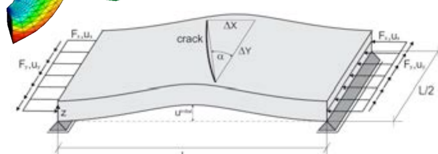
jager, schmalholz, schmid, kuhl [2008]

discontinuous kinematics

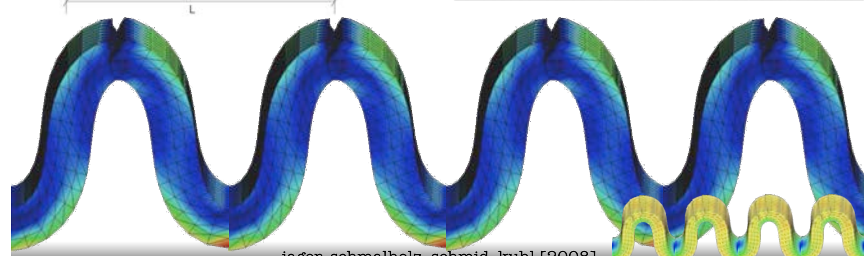
example 03: discontinuous elements



brittle fracture during folding of rocks



loadcase	I	II	III	VI	V	VI	VII
shear-to-compression ratio	0	1/4	1/2	1	2	3	4
α @ crack 1	0.0	1.6	3.4	5.2	8.5	11.6	51.7
α @ crack 2	0.0	1.7	3.2	4.0	8.0	11.5	51.4
α @ crack 3	0.0	0.4	2.2	4.1	8.0	11.4	50.8
α @ crack 4	0.0	1.6	2.4	5.2	8.8	16.5	63.9
α @ crack 5	0.0	1.5	2.1	4.1	8.9	13.8	50.4
α @ crack 6	0.0	2.7	4.5	8.8	16.3	26.1	54.2
α @ crack 7	0.0	3.6	5.5	8.6	12.4	20.8	49.6
average α	0.0	1.9	3.3	5.7	10.1	15.9	53.1



jager, schmalholz, schmid, kuhl [2008]

discontinuous kinematics