

01. motivation



me338 - continuum mechanics

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me338 - goals

although the basic concepts of continuum mechanics have been established more than five decades ago, the 21st century faces many new and exciting potential applications of continuum mechanics that go way beyond the standard classical theory. when applying continuum mechanics to these challenging new phenomena, it is critical to understand the main three ingredients of continuum mechanics: the **kinematic equations**, the **balance equations**, and the **constitutive equations**. after a brief summary of the relevant equations in tensor algebra and analysis, we will introduce the basic concepts of finite strain kinematics. we will then discuss the concept of stress, followed by the balance equations for **mass**, **momentum**, **moment of momentum**, **energy** and **entropy**. while all these equations are general and valid for any kind of material, the last set of equations, the constitutive equations, specifies particular subclasses of materials. we will discuss constitutive equations for **hyperelastic** materials, both **isotropic** and **anisotropic**, and for **inelastic** materials with internal variables. last, we will address these considerations in the context of variational principles.

introduction

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me338 - suggested reading

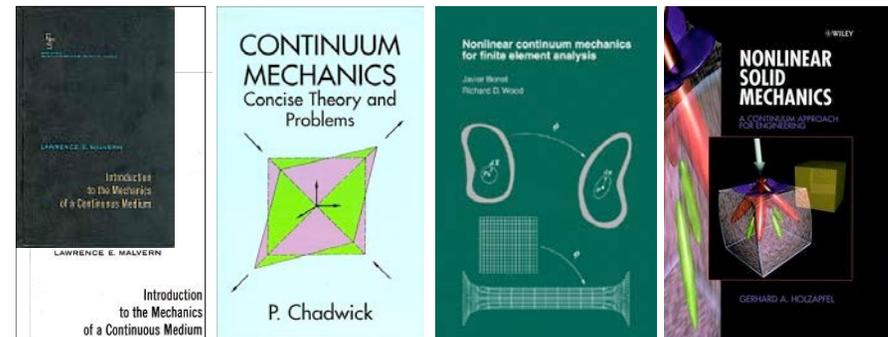
- murnaghan fd: finite deformation of an elastic solid, john wiley & sons, 1951
- eringen ac: nonlinear theory of continuous media, mc graw-hill, 1962
- truesdell c, noll, w: the non-linear field theories of mechanics, springer, 1965
- eringen ac: mechanics of continua, john wiley & sons, 1967
- **malvern le: introduction to the mechanics of a continuous medium, prentice hall, 1969**
- oden jt: finite elements of nonlinear continua, dover reprint, 1972
- **chadwick p: continuum mechanics - concise theory and problems, dover reprint, 1976**
- ogden, rw: non-linear elastic deformations, dover reprint, 1984
- maugin ga: the thermodynamics of plasticity and fracture, cambridge university press, 1992
- spencer ajm: continuum mechanics, dover reprint, 1992
- robers aj: one-dimensional introduction to continuum mechanics, world scientific, 1994
- **bonet j, wood rd: nonlinear continuum mechanics for fe analysis, cambridge university, 1997**
- silhavy m: the mechanics and thermodynamics of continuous media, springer, 1997
- **holzafel ga: nonlinear solid mechanics, john wiley & sons, 2000**
- haupt p: continuum mechanics and theory of materials, springer, 2000
- podio-guidugli p: a primer in elasticity, kluwer academic press, 2000
- liu is: continuum mechanics, springer, 2002
- reddy jn: an introduction to continuum mechanics, cambridge university press, 2007



introduction

3

me338 - suggested reading



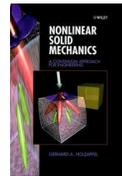
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introduction

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me338 - syllabus

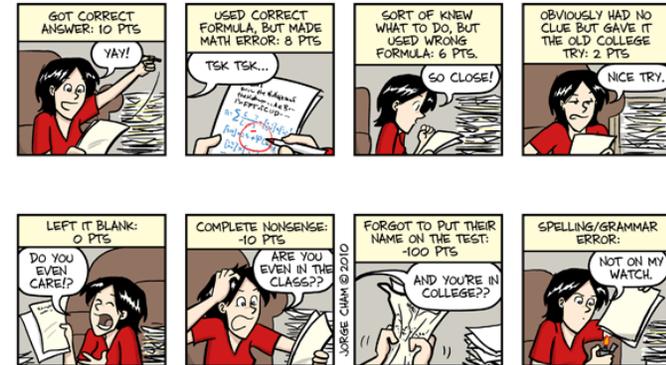
day	date	topic	chapters	pages
tue	sep 24	why continuum mechanics?		
thu	sep 26	introduction to vectors and tensors	1.1-1.5	1-32
tue	oct 01	introduction to vectors and tensors	1.6-1.9	32-55
thu	oct 03	kinematics	2.1-2.4	55-76
tue	oct 08	kinematics	2.5-2.8	76-109
thu	oct 10	concept of stress	3.1-3.4	109-131
tue	oct 15	balance principles	4.1-4.4	131-161
thu	oct 17	balance principles	4.5-4.7	161-179
tue	oct 22	aspects of objectivity	5.1-5.4	179-205
thu	oct 24	hyperelastic materials	6.1-6.2	205-222
tue	oct 29	hyperelastic materials	6.3-6.5	222-252
thu	nov 31	hyperelastic materials	6.6-6.8	252-278
tue	nov 05	concept of internal variables	6.9-6.10	285-295
thu	nov 07	isotropic damage	6.11	295-304
tue	nov 12	thermodynamics of materials	7.1-7.6	305-337
thu	nov 14	thermodynamics of materials	7.7-7.9	337-371
tue	nov 19	final prep		
thu	nov 21	final		
tue	dec 03	selected topics		
thu	dec 05	selected topics		
thu	dec 05	written part of final projects due		



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me338 - grading

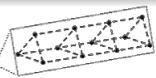
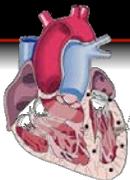


- 30 % homework - 3 homework assignments, 10% each
- 50 % final - closed book, closed notes, one single page cheat sheet
- 20 % final project - written evaluation of a manuscript and its discussion in class

introduction

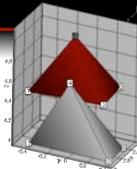
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me338 - homework



Calculate strains in the beating heart
Given measured marker coordinates

$$\begin{aligned}
 X_1 &= [+3.00 \ -0.40 \ +4.00]^T & x_1 &= [+2.80 \ -0.51 \ +4.41]^T \\
 X_2 &= [+3.00 \ +0.40 \ +4.00]^T & x_2 &= [+2.79 \ +0.24 \ +4.44]^T \\
 X_3 &= [+2.70 \ +0.40 \ +4.00]^T & x_3 &= [+2.46 \ +0.25 \ +4.48]^T \\
 X_4 &= [+3.00 \ +0.00 \ +4.70]^T & x_4 &= [+2.80 \ -0.05 \ +5.08]^T
 \end{aligned}$$



- [01] Determine three vectors dX_i that span the tetrahedron at end diastole.
- [02] Determine the same three vectors dx_i that span the tetrahedron at end systole.
- [03] Determine the deformation gradient F that maps all diastolic line elements dX_i onto the systolic line elements dx_i .
- [04] Control your results by calculating $dx_i = F \cdot dX_i$.
- [05] Determine the systolic fiber direction $n^{fib} = F \cdot N^{fib}$.
- [06] Determine the fiber stretch $\lambda = \sqrt{n^{fib} \cdot n^{fib}}$.
- [07] Determine the second Green Lagrange strain tensor $E = 1/2 [F \cdot F - I]$.
- [08] Determine the displacement gradient tensor $H = F - I$.
- [09] Linearize the Green Lagrange strain tensor E with the help of the Gateaux derivative to obtain the small strain tensor $\epsilon = 1/2 [H + H^T]$.
- [10] Determine the volume dilation $e = \text{tr}(\epsilon)$.
- [11] Last, determine the normal strain $\epsilon_n = N^{fib} \cdot \epsilon \cdot N^{fib}$.

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me338 - homework



introduction

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continuum mechanics

continuum mechanics [kən'tɪn.ju.əm mə'kæn.ɪks] is a branch of physics (specifically mechanics) that deals with continuous matter. the fact that matter is made of atoms and that it commonly has some sort of heterogeneous microstructure is ignored in the simplifying approximation that physical quantities, such as energy and momentum, can be handled in the infinitesimal limit. differential equations can thus be employed in solving problems in continuum mechanics.



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continuum mechanics

continuum mechanics [kən'tɪn.ju.əm mə'kæn.ɪks] is the branch of mechanics concerned with the stress in solids, liquids and gases and the deformation or flow of these materials. the adjective continuous refers to the simplifying concept underlying the analysis: we disregard the molecular structure of matter and picture it as being without gaps or empty spaces. we suppose that all the mathematical functions entering the theory are continuous functions. this hypothetical continuous material we call a continuum.



malvern 'introduction to the mechanics of a continuous medium' [1969]

introduction

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continuum mechanics

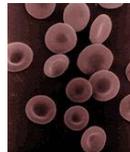
continuum hypothesis [kən'tɪn.ju.əm haɪ'pɔːθ.ə.sɪs] we assume that the characteristic length scale of the microstructure is much smaller than the characteristic length scale of the overall problem, such that the properties at each point can be understood as averages over a characteristic length scale

$$l^{\text{micro}} \ll l^{\text{averg}} \ll l^{\text{conti}}$$

example: biomechanics

$$l^{\text{micro}} = l^{\text{cells}} \approx 10\mu\text{m}$$

$$l^{\text{conti}} = l^{\text{tissue}} \approx 10\text{cm}$$



we can apply the continuum hypothesis to analyze biological structures

introduction

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the potato equations

- **kinematic equations - what 's strain?** $\epsilon = \frac{\Delta l}{l}$
general equations that characterize the deformation of a physical body without studying its physical cause
- **balance equations - what 's stress?** $\sigma = \frac{F}{A}$
general equations that characterize the cause of motion of any body
- **constitutive equations - how are they related?** $\sigma = E \epsilon$
material specific equations that complement the set of governing equations

continuum mechanics

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the potato equations

- **kinematic equations - why not** $\epsilon = \frac{\Delta l}{l}$?
 inhomogeneous deformation » non-constant
 finite deformation » non-linear $\mathbf{F} = \nabla_X \varphi$
 inelastic deformation » growth tensor $\mathbf{F} = \mathbf{F}_e \cdot \mathbf{F}_g$
- **balance equations - why not** $\sigma = \frac{F}{A}$? $\text{Div}(\mathbf{P}) + \rho \mathbf{b}_0 = 0$
 equilibrium in deformed configuration » multiple stresses
- **constitutive equations - why not** $\sigma = E \epsilon$?
 finite deformation » non-linear $\mathbf{P} = \mathbf{P}(\mathbf{F})$
 inelastic deformation » internal variables $\mathbf{P} = \mathbf{P}(\rho, \mathbf{F}, \mathbf{F}_g)$

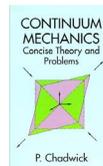
kinematic equations

kinematic equations [kmə'mætɪk ɪ'kweɪ.ʒəns] describe the motion of objects without the consideration of the masses or forces that bring about the motion. the basis of kinematics is the choice of coordinates. the 1st and 2nd time derivatives of the position coordinates give the velocities and accelerations. the difference in placement between the beginning and the final state of two points in a body expresses the numerical value of strain. strain expresses itself as a change in size and/or shape.



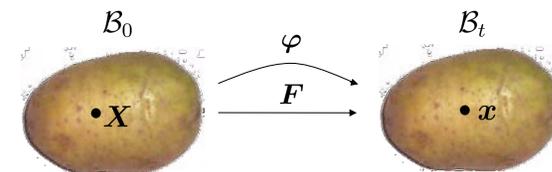
kinematic equations

kinematics [kmə'mætɪks] is the study of motion per se, regardless of the forces causing it. the primitive concepts concerned are position, time and body, the latter abstracting into mathematical terms intuitive ideas about aggregations of matter capable of motion and deformation.



chadwick 'continuum mechanics' [1976]

potato kinematics



- nonlinear deformation map φ
 $x = \varphi(X, t)$ with $\varphi : \mathcal{B}_0 \times \mathbb{R} \rightarrow \mathcal{B}_t$
- spatial derivative of φ - deformation gradient
 $d\mathbf{x} = \mathbf{F} \cdot d\mathbf{X}$ with $\mathbf{F} : T\mathcal{B}_0 \rightarrow T\mathcal{B}_t$ $\mathbf{F} = \left. \frac{\partial \varphi}{\partial \mathbf{X}} \right|_{t \text{ fixed}}$

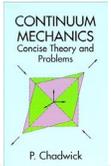
balance equations

balance equations ['bæl.əns r'kwɛɪ.ʒəns] of mass, momentum, angular momentum and energy, supplemented with an entropy inequality constitute the set of conservation laws. the law of **conservation of mass/matter** states that the **mass of a closed system** of substances will remain **constant**, regardless of the processes acting inside the system. the principle of conservation of momentum states that the total momentum of a closed system of objects is constant.



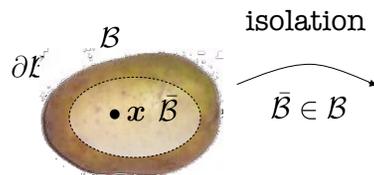
balance equations

balance equations ['bæl.əns r'kwɛɪ.ʒəns] of mass, linear momentum, angular momentum and energy **apply to all material bodies**. each one gives rise to a field equation, holding on the configurations of a body in a sufficiently smooth motion and a jump condition on surfaces of discontinuity. like position, time and body, the concepts of mass, force, heating and internal energy which enter into the formulation of the balance equations are regarded as having primitive status in continuum mechanics.



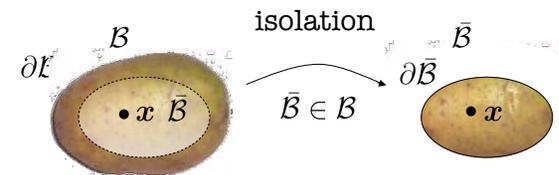
chadwick 'continuum mechanics' [1976]

potato balance equations



[1] isolate subset \bar{B} from B

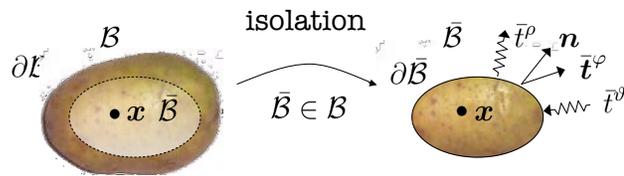
potato balance equations



[1] isolate subset \bar{B} from B

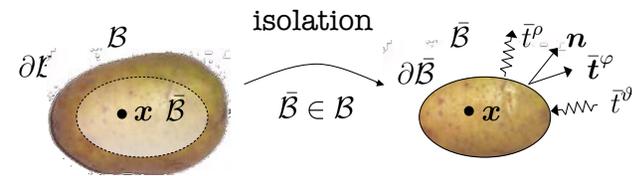
[2] **characterize** influence of remaining body through phenomenological quantities - contact fluxes \bar{t}^p, \bar{t}^e & \bar{t}^0

potato balance equations



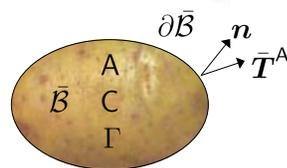
- [1] isolate subset \bar{B} from B
- [2] characterize influence of remaining body through phenomenological quantities - contact fluxes $\bar{t}^\rho, \bar{t}^\varphi$ & \bar{t}^θ
- [3] **define** basic physical quantities - mass, linear and angular momentum, energy

potato balance equations



- [1] isolate subset \bar{B} from B
- [2] characterize influence of remaining body through phenomenological quantities - contact fluxes $\bar{t}^\rho, \bar{t}^\varphi$ & \bar{t}^θ
- [3] define basic physical quantities - mass, linear and angular momentum, energy
- [4] **postulate** balance of these quantities

balance equations



general format

- A... balance quantity
- B... flux $B \cdot n = \bar{T}^A$
- C... source
- Γ ... production

$$D_t A = \text{Div}(B) + C + \Gamma$$

constitutive equations

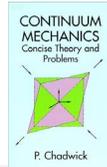
constitutive equations [kən'stɪ.tu.tɪv ɪ'kwer.ɪ.ʒəns] in structural analysis, constitutive relations **connect applied stresses** or forces to **strains** or deformations. the constitutive relations for linear materials are linear. more generally, in physics, a constitutive equation is a relation between two physical quantities (often tensors) that is specific to a material, and does not follow directly from physical law. some constitutive equations are **simply phenomenological**; others are **derived from first principles**.



constitutive equations

constitutive equations [kən'stɪ.tu.tɪv ɪ'kwel.ɪʒəns] or equations of state bring in the **characterization of particular materials** within continuum mechanics. mathematically, the purpose of these relations is to supply connections between kinematic, mechanical and thermal fields. physically, constitutive equations represent the various forms of **idealized material response** which serve as **models** of the behavior of actual substances.

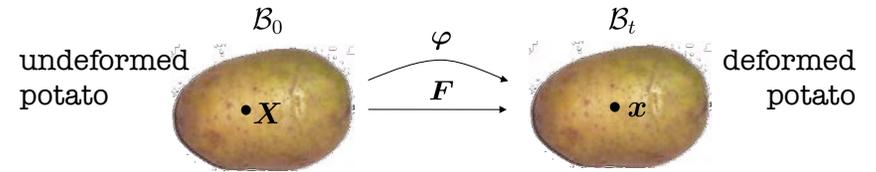
chadwick 'continuum mechanics' [1976]



constitutive equations

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constitutive equations



- free energy $\psi_0^{\text{neo}} = \frac{1}{2} \lambda_0 \ln^2(\det(\mathbf{F})) + \frac{1}{2} \mu_0 [\mathbf{F}^t \cdot \mathbf{F} : \mathbf{I} - n^{\text{dim}} - 2 \ln(\det(\mathbf{F}))]$

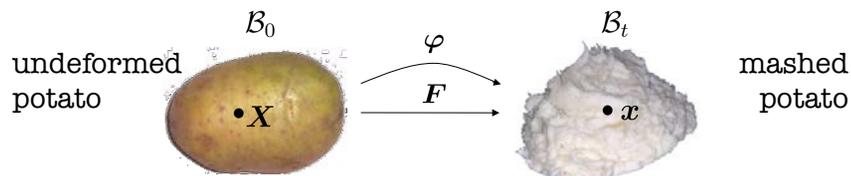
- definition of stress - neo hookean elasticity

$$\mathbf{P}^{\text{neo}} = D_{\mathbf{F}} \psi_0^{\text{neo}} = \mu_0 \mathbf{F} + [\lambda_0 \ln(\det(\mathbf{F})) - \mu_0] \mathbf{F}^{-t}$$

continuum mechanics

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constitutive equations



- free energy ~~$\psi_0^{\text{neo}} = \frac{1}{2} \lambda_0 \ln^2(\det(\mathbf{F})) + \frac{1}{2} \mu_0 [\mathbf{F}^t \cdot \mathbf{F} : \mathbf{I} - n^{\text{dim}} - 2 \ln(\det(\mathbf{F}))]$~~

- definition of stress - neo hookean elasticity

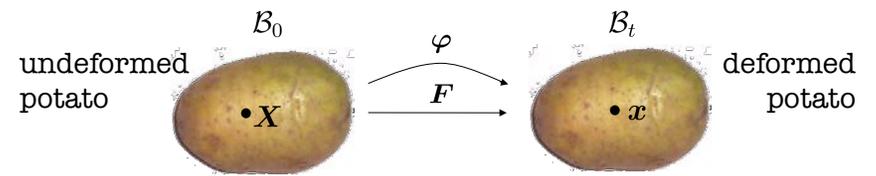
~~$$\mathbf{P}^{\text{neo}} = D_{\mathbf{F}} \psi_0^{\text{neo}} = \mu_0 \mathbf{F} + [\lambda_0 \ln(\det(\mathbf{F})) - \mu_0] \mathbf{F}^{-t}$$~~

- remember! mashing potatoes is not an elastic process!

continuum mechanics

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constitutive equations



- free energy $\psi_0^{\text{neo}} = \frac{1}{2} \lambda_0 \ln^2(\det(\mathbf{F})) + \frac{1}{2} \mu_0 [\mathbf{F}^t \cdot \mathbf{F} : \mathbf{I} - n^{\text{dim}} - 2 \ln(\det(\mathbf{F}))]$

- large strain - lamé parameters and bulk modulus

$$\lambda = \frac{E\nu}{[1+\nu][1-2\nu]} \quad \mu = \frac{E}{2[1+\nu]} \quad \kappa = \frac{E}{3[1-2\nu]}$$

- small strain - young's modulus and poisson's ratio

$$E = 3\kappa[1-2\nu] \quad \nu = \frac{3\kappa-2\mu}{2[3\kappa+\mu]}$$

continuum mechanics

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living matter lab

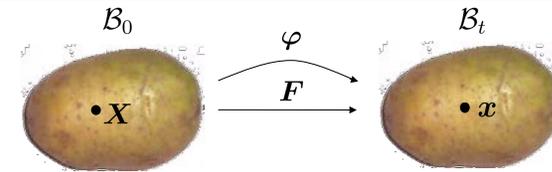


adrian buganza tepole, mona eskandari, nele famaey (ku leuven, belgium), martin genet, serdar goktepe (metu ankara, turkey), maria holland, corey murphey, martin pfaller, manu rausch, pablo saez, tyler shultz, alkis tsamis (u of pittsburgh), jon wong, alex zollner

<http://biomechanics.stanford.edu>

why continuum mechanics is cool 29

from continuous governing equations...



- kinematic equations

$$\mathbf{x} = \boldsymbol{\varphi}(\mathbf{X}, t) \text{ and } \mathbf{F} = \frac{\partial \boldsymbol{\varphi}}{\partial \mathbf{X}}$$
- balance equations

$$\mathbf{0} = \text{Div } \mathbf{P} + \mathbf{B}$$
- constitutive equations

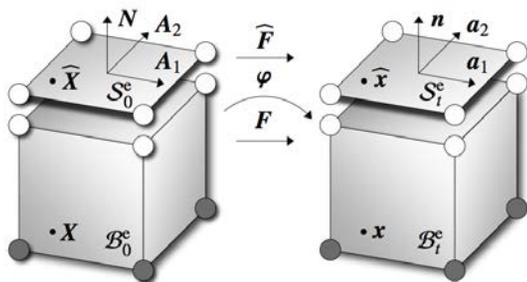
$$\boldsymbol{\psi} = \boldsymbol{\psi}(\mathbf{F}, \mathbf{F}^g) = \boldsymbol{\psi}(\mathbf{F}^e)$$

why continuum mechanics is cool 30

... via finite element discretization ...



volume elements + surface elements



why continuum mechanics is cool 31

... to discrete governing equations

- test and trial functions

$$\delta \boldsymbol{\varphi} = \sum_{i=1}^{n_{bn}} N^i \delta \boldsymbol{\varphi}_i \quad \boldsymbol{\varphi} = \sum_{j=1}^{n_{bn}} N^j \boldsymbol{\varphi}_j$$
- test and trial function gradients

$$\delta \mathbf{F} = \nabla \delta \boldsymbol{\varphi} = \sum_{i=1}^{n_{bn}} \delta \boldsymbol{\varphi}_i \otimes \nabla N^i \quad \mathbf{F} = \nabla \boldsymbol{\varphi} = \sum_{j=1}^{n_{bn}} \boldsymbol{\varphi}_j \otimes \nabla N^j$$
- discrete residual

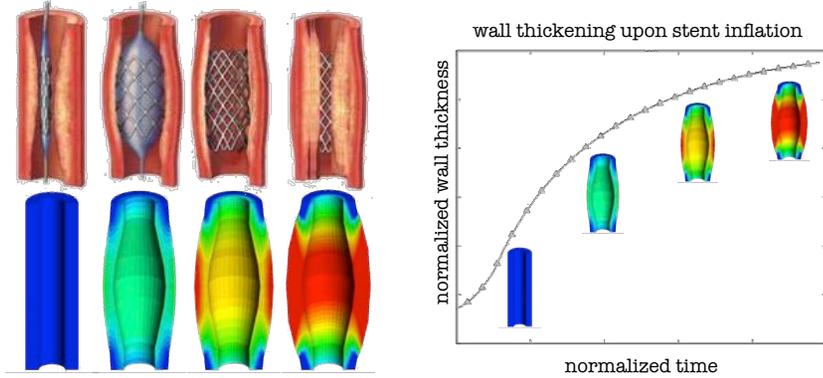
$$\mathbf{R}_I = \mathbf{A} \int_{\mathcal{B}_0^c} \nabla N^i \cdot \mathbf{P} \, dV_e \doteq \mathbf{0}$$
- consistent linearization

$$\mathbf{K}_{IJ} = \mathbf{A} \int_{\mathcal{B}_0^c} [I \cdot \nabla N^i] : \mathbb{A} \cdot \nabla N^j \, dV_e$$



why continuum mechanics is cool 32

arterial growth and in-stent restenosis



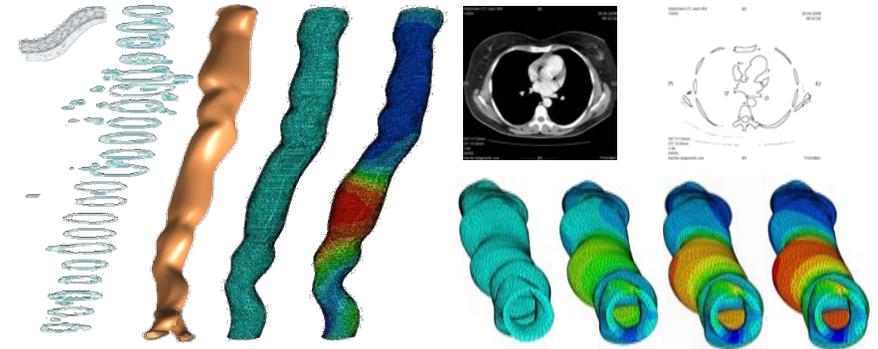
chronic wall thickening and gradual renarrowing

himpel, kuhl, menzel, steinmann [2005]

example: vascular system

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arterial growth and in-stent restenosis



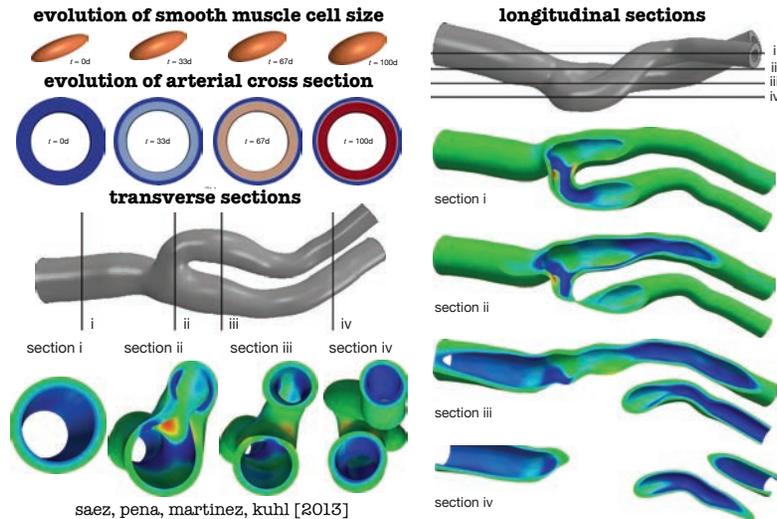
chronic wall thickening and gradual renarrowing

kuhl, maas, himpel, menzel [2007]

example: vascular system

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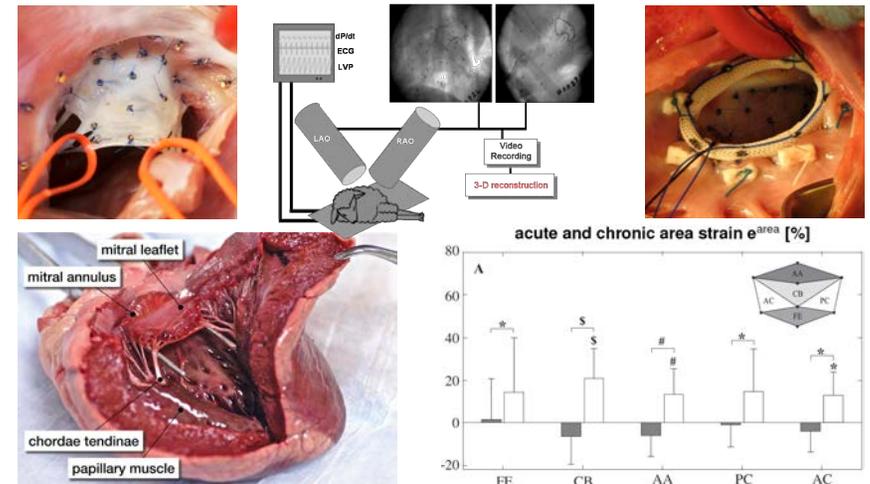
hypertension in human carotid artery



saez, pena, martinez, kuhl [2013]

example: vascular system

mitral valve growth in regurgitation

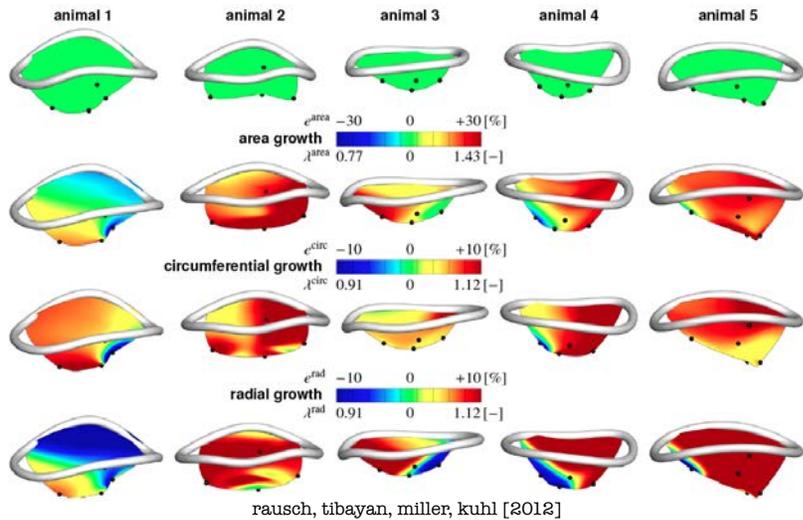


goktepe, bothe, kvitting, swanson, ingels, miller, kuhl [2010], rausch, tibayan, miller, kuhl [2012]

example: mitral valve

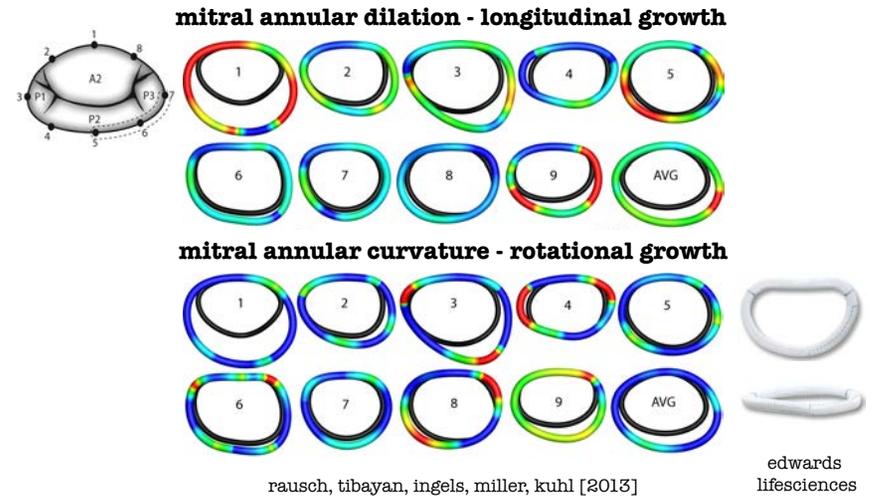
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mitral valve growth in regurgitation



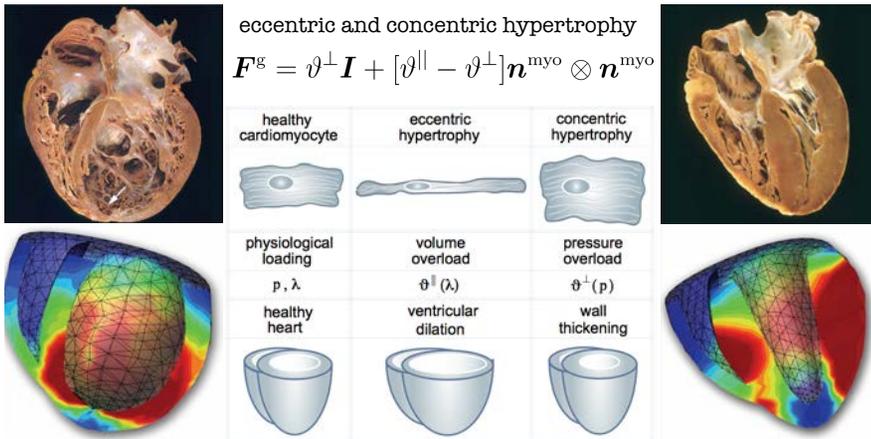
example: mitral valve

mitral valve growth in regurgitation



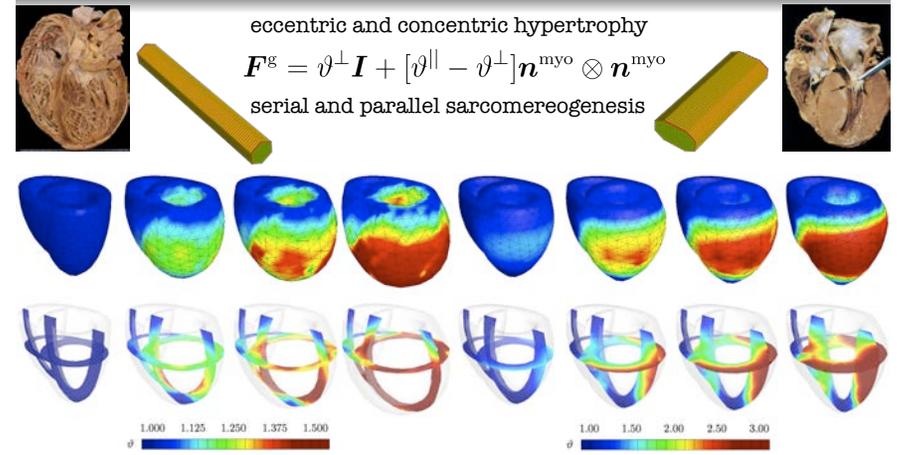
example: mitral valve

cardiac growth in dilation and hypertrophy



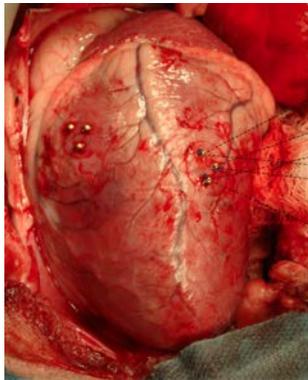
example: cardiac system

cardiac growth in dilation and hypertrophy



example: cardiac system

fiber growth of the heart



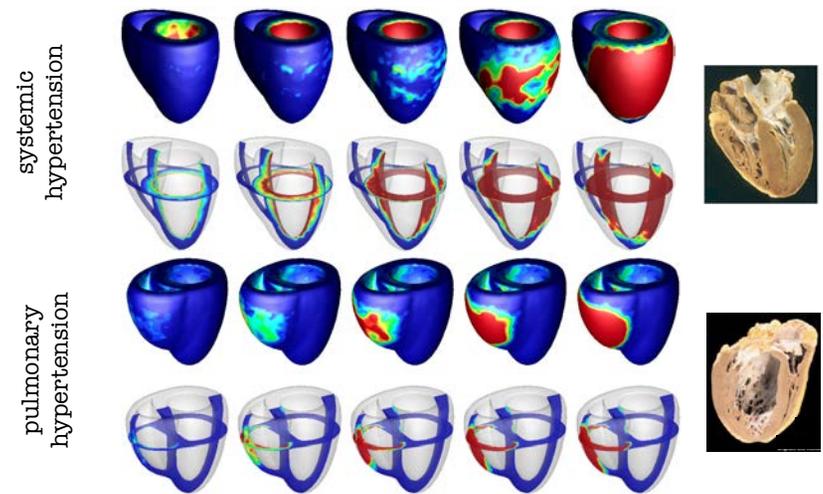
deformation
 $\varphi(X, t) = \sum_{l=1}^{n_{apx}} c_l(t) N_l(X)$
 deformation gradient
 $F(X, t) = \sum_{l=1}^{n_{apx}} c_l(t) \otimes \nabla N_l(X)$
 volume changes
 $J(X, t) = \det(F(X, t))$
 fiber stretch
 $\lambda_{FF}(X, t) = [f(X) \cdot F^t(X, t) \cdot F(X, t) \cdot f(X)]^{1/2}$

J^E	0.74±0.19	0.00	0.82±0.19	0.01	0.89±0.21	0.10
λ_{FF}^E	1.03±0.12	0.49	1.04±0.16	0.36	1.08±0.11	0.04

tsamis, cheng, nguyen, langer, miller, kuhl [2012]

example: cardiac system

cross-fiber growth of the heart



rausch, dam, goktepe, abilez, kuhl [2011]

example: cardiac system

skin growth in tissue expansion

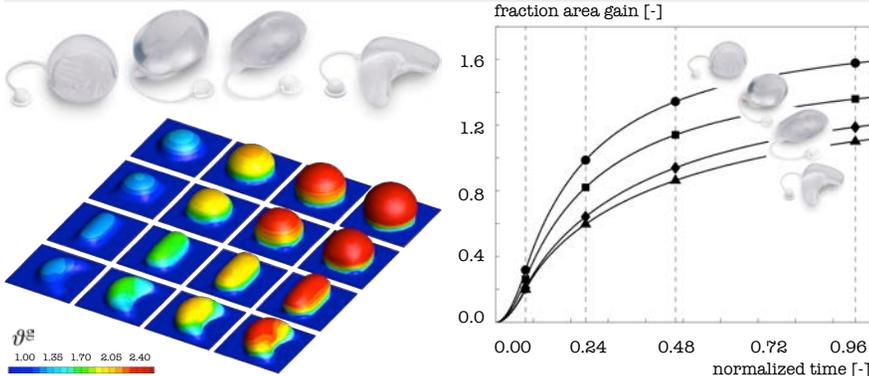


figure. tissue expander inflation. spatio-temporal evolution of area growth. under the same pressure applied to the same base surface area, the circular expander induces the largest amount of growth followed by the square, the rectangular, and the crescent-shaped expanders.

buganza tepole, ploch, wong, gosain, kuhl [2011]

example: skin

skin growth in plastic surgery

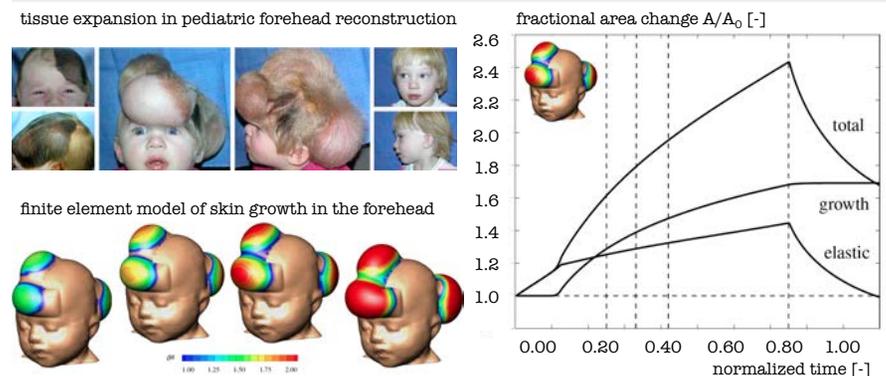


figure. skin expansion in pediatric forehead reconstruction. case study: simultaneous forehead, anterior and posterior scalp expansion, right. the initial area of 149.4cm² increases gradually as the grown skin area increases to 190.2cm², 207.4cm², 220.4cm², and finally 251.2cm², from bottom left to right.

zollner, buganza tepole, gosain, kuhl [2012]

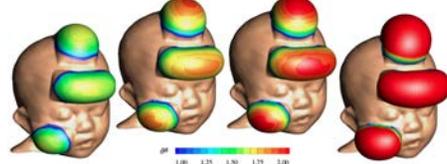
example: skin

skin growth in plastic surgery

tissue expansion in pediatric forehead reconstruction



finite element model of skin growth in the forehead



fractional area change A/A_0 [-]

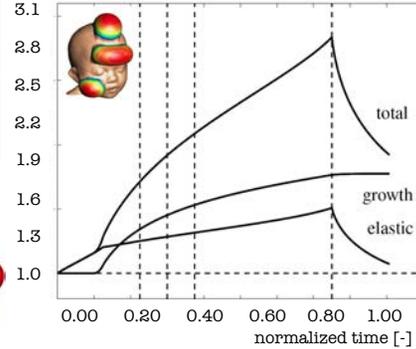


figure. skin expansion in pediatric forehead reconstruction. case study: simultaneous forehead, anterior and posterior scalp expansion, right. the initial area 128.7cm² increases gradually as the grown skin area increases to 176.0 cm², 191.3 cm², 202.1 cm², and finally 227.1 cm², from bottom left to right.

zollner, buganza tepole, gosain, kuhl [2012]

example: skin

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skin growth in plastic surgery

tissue expansion in pediatric defect repair



finite element model of skin growth in the skull

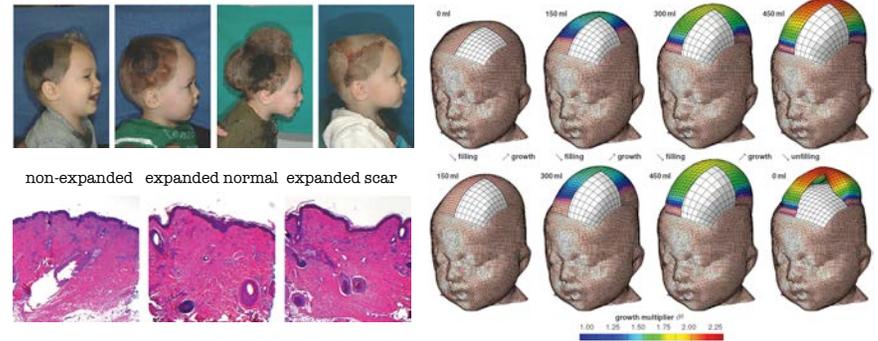


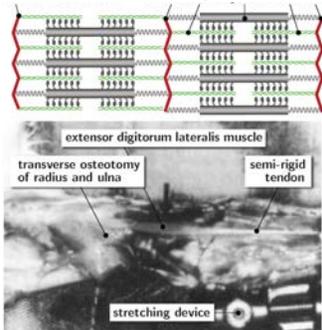
figure. skin expansion creates new skin with the same histological appearance as the native skin: the epidermis displays a similar wrinkling pattern and thickness; the dermis displays the same thickness; cell-to-matrix volume ratios and collagenous microstructure are similar.

zollner, holland, honda, gosain, kuhl [2013]

example: skin

muscle fiber growth

Z-line sarcomere Z-line myosin actin Z-line



serial sarcomere number vs time

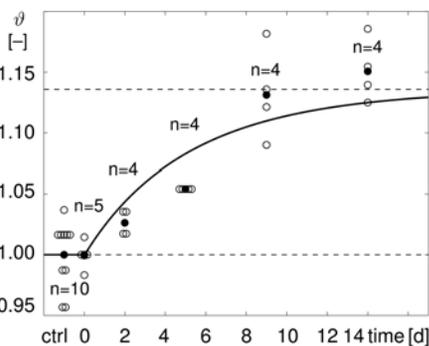


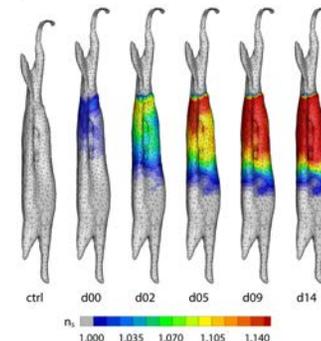
figure. temporal evolution of serial sarcomere number in chronically stretched skeletal muscle. upon stretching the extensor digitorum lateralis muscle by 1.14, the sarcomere number increases gradually from 1.00 to 1.14 within two weeks, bringing the sarcomere length back to its initial value. computationally predicted sarcomere numbers, solid line, agree nicely with experimentally measured sarcomere numbers, white circles, and their mean values, black circles.

example: skeletal muscle

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muscle fiber growth

gradual increase in sarcomere number



serial sarcomere number vs time

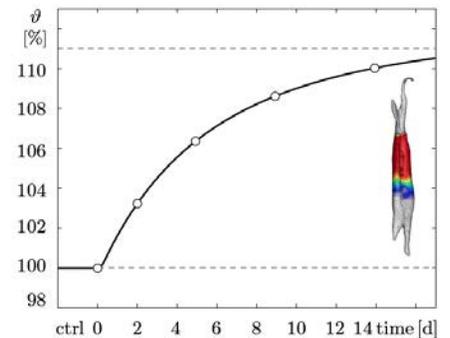


figure. spatio-temporal evolution of serial sarcomere number in chronically stretched skeletal muscle. upon stretching the biceps brachii muscle by 1.14, the serial sarcomere number increases gradually from 1.00 to 1.14 within two weeks, bringing the sarcomere length back to its initial value of 1.00. the serial sarcomere number is a measure for the inelastic fiber stretch.

zollner, bol, abilez, kuhl [2012]

example: skeletal muscle

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muscle fiber growth

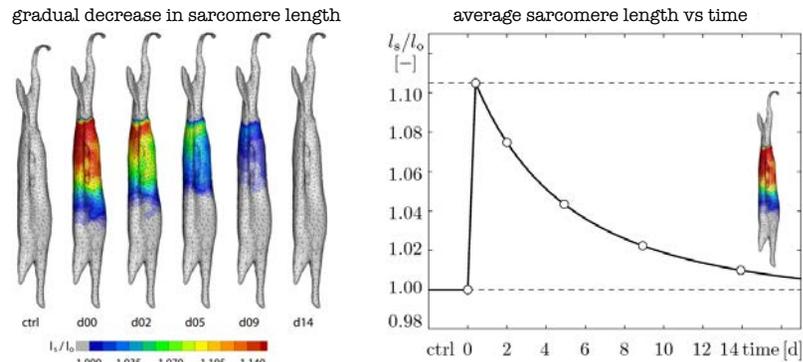


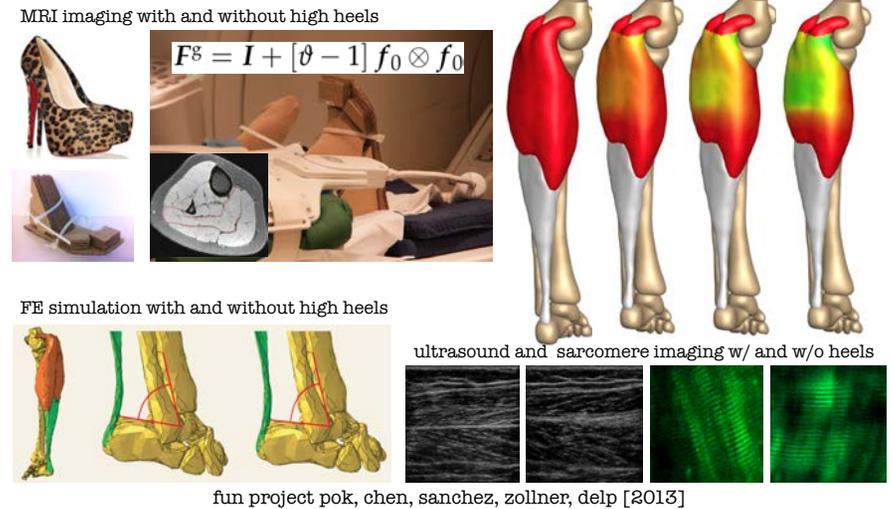
figure. spatio-temporal evolution of sarcomere length in chronically stretched skeletal muscle. upon stretching the biceps brachii muscle by 1.14, the average sarcomere length decreases gradually from 1.14 to 1.00 within two weeks, bringing the sarcomere length back to its initial value of 1.00. the sarcomere length is a measure for the elastic fiber stretch.

zollner, bol, abilez, kuhl [2012]

example: skeletal muscle

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muscle fiber shortening with high heels



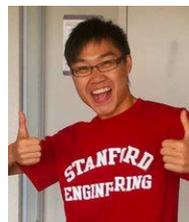
fun project pok, chen, sanchez, zollner, delp [2013]

example: high heels

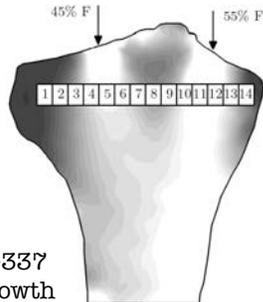
density growth of the tibia

Computational modeling of bone density profiles in response to gait: A subject-specific approach

Henry Pang¹, Abhishek P. Shiwalkar¹, Chris M. Madormo¹, Rebecca E. Taylor¹, Thomas P. Andriacchi^{1,2}, Ellen Kuhl^{1,3,4}



class project me337
mechanics of growth



trial	I	II	III	mean
max knee force [N]	552	549	579	560
max knee force [% BW]	94.9	94.4	99.4	96.2

example: bone

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density growth of the tibia

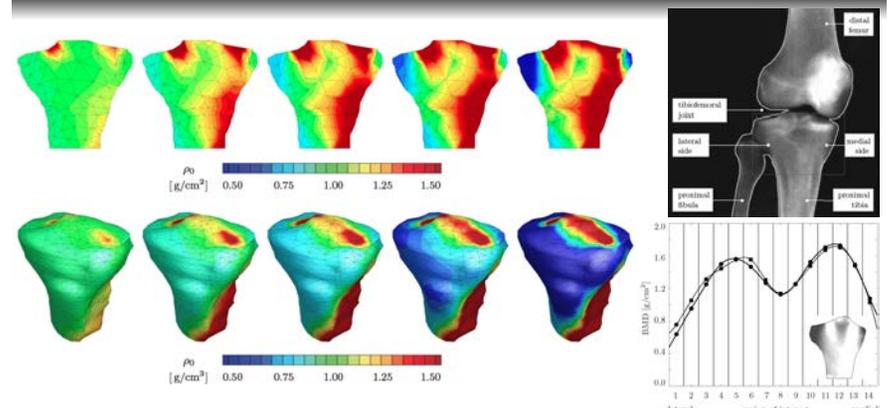


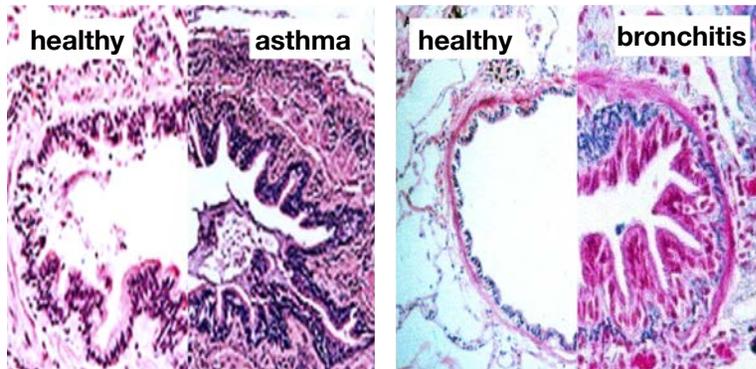
figure. regional variation of bone mineral density in fourteen regions of interest. squares and dotted lines indicate the experimentally measured bone mineral density from dual-energy X-ray absorptiometry. circles and solid lines indicate the computationally predicted bone density.

pang, shiwalkar, madormo, taylor, andriacchi, kuhl [2012]

example: bone

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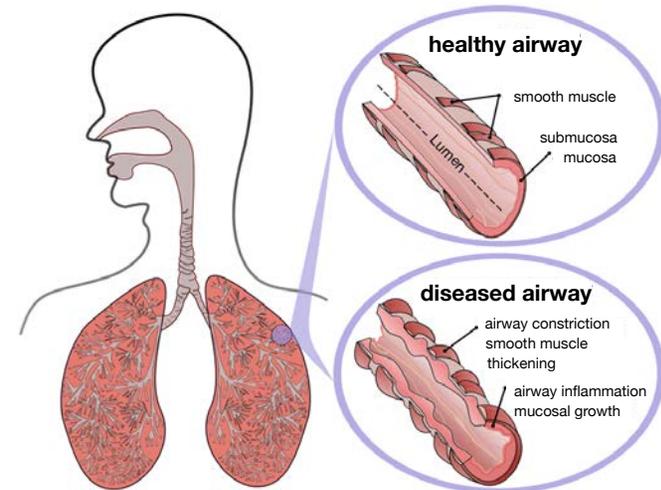
constrained growth in asthma & bronchitis



wiggs, hrousis, drazen, kamm [1997], kamm [1999], jin, cai, suo [2011], moulton, goriely [2011], li, cao, feng, gao [2011], cao, li, feng [2012], papastavrou, steinmann, kuhl [2013], eskandari, pfaller, kuhl [2013]

example: asthma and bronchitis

constrained growth in asthma & bronchitis



eskandari, pfaller, kuhl [2013]

example: asthma and bronchitis

constrained growth in asthma & bronchitis

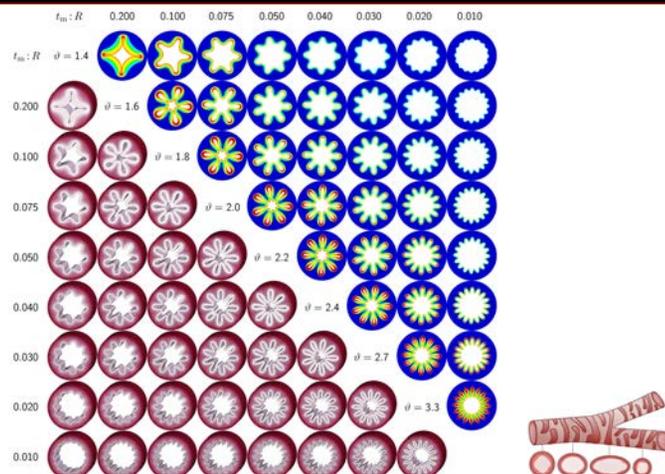


figure. sensitivity with respect to thickness. smaller bronchi with large relative mucosal thickness are at higher risk of to airflow obstruction than larger bronchi with small mucosal thickness.

example: asthma and bronchitis

cortical folding in the developing brain

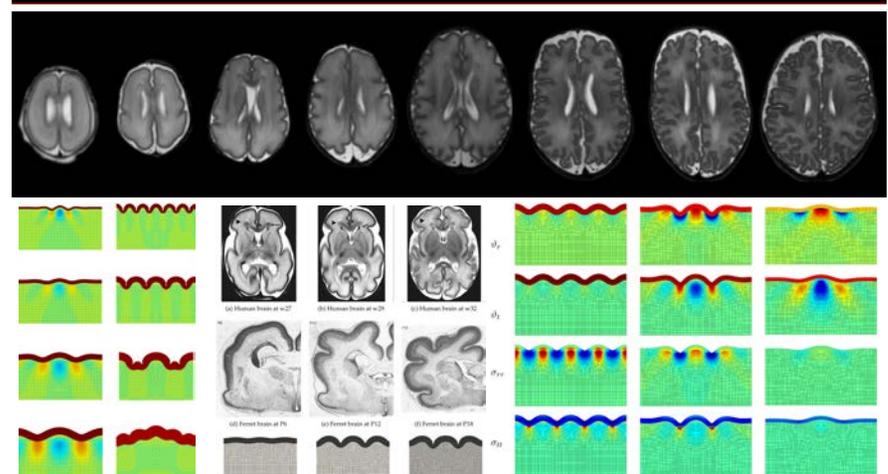


figure. progressive cortical folding in the developing human brain. malforming might be associated with microgyria, lissencephaly, schizophrenia, and autism. srinivasan [2008], lettau, kuhl [2013]

example: brain development