2 Homework - ME338

due 10/24/13, 11:00am, 200-305

You can drop off late homework in a box in front of Durand 217. Please mark clearly with date and time @drop off. We will take off 1/10 of points for each 24 hours late.

Kinematics

Despite tremendous research during the past 20 years, heart failure remains one of the most common, costly, disabling, and deadly medical conditions affecting more than 25 million people worldwide. In an attempt to better understand heart disease and to quantify strain profiles in the beating heart, researchers in the lab of Prof. D. Craig Miller in the Department of Cardiothoracic Surgery in the School of Medicine at Stanford have developed a novel technique to measure infarct-induced changes of ventricular wall kinematics. They insert a transmural beadset into the wall of the heart, see Figure 1, and acquire videofluoroscopic images at 60 frames per second using a biplane videofluoroscopy system. To obtain a fully three-dimensional picture of the deformation, they merge the marker coordinates from two biplane views using semi-automated image processing and digitization software. To determine local muscle fiber directions, they cut transmural rectangular blocks of tissue from the regions with the implanted markers and slice them into 1mm thick transmural sections. This allows them to measure the muscle fiber orientation at each point through tissue histology. **The objective of this homework is to quantify ventricular wall strains and characterize muscle fiber contraction in vivo using these experimental data.** This homework can help to answer the following scientific and clinical questions: Does fiber contraction display regional and transmural variations? How do contraction profiles change in regions close to an infarct? Can passive support prevent these changes? Is the fiber contraction of the non-infarcted myocardium inherently normal? Does the failing heart dilate uniformly or is dilation distributed heterogeneously?

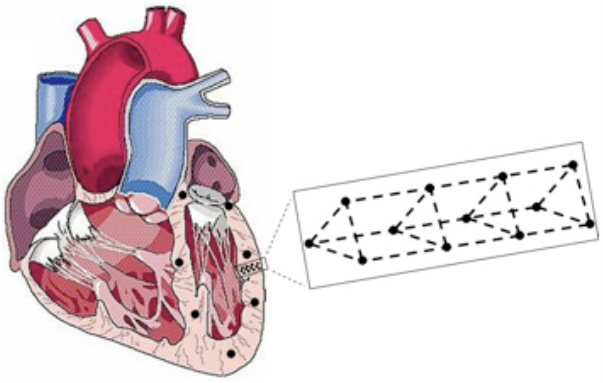

Fig. 1 Heart with ventricular markers and beadset

Figure 2 shows a typical data set of four markers of the beadset array. The red tetrahedron shows the marker positions $\boldsymbol{X}$ at end diastole, i.e., at the end of the cardiac filling phase. The blue tetrahedron illustrates the marker positions $\boldsymbol{x}$ at end systole, i.e., at the end of the cardiac contraction phase. You can assume that the tetrahedra in Figure 2 are oriented such that their $\boldsymbol{e}_3$ (or $\boldsymbol{z}$) axis is aligned with in the longitudinal direction of the heart, and that $\boldsymbol{e}_1$ (or $\boldsymbol{x}$) and $\boldsymbol{e}_2$ (or $\boldsymbol{y}$) are the radial/transmural and circumferential directions, respectively. Just to give you an idea, the left ventricular chamber is about 8 cm long and has a maximum radius of approximately 4 cm. The measured marker coordinates are given below.

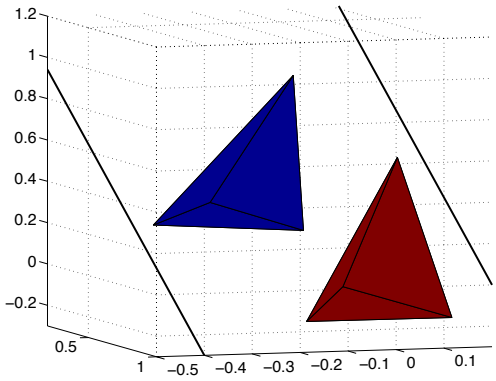

Fig. 2 Material and spatial configurations

$$\boldsymbol{X}_0 = [\ +0.25\ -0.27\ -0.25\ ]^t \qquad \boldsymbol{x}_0 = [\ +0.01\ -0.40\ +0.15\ ]^t$$
$$\boldsymbol{X}_1 = [\ +0.25\ +0.53\ -0.25\ ]^t \qquad \boldsymbol{x}_1 = [\ +0.00\ +0.32\ +0.15\ ]^t$$
$$\boldsymbol{X}_2 = [\ -0.05\ +0.52\ -0.25\ ]^t \qquad \boldsymbol{x}_2 = [\ -0.33\ +0.38\ +0.21\ ]^t$$
$$\boldsymbol{X}_3 = [\ +0.25\ +0.13\ +0.45\ ]^t \qquad \boldsymbol{x}_3 = [\ +0.05\ +0.07\ +0.85\ ]^t$$

The histologically measured fiber orientation angle for the displayed tetrahedra, measured clockwise against the horizontal axes, is $\alpha = 45.4^0$, indicated through the two black lines in Figure 2. For simplicity, you can assume that the muscle fibers are planar in the transmural plane. The cardiac muscle fiber direction $\boldsymbol{N}^{\text{fib}}$ in the relaxed diastolic configuration is thus given through the following unit vector.

$$\boldsymbol{N}^{\text{fib}} = [\,0.0\ -\cos(\alpha)\ +\sin(\alpha)\,]^t \qquad \text{with} \qquad \alpha = 45.4^0$$

The overall objective is this homework is to determine the muscle fiber contraction in a beating heart. This is pretty straightforward if you solve the following substeps.

[1] Determine three vectors $\mathrm{d}\boldsymbol{X}_i$ that span the tetrahedron at end diastole.
Take an arbitrary point of the tetrahedron as origin, e.g., $\boldsymbol{X}_0$, and calculate the three vectors $\mathrm{d}\boldsymbol{X}_1$, $\mathrm{d}\boldsymbol{X}_2$, and $\mathrm{d}\boldsymbol{X}_3$ from the origin to any other point using the coordinates $\boldsymbol{X}$ at end diastole such that $\mathrm{d}\boldsymbol{X}_i = \boldsymbol{X}_i - \boldsymbol{X}_0$ for $i = 1, 2, 3$.

[2] Determine the same three vectors $\mathrm{d}\boldsymbol{x}_i$ that span the tetrahedron at end systole.
Take the same point as origin, e.g., $\boldsymbol{x}_0$, and calculate the vectors $\mathrm{d}\boldsymbol{x}_1$, $\mathrm{d}\boldsymbol{x}_2$, and $\mathrm{d}\boldsymbol{x}_3$ from the origin to any other point using the coordinates $\boldsymbol{x}$ at end systole such that $\mathrm{d}\boldsymbol{x}_i = \boldsymbol{x}_i - \boldsymbol{x}_0$ for $i = 1, 2, 3$.

[3] Determine the deformation gradient tensor $\boldsymbol{F}$ that maps all diastolic line elements as $\mathrm{d}\boldsymbol{X}_i$ onto the systolic line elements $\mathrm{d}\boldsymbol{x}_i$.
$\boldsymbol{F} = \partial\boldsymbol{x} \,/\, \partial\boldsymbol{X}$ is called the deformation gradient and it is the key kinematic quantity to characterize finite strain kinematics. It maps line elements according to $\mathrm{d}\boldsymbol{x}_i = \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{X}_i$.

Applying this mapping to all three line elements $\mathrm{d}\boldsymbol{X}_i$ defines three vector valued equations, i.e., nine equations to solve for the nine components of $\boldsymbol{F}$. To obtain a more compact notation, rearrange all diastolic line elements from [1] and all systolic line elements from [2] in 3×3 matrices $\boldsymbol{C} := [\ \mathrm{d}\boldsymbol{X}_1;\ \mathrm{d}\boldsymbol{X}_2;\ \mathrm{d}\boldsymbol{X}_3\ ]$ and $\boldsymbol{c} := [\ \mathrm{d}\boldsymbol{x}_1;\ \mathrm{d}\boldsymbol{x}_2;\ \mathrm{d}\boldsymbol{x}_3\ ]$. Now, determine the deformation gradient $\boldsymbol{F}$ by solving the equation $\boldsymbol{F} \cdot \boldsymbol{C} = \boldsymbol{c}$.

[4] Control your results by calculating $\mathrm{d}\boldsymbol{x}_i = \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{X}_i$.
Do the calculated systolic line elements $\mathrm{d}\boldsymbol{x}_i$ match the ones you had calculated in [2]?

[5] Determine the systolic fiber direction $\boldsymbol{n}^{\text{fib}} = \boldsymbol{F} \cdot \boldsymbol{N}^{\text{fib}}$.
The deformation gradient can be used to map the measured diastolic fiber direction $\boldsymbol{N}^{\text{fib}}$ onto the systolic fiber direction $\boldsymbol{n}^{\text{fib}}$. Determine $\boldsymbol{n}^{\text{fib}}$ and comment on how $\boldsymbol{N}^{\text{fib}}$ and $\boldsymbol{n}^{\text{fib}}$ deviate.

[6] Determine the fiber stretch $\lambda = \sqrt{\boldsymbol{n}^{\text{fib}} \cdot \boldsymbol{n}^{\text{fib}}}$.
Since the fiber orientation $\boldsymbol{N}^{\text{fib}}$ was given as a unit vector, the length of the systolic vector $\boldsymbol{n}^{\text{fib}} = \boldsymbol{F} \cdot \boldsymbol{N}^{\text{fib}}$ corresponds to the relative change in fiber length, i.e., the fiber stretch in the finite strain setting, $\lambda = \sqrt{\boldsymbol{n}^{\text{fib}} \cdot \boldsymbol{n}^{\text{fib}}} = \sqrt{\boldsymbol{N}^{\text{fib}} \cdot \boldsymbol{F}^{\text{t}} \cdot \boldsymbol{F} \cdot \boldsymbol{N}^{\text{fib}}}$.

[7] Determine the second order Green Lagrange strain tensor $\boldsymbol{E} = \frac{1}{2}\,[\,\boldsymbol{F}^{\text{t}} \cdot \boldsymbol{F} - \boldsymbol{I}\,]$
$\boldsymbol{E}$ is called the Green Lagrange strain tensor and it is used to characterize strains in the undeformed configuration in a finite strain setting.

[8] Determine the displacement gradient tensor $\boldsymbol{H} = \boldsymbol{F} - \boldsymbol{I}$.
$\boldsymbol{H} = \nabla \boldsymbol{u}$ is the nonsymmetric displacement gradient tensor which can also be expressed as $\boldsymbol{H} = \partial \boldsymbol{u} \,/\, \partial \boldsymbol{X} = \partial [\boldsymbol{x} - \boldsymbol{X}] \,/\, \partial \boldsymbol{X} = \boldsymbol{F} - \boldsymbol{I}$.

[9] Linearize the Green Lagrange strain tensor $\boldsymbol{E}$ with the help of the Gateaux derivative to obtain the small strain tensor $\boldsymbol{\epsilon} = \frac{1}{2}\,(\boldsymbol{H} + \boldsymbol{H}^{\text{t}})$.
Linearize $\boldsymbol{E}$ formally, then calculate $\boldsymbol{\epsilon}$, compare the small strain approximation $\boldsymbol{\epsilon}$ with the large strain Green Lagrange tensor $\boldsymbol{E}$, and comment on your results.

[10] Determine the volume dilation $e = \mathbf{tr}(\boldsymbol{\epsilon})$.
Comment on whether the tissue behaves compressible or incompressible and on whether this was what you would have expected.

[11] Last, determine the normal strain $\epsilon_n = \boldsymbol{N}^{\text{fib}} \cdot \boldsymbol{\epsilon} \cdot \boldsymbol{N}^{\text{fib}}$.
Compare the large strain fiber stretch λ to the small strain approximation of the fiber contraction ϵ_n. Comment on your results. Do you think they are reasonable? Find evidence in the literature about the amount of maximum cardiomyocyte contraction.

You can use MATLAB to solve the matrix and vector operations. If you choose to do so, you must deliver a printout of your MATLAB code with the homework.

Additional reading Tsamis A, Bothe W, Kvitting JP, Swanson JC, Miller DC, Kuhl E. Active contraction of cardiac muscle: In vivo characterization of mechanical activation sequences in the beating heart. J Mech Behavior Biomed Mat, 2011;4:1167-1176.