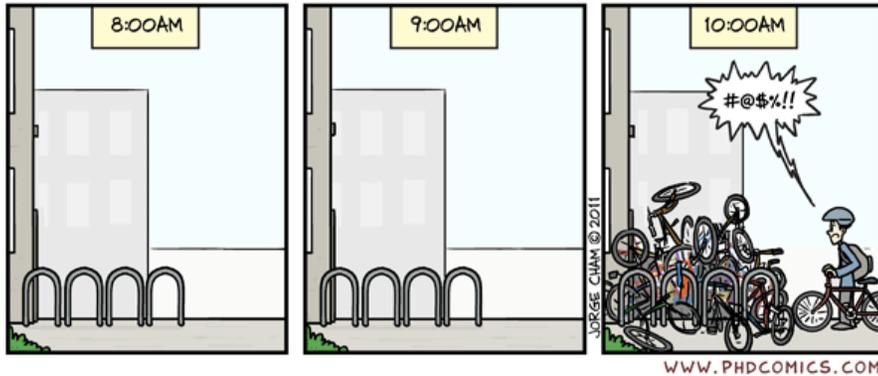


07 - balance principles



holzappel 'nonlinear solid mechanics' [2000], chapter 4, pages 131-161

07 - balance principles

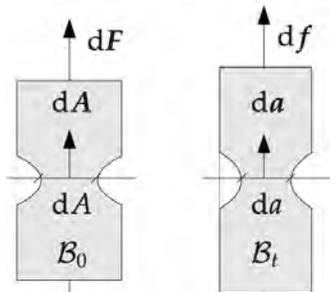
me338 - syllabus

day	date	topic	chapters	pages
tue	sep 25	why continuum mechanics?		
thu	sep 27	introduction to vectors and tensors	1.1-1.5	1-32
tue	oct 02	introduction to vectors and tensors	1.6-1.9	32-55
thu	oct 04	kinematics	2.1-2.4	55-76
tue	oct 09	kinematics	2.5-2.8	76-109
thu	oct 11	concept of stress	3.1-3.4	109-131
tue	oct 16	balance principles	4.1-4.4	131-161
thu	oct 18	balance principles	4.5-4.7	161-179
tue	oct 23	aspects of objectivity	5.1-5.4	179-205
thu	oct 25	hyperelastic materials	6.1-6.2	205-222
tue	oct 30	hyperelastic materials	6.3-6.5	222-252
thu	nov 01	hyperelastic materials	6.6-6.8	252-278
tue	nov 06	hyperelastic materials	6.9-6.11	278-305
thu	nov 08	thermodynamics of materials	7.1-7.6	305-337
tue	nov 13	midterm prep		
thu	nov 15	midterm		
tue	nov 27	thermodynamics of materials	7.7-7.9	337-371
thu	nov 29	variational principles	8.1-8.3	371-392
tue	dec 04	variational principles	8.4-8.6	392-414
thu	dec 06	final project discussion		



07 - balance principles

stress tensors



cauchy / true stress
relates spatial force to spatial area
 $df = t da = \sigma \cdot n da = \sigma \cdot da$

first piola kirchhoff / nominal stress
relates spatial force to material area

$$df = t da = \sigma \cdot n da = \sigma \cdot da = J \sigma \cdot F^{-t} \cdot dA = P \cdot dA$$

second piola kirchhoff stress
relates material force to material area

$$dF = F^{-1} \cdot df = F^{-1} \cdot P \cdot dA = J F^{-1} \cdot \sigma \cdot F^{-t} \cdot dA = S \cdot dA$$

06 - concept of stress

stress tensors

cauchy / true stress
relates spatial force to spatial area
 $t = \sigma \cdot n \quad t_i = \sigma_{ij} n_j$

first piola kirchhoff / nominal stress
relates spatial force to material area
 $P = J \sigma \cdot F^{-t} \quad P_{ij} = J \sigma_{ik} F_{kj}^{-t}$

second piola kirchhoff stress
relates material force to material area
 $S = F^{-1} \cdot P = J F^{-1} \cdot \sigma \cdot F^{-t}$

$$S_{IJ} = F_{Ik}^{-1} P_{kj} = J F_{Ik}^{-1} \sigma_{kl} F_{lj}^{-t}$$

06 - concept of stress

stress tensors



gustav robert kirchhoff
[1824-1887]

first piola kirchhoff

$$P = F \cdot S$$

$$P = J \sigma \cdot F^{-t}$$

second piola kirchhoff

$$S = F^{-1} \cdot P$$

$$S = J F^{-1} \cdot \sigma \cdot F^{-t}$$



augustin louis cauchy
[1789-1857]

cauchy

$$\sigma = \frac{1}{J} P \cdot F^t$$

$$\sigma = \frac{1}{J} F \cdot S \cdot F^t$$

06 - concept of stress

5

transport mechanisms

covariant / strains

$$E = F^t \cdot e \cdot F \quad \leftarrow \text{pull back} \quad e = F^{-t} \cdot E \cdot F^{-1}$$

$$E_{IJ} = F_{Ik}^t e_{kl} F_{lJ} \quad \text{push forward} \rightarrow \quad e_{ij} = F_{iK}^{-t} E_{KL} F_{Lj}^{-1}$$

contravariant / stresses

$$S = J F^{-1} \cdot \sigma \cdot F^{-t} \quad \leftarrow \text{pull back} \quad \sigma = \frac{1}{J} F \cdot S \cdot F^t$$

$$S_{IJ} = J F_{Ik}^{-1} \sigma_{kl} F_{lJ}^{-t} \quad \text{push forward} \rightarrow \quad \sigma_{ij} = \frac{1}{J} F_{iK} S_{KL} F_{Lj}^t$$

06 - concept of stress

6

balance equations

balance equations ['bæl.əns r'kwel.jəns] of mass, momentum, angular momentum and energy, supplemented with an entropy inequality constitute the set of conservation laws. the law of **conservation of mass/matter** states that the **mass of a closed system** of substances will remain **constant**, regardless of the processes acting inside the system. the principle of conservation of momentum states that the total momentum of a closed system of objects is constant.



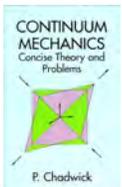
WIKIPEDIA
The Free Encyclopedia

07 - balance principles

balance equations

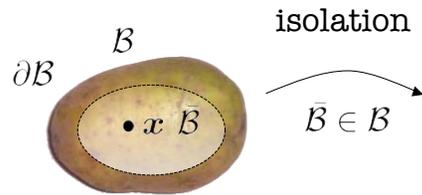
balance equations ['bæl.əns r'kwel.jəns] of mass, linear momentum, angular momentum and energy **apply to all material bodies**. each one gives rise to a field equation, holding on the configurations of a body in a sufficiently smooth motion and a jump condition on surfaces of discontinuity. like position, time and body, the concepts of mass, force, heating and internal energy which enter into the formulation of the balance equations are regarded as having primitive status in continuum mechanics.

chadwick 'continuum mechanics' [1976]



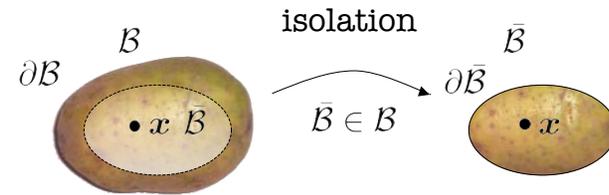
07 - balance principles

potato balance equations



[1] isolate subset \bar{B} from B

potato balance equations



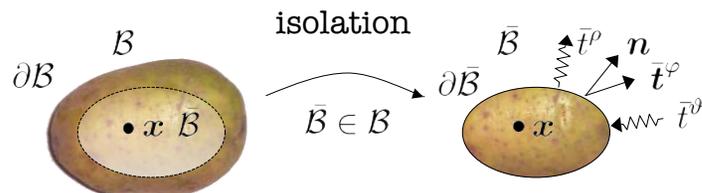
[1] isolate subset \bar{B} from B

[2] **characterize** influence of remaining body through phenomenological quantities - contact fluxes $\bar{t}^\rho, \bar{t}^\varphi$ & \bar{t}^θ

07 - balance principles

07 - balance principles

potato balance equations

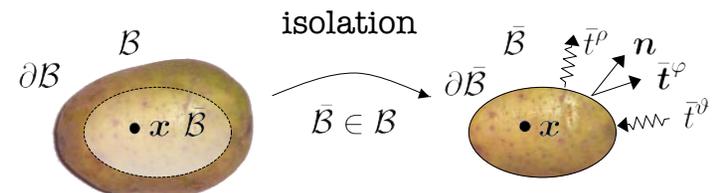


[1] isolate subset \bar{B} from B

[2] characterize influence of remaining body through phenomenological quantities - contact fluxes $\bar{t}^\rho, \bar{t}^\varphi$ & \bar{t}^θ

[3] **define** basic physical quantities - mass, linear and angular momentum, energy

potato balance equations



[1] isolate subset \bar{B} from B

[2] characterize influence of remaining body through phenomenological quantities - contact fluxes $\bar{t}^\rho, \bar{t}^\varphi$ & \bar{t}^θ

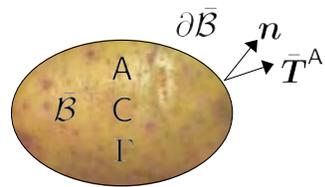
[3] define basic physical quantities - mass, linear and angular momentum, energy

[4] **postulate** balance of these quantities

07 - balance principles

07 - balance principles

generic balance equation



general format

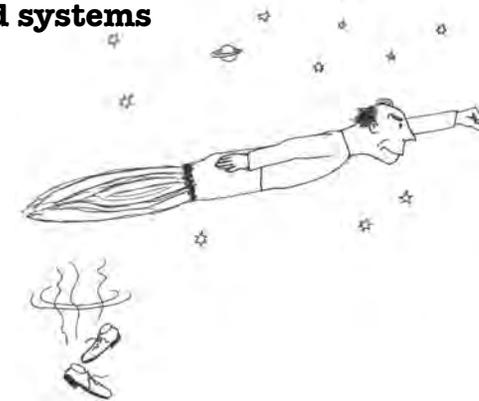
A... balance quantity
 B... flux $\mathbf{B} \cdot \mathbf{n} = \bar{T}^A$
 C... source
 Γ... production

$$D_t A = \text{Div}(\mathbf{B}) + C + \Gamma$$

07 - balance principles

balance of mass

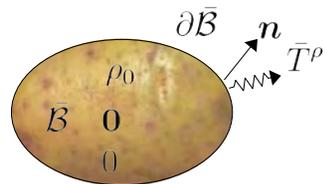
- here: **closed systems**



- unlike open systems **closed systems** have a **constant mass**
- examples of open systems:
rocket propulsion and **biological growth** (me33?)

07 - balance principles

balance of mass

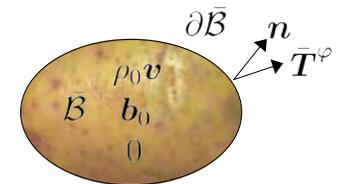


ρ_0 ... density
 0 ... no mass flux $\bar{T}^\rho = 0$
 0 ... no mass source
 0 ... no mass production

continuity equation $D_t \rho_0 = 0$

07 - balance principles

balance of (linear) momentum



$\rho_0 v$... linear momentum density
P ... momentum flux - stress $\mathbf{P} \cdot \mathbf{n} = \bar{T}^\rho$
 b_0 ... momentum source - force
 0 ... no momentum production

equilibrium equation $D_t(\rho_0 v) = \text{Div}(\mathbf{P}) + b_0$

07 - balance principles

compare



First published in 1679, Isaac Newton's "Procrastinare Unnaturalis Principia Mathematica" is often considered one of the most important single works in the history of science. Its Second Law is the most powerful of the three, allowing mathematical calculation of the duration of a doctoral degree.

SECOND LAW

"The age, a , of a doctoral process is directly proportional to the flexibility, f , given by the advisor and inversely proportional to the student's motivation, m "

Mathematically, this postulate translates to:

$$age_{\text{PhD}} = \frac{\text{flexibility}}{\text{motivation}}$$

$$a = F / m$$

$$\therefore F = m a$$

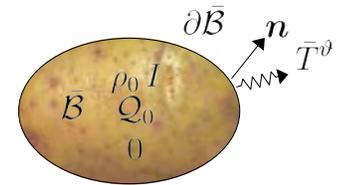
This Law is a quantitative description of the effect of the forces experienced by a grad student. A highly motivated student may still remain in grad school given enough flexibility. As motivation goes to zero, the duration of the PhD goes to infinity.

PHD.STANFORD.EDU
JORGE.CHAM@THE STANFORD DAILY

$$D_t(\rho_0 v) = \text{Div}(P) + b_0 \quad \text{mass point} \quad m D_t v = m a = F$$

07 - balance principles

balance of (internal) energy



- $\rho_0 I$... internal energy density
- Q ... heat flux $-Q \cdot n = \bar{T}^\theta$
- Q_0 ... heat source
- 0 ... no heat production

$$\text{energy equation} \quad D_t(\rho_0 I) = \underbrace{P : D_t F}_{\text{internal mechanical power}} - \underbrace{v \cdot b_0}_{\text{external thermal power}} + \text{Div}(-Q) + Q_0$$

07 - balance principles