08 - balance principles

The Four Stages of Data Loss: Dealing with accidental deletion of months of hard-earned data.

- Stage 1: Denial
- Stage 2: Anger
- Stage 3: Depression
- Stage 4: Acceptance

Holzapfel "Nonlinear Solid Mechanics" [2000], Chapter 4, Pages 131-179

Homework 02

Videofluoroscopic markers

Surgically implanted epicardial markers and transmural bead set

4D coordinates from in vivo biplane videofluoroscopic marker images

Itoh, Bothe, Swanson Birchill, Escobar Kvitting, Nguyen, Langer, Rodriguez, Criscione, Ingels, Miller

How much does the heart contract?

\[ \varphi(X, t) = \sum_{i=1}^{n_{apx}} c_i(t) N_i(X) \]

Deformation gradient

\[ F(X, t) = \sum_{i=1}^{n_{apx}} c_i(t) \otimes \nabla N_i(X) \]

Green Lagrange strains

\[ E(X, t) = \frac{1}{2} \left[ F^t \cdot F - I \right] \]

Fiber strain

\[ E_{ff}(X, t) = f(X) \cdot E(X, t) \cdot f(X) \]

Tsamis, Bothe, Kvitting, Swanson, Miller, Kuhl [2011]
how much does the heart contract?

mean anterior-basal fiber strain

mean lateral-equatorial fiber strain

tsamis, bothe, kvitting, swanson, miller, kuhl [2011]

how much does the heart contract?

tsamis, cheng, nguyen, langer, miller, kuhl [2012]

how much does the heart grow?

definition

\[ \phi (X, t) = \sum_{j=1}^{n_{\text{apx}}} c_j(t) n_j(X) \]

definition gradient

\[ F(X, t) = \sum_{j=1}^{n_{\text{apx}}} c_j(t) \otimes \nabla n_j(X) \]

volume changes

\[ J(X, t) = \det (F(X, t)) \]

fiber stretch

\[ \lambda_{\text{FF}}(X, t) = \left[ f(X) \cdot F^\text{T}(X, t) \cdot F(X, t) \cdot f(X) \right]^{1/2} \]

how much does the heart grow?

tsamis, cheng, nguyen, langer, miller, kuhl [2012]

<table>
<thead>
<tr>
<th></th>
<th>epi 20% depth</th>
<th>p</th>
<th>mid 50% depth</th>
<th>p</th>
<th>endo 80% depth</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>F_{SC}</td>
<td>1.00±0.12</td>
<td>0.96</td>
<td>1.03±0.14</td>
<td>0.46</td>
<td>1.02±0.10</td>
<td>0.44</td>
</tr>
<tr>
<td>F_{CL}</td>
<td>0.04±0.14</td>
<td>0.42</td>
<td>0.01±0.10</td>
<td>0.77</td>
<td>0.01±0.09</td>
<td>0.61</td>
</tr>
<tr>
<td>F_{CR}</td>
<td>-0.07±0.29</td>
<td>0.46</td>
<td>-0.03±0.16</td>
<td>0.61</td>
<td>0.05±0.14</td>
<td>0.29</td>
</tr>
<tr>
<td>F_{EC}</td>
<td>-0.02±0.17</td>
<td>0.75</td>
<td>-0.04±0.13</td>
<td>0.33</td>
<td>-0.04±0.11</td>
<td>0.24</td>
</tr>
<tr>
<td>F_{EC}</td>
<td>1.10±0.15</td>
<td>0.06</td>
<td>1.10±0.13</td>
<td>0.03</td>
<td>1.11±0.11</td>
<td>0.01</td>
</tr>
<tr>
<td>F_{EL}</td>
<td>0.02±0.16</td>
<td>0.71</td>
<td>0.10±0.20</td>
<td>0.11</td>
<td>0.18±0.34</td>
<td>0.12</td>
</tr>
<tr>
<td>F_{ER}</td>
<td>-0.01±0.09</td>
<td>0.64</td>
<td>-0.03±0.17</td>
<td>0.54</td>
<td>-0.05±0.19</td>
<td>0.41</td>
</tr>
<tr>
<td>F_{RR}</td>
<td>0.00±0.05</td>
<td>0.86</td>
<td>-0.00±0.09</td>
<td>0.96</td>
<td>-0.01±0.11</td>
<td>0.67</td>
</tr>
<tr>
<td>F_{RR}</td>
<td>0.68±0.15</td>
<td>0.00</td>
<td>0.73±0.15</td>
<td>0.00</td>
<td>0.77±0.22</td>
<td>0.01</td>
</tr>
<tr>
<td>J_E</td>
<td>0.74±0.19</td>
<td>0.00</td>
<td>0.82±0.19</td>
<td>0.01</td>
<td>0.89±0.21</td>
<td>0.10</td>
</tr>
<tr>
<td>\lambda_{\text{FF}}</td>
<td>1.03±0.12</td>
<td>0.49</td>
<td>1.04±0.16</td>
<td>0.36</td>
<td>1.08±0.11</td>
<td>0.04</td>
</tr>
</tbody>
</table>

tsamis, cheng, nguyen, langer, miller, kuhl [2012]

longitudinal growth by more than 10%
radial thinning by more than 20%
volume decrease by more than 15%
fiber lengthening by more than 5%

<table>
<thead>
<tr>
<th></th>
<th>epi 20% depth</th>
<th>p</th>
<th>mid 50% depth</th>
<th>p</th>
<th>endo 80% depth</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>F_{SC}</td>
<td>1.00±0.12</td>
<td>0.96</td>
<td>1.03±0.14</td>
<td>0.46</td>
<td>1.02±0.10</td>
<td>0.44</td>
</tr>
<tr>
<td>F_{CL}</td>
<td>0.04±0.14</td>
<td>0.42</td>
<td>0.01±0.10</td>
<td>0.77</td>
<td>0.01±0.09</td>
<td>0.61</td>
</tr>
<tr>
<td>F_{CR}</td>
<td>-0.07±0.29</td>
<td>0.46</td>
<td>-0.03±0.16</td>
<td>0.61</td>
<td>0.05±0.14</td>
<td>0.29</td>
</tr>
<tr>
<td>F_{EC}</td>
<td>-0.02±0.17</td>
<td>0.75</td>
<td>-0.04±0.13</td>
<td>0.33</td>
<td>-0.04±0.11</td>
<td>0.24</td>
</tr>
<tr>
<td>F_{EC}</td>
<td>1.10±0.15</td>
<td>0.06</td>
<td>1.10±0.13</td>
<td>0.03</td>
<td>1.11±0.11</td>
<td>0.01</td>
</tr>
<tr>
<td>F_{EL}</td>
<td>0.02±0.16</td>
<td>0.71</td>
<td>0.10±0.20</td>
<td>0.11</td>
<td>0.18±0.34</td>
<td>0.12</td>
</tr>
<tr>
<td>F_{ER}</td>
<td>-0.01±0.09</td>
<td>0.64</td>
<td>-0.03±0.17</td>
<td>0.54</td>
<td>-0.05±0.19</td>
<td>0.41</td>
</tr>
<tr>
<td>F_{RR}</td>
<td>0.00±0.05</td>
<td>0.86</td>
<td>-0.00±0.09</td>
<td>0.96</td>
<td>-0.01±0.11</td>
<td>0.67</td>
</tr>
<tr>
<td>F_{RR}</td>
<td>0.68±0.15</td>
<td>0.00</td>
<td>0.73±0.15</td>
<td>0.00</td>
<td>0.77±0.22</td>
<td>0.01</td>
</tr>
<tr>
<td>J_E</td>
<td>0.74±0.19</td>
<td>0.00</td>
<td>0.82±0.19</td>
<td>0.01</td>
<td>0.89±0.21</td>
<td>0.10</td>
</tr>
<tr>
<td>\lambda_{\text{FF}}</td>
<td>1.03±0.12</td>
<td>0.49</td>
<td>1.04±0.16</td>
<td>0.36</td>
<td>1.08±0.11</td>
<td>0.04</td>
</tr>
</tbody>
</table>

tsamis, cheng, nguyen, langer, miller, kuhl [2012]
08 - balance principles

[1] isolate subset $\tilde{B}$ from $B$

[2] characterize influence of remaining body through phenomenological quantities - contact fluxes $\vec{v}^p$, $\vec{t}^\varphi$ & $\vec{t}^\psi$

[3] define basic physical quantities - mass, linear and angular momentum, energy

[4] postulate balance of these quantities
**08 - balance principles**

**balance of mass**

- mass is constant
  
  \[
  \dot{m} = D_t m = D_t \int_{B_0} \rho_0 \, dV \div 0 = D_t \int_{B_t} \rho_t \, dv = \text{const} \quad \dot{m} = D_t m = D_t \int_{B_t} \rho_t \, dv \div 0 = \int_{B_0} D_t (J \rho_t) \, dV \div 0
  \]

- material density is constant
  
  \[
  \dot{\rho}_0 = 0 \rightarrow \rho_0 = \text{const} \quad \dot{\rho}_t + \rho_t \, \text{div}(v) = 0
  \]

\[
D_t (J \rho_t) = J D_t \rho_t + \rho_t D_t J = J D_t \rho_t + \rho_t J \text{div}(v) = J \left[ \dot{\rho}_t + \rho_t \text{div}(v) \right]
\]

- unlike open systems, **closed systems** have a constant mass

- examples of open systems:
  - rocket propulsion
  - biological growth (me337)
**balance of mass**

Global balance of mass

\[ \dot{m} = \int_{B_0} \dot{\rho}_0 \, dV = 0 \quad \dot{m} = \int_{B_0} \dot{\rho}_t + \rho_t \, \text{div}(\mathbf{v}) \, dV = 0 \]

Local balance of mass

\[ \dot{\rho}_0 = 0 \quad \dot{\rho}_t + \rho_t \, \text{div}(\mathbf{v}) = 0 \]

---

**balance of (linear) momentum**

Linear momentum density

\[ \dot{L} = D_t \int_{B_0} \rho_0 \mathbf{v} \, dV = \int_{B_0} \rho_0 \dot{\mathbf{v}} \, dV = \mathbf{F} = \int_{\partial B_t} \mathbf{T} \, dA + \int_{B_t} b_t \, dA \]

Global balance of linear momentum

\[ \int_{B_0} \rho_0 \dot{\mathbf{v}} \, dV = \int_{B_0} \text{Div}(\mathbf{P}) \, dV + \int_{B_0} b_0 \, dV \]

Local balance of linear momentum

\[ \rho_0 \dot{\mathbf{v}} = \text{Div}(\mathbf{P}) + b_0 \quad \rho_t \dot{\mathbf{v}} = \text{div}(\sigma) + b_t \]

\[ \rho_0 = J \rho_t \quad \mathbf{P} = J \sigma \cdot F^{-t} \quad b_0 = J b_t \]
balance of (linear) momentum

\[ \rho_0 \mathbf{v} \quad \text{linear momentum density} \]
\[ P \quad \text{momentum flux - stress} \quad P \cdot n = \mathbf{T}^\phi \]
\[ b_0 \quad \text{momentum source - force} \]
\[ 0 \quad \text{no momentum production} \]

**equilibrium equation**

\[ \rho_0 \mathbf{v} = \text{Div}(P) + b_0 \]

---

compare

First published in 1679, Isaac Newton's "Principia Mathematica" is often considered one of the most important single works in the history of science. Its Second Law is the most powerful of the three, allowing mathematical calculation of the duration of a doctoral degree.

**SECOND LAW**

"The age, \( \mathbf{a} \), of a doctoral process is directly proportional to the flexibility, \( f \), given by the advisor and inversely proportional to the student's motivation, \( m \)."

Mathematically, this postulate translates to:

\[ \mathbf{a} = \frac{f}{m} \]

- \( \mathbf{F} = \mathbf{m} \mathbf{a} \)

This law is a quantitative description of the effect of the forces experienced by a graduate student. A highly motivated student may still remain in grad school given enough flexibility. As motivation goes to zero, the duration of the PhD goes to infinity.

---

**08 - balance principles**

**balance of (internal) energy**

**global balance of internal energy**

\[ \mathbf{D}_t \int_{\mathcal{B}_0} E_0 \, d\mathbf{V} = \int_{\mathcal{B}_0} P : \mathbf{\dot{F}} \, d\mathbf{V} - \int_{\mathcal{B}_0} \text{Div}(Q) \, d\mathbf{V} + \int_{\mathcal{B}_0} R_0 \, d\mathbf{V} \]

\[ \mathbf{D}_t \int_{\mathcal{B}_t} E_\mathbf{t} \, d\mathbf{V} = \int_{\mathcal{B}_t} \sigma : \mathbf{d} \, d\mathbf{V} - \int_{\mathcal{B}_t} \text{Div}(q) \, d\mathbf{V} + \int_{\mathcal{B}_t} R_t \, d\mathbf{V} \]

**local balance of internal energy**

\[ \dot{E}_0 = P : \mathbf{\dot{F}} - \text{Div}(Q) + R_0 \quad \dot{E}_\mathbf{t} = \sigma : \mathbf{d} - \text{Div}(q) + R_t \]

\[ E_0 = J E_t \quad Q = J q \cdot \mathbf{F}^{-t} \quad R_0 = J R_t \]

**energy equation**

\[ \dot{E}_0 = P : \mathbf{\dot{F}} - \text{Div}(Q) + R_0 \]

- internal mechanical power
- external thermal power

---

**08 - balance principles**

**mass point**

\[ m \mathbf{D}_t \mathbf{v} = m \mathbf{a} = \mathbf{F} \]