holzapfel 'nonlinear solid mechanics' [2000], chapter 4, pages 131-161

07 - balance principles

stress tensors

caulcy / true stress
relates spatial force to spatial area
\[ df = t \, da = \sigma \cdot n \, da = \sigma \cdot da \]

first piola kirchhoff / nominal stress
relates spatial force to material area
\[ df = t \, da = \sigma \cdot n \, da = \sigma \cdot da = \int \sigma \cdot F^{-t} \cdot dA = P \cdot dA \]

second piola kirchhoff stress
relates material force to material area
\[ dF = F^{-1} \cdot df = F^{-1} \cdot P \cdot dA = \int F^{-1} \cdot \sigma \cdot F^{-t} \cdot dA = S \cdot dA \]
stress tensors

first piola kirchhoff
\[ P = F \cdot S \]
\[ P = J \sigma \cdot F^{-t} \]

second piola kirchhoff
\[ S = F^{-1} \cdot P \]
\[ S = J F^{-1} \cdot \sigma \cdot F^{-t} \]

cauchy
\[ \sigma = \frac{1}{J} P \cdot F^{-t} \]
\[ \sigma = \frac{1}{J} F \cdot S \cdot F^t \]

covariant / strains
\[ E = F^t \cdot e \cdot F \]
\[ E_{ij} = F_{ik}^t e_{kl} F_{lj} \]
\[ e_{ij} = F_{ik}^{-t} E_{KL} F_{Lj}^{-1} \]

contravariant / stresses
\[ S = J F^{-1} \cdot \sigma \cdot F^{-t} \]
\[ S_{ij} = J F_{ik}^{-1} \sigma_{kl} F_{lj}^{-t} \]
\[ \sigma_{ij} = \frac{1}{J} F_{IK} S_{KL} F_{Lj}^t \]

06 - concept of stress

transport mechanisms

balance equations

平衡方程 [bæləns ɪˈkwɛr.ˌmənəs] of mass, momentum, angular momentum and energy, supplemented with an entropy inequality constitute the set of conservation laws. The law of conservation of mass/matter states that the mass of a closed system of substances will remain constant, regardless of the processes acting inside the system. The principle of conservation of momentum states that the total momentum of a closed system of objects is constant.

chadwick ‘continuum mechanics’ [1976]
07 - balance principles

[1] isolate subset \( \bar{B} \) from \( B \)

[2] characterize influence of remaining body through phenomenological quantities - contact fluxes \( \bar{t}^\nu, \bar{t}^\rho \& \bar{\bar{t}}^\rho \)

[3] define basic physical quantities - mass, linear and angular momentum, energy

[4] postulate balance of these quantities
07 - balance principles

**balance of mass**

- unlike open systems **closed systems** have a constant mass
- examples of open systems: rocket propulsion and biological growth (me337)

**balance of mass**

- $07 - \text{balance principles}$

**generic balance equation**

\[
\begin{align*}
A &... \text{balance quantity} \\
B &... \text{flux} \quad B \cdot n = \bar{T}^A \\
C &... \text{source} \\
\Gamma &... \text{production}
\end{align*}
\]

\[D_tA = \text{Div}(B) + C + \Gamma\]

**balance of mass**

\[
\begin{align*}
\rho_0 &... \text{density} \\
0 &... \text{no mass flux} \quad \bar{T}^\rho = 0 \\
0 &... \text{no mass source} \\
0 &... \text{no mass production}
\end{align*}
\]

\[D_t\rho_0 = 0\]

**balance of (linear) momentum**

\[
\begin{align*}
\rho_0 v &... \text{linear momentum density} \\
P &... \text{momentum flux - stress} \quad P \cdot n = \bar{T}^\rho \\
b_0 &... \text{momentum source - force} \\
0 &... \text{no momentum production}
\end{align*}
\]

\[D_t(\rho_0 v) = \text{Div}(P) + b_0\]
First published in 1679, Isaac Newton’s "Philosophiae Naturalis Principia Mathematica" is often considered one of the most important single works in the history of science. Its Second Law is the most powerful of the three, allowing mathematical calculation of the duration of a doctoral degree.

SECOND LAW
The age, \( a \), of a doctoral process is directly proportional to the flexibility, \( F \), given by the advisor and inversely proportional to the student’s motivation, \( m \).

Mathematically, this postulate translates to:

\[
\frac{a}{m} = \frac{F}{\text{motivation}}
\]

\[
a = \frac{F}{m}
\]

\[
\therefore F = ma
\]

This Law is a quantitative description of the effect of the forces experienced by a grad student. A highly motivated student may still remain in grad school given enough flexibility. As motivation goes to zero, the duration of the PhD goes to infinity.

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\[D_t (\rho_0 \nu) = \text{Div}(P) + b_0\]

mass point

\[m D_t \nu = m a = F\]

\[\rho_0 \, I\] ... internal energy density

\[Q\] ... heat flux

\[-Q \cdot n = T^\theta\]

\[Q_0\] ... heat source

\[\Gamma\] ... no heat production

\[\textbf{energy equation}\]

\[D_t(\rho_0 I) = P : D_t F - v \cdot b_0 + \text{Div}(-Q) + Q_0\]

internal mechanical power

external thermal power