holzapfel 'nonlinear solid mechanics' [2000], chapter 2.5-2.8, pages 76-109

kinematic equations

kinematic equations describe the motion of objects without the consideration of the masses or forces that bring about the motion. the basis of kinematics is the choice of coordinates. the 1st and 2nd time derivatives of the position coordinates give the velocities and accelerations. the difference in placement between the beginning and the final state of two points in a body expresses the numerical value of strain. strain expresses itself as a change in size and/or shape.

me338 - syllabus

day    date    topic                                      chapters  pages
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tue    sep  25    why continuum mechanics?                 1.1-1.5    1-32
thu    sep  27    introduction to vectors and tensors     1.6-1.9    32-55
tue    oct  02    introduction to vectors and tensors     1.6-1.9    32-55
thu    oct  04    kinematics                             7.1-2.8    55-76
thu    oct  09    kinematics                             7.1-2.8    55-76
thu    nov  11    concept of stress                     3.1-3.4    109-131
tue    oct  16    balance principles                     4.1-4.4    131-161
thu    oct  18    balance principles                     4.5-4.7    161-179
tue    oct  23    aspects of objectivity                 5.1-5.4    179-205
thu    nov  25    hyperelastic materials                 6.1-6.3    205-222
tue    nov  30    hyperelastic materials                 6.3-6.5    222-252
thu    nov  01    hyperelastic materials                 6.6-6.8    252-278
tue    nov  06    hyperelastic materials                 6.9-6.11   278-305
thu    nov  08    thermodynamics of materials             7.1-7.6    305-337
tue    nov  13    midterm prep                            7.7-7.9    337-371
thu    nov  15    midterm                                7.7-7.9    337-371
thu    nov  27    thermodynamics of materials             8.1-8.3    371-392
thu    dec  04    variational principles                 8.4-8.6    392-414
thu    dec  06    final project discussion               8.4-8.6    392-414

kinematics is the study of motion per se, regardless of the forces causing it. the primitive concepts concerned are position, time and body, the latter abstracting into mathematical terms intuitive ideas about aggregations of matter capable of motion and deformation.

chadwick 'continuum mechanics' [1976]
motion

\[ x = \varphi(X, t) \quad \text{with} \quad \varphi : B_0 \times \mathbb{R} \to B_t \]

- spatial deformation map

\[ X = \Phi(x, t) \quad \text{with} \quad \Phi : B_t \times \mathbb{R} \to B_0 \]

- material deformation map

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koan 29, the mumon koan, 13th century

Two monks were arguing about the temple flag waving in the wind. One said, 'The flag moves.' The other said, 'The wind moves.' They argued back and forth but could not agree. The sixth ancestor said, 'Gentlemen! It is not the wind that moves; it is not the flag that moves; it is your mind that moves.' The two monks were struck with awe.
derivatives

- material time derivative of a material field
  \[ \dot{f}(X, t) = \frac{Df(X, t)}{Dt} = \left( \frac{\partial f(X, t)}{\partial t} \right) |_x \]
- material gradient of a material field
  \[ \text{Grad} f(X, t) = \frac{\partial f(X, t)}{\partial X} |_t \]

spatial time derivative of a spatial field
\[ \frac{\partial f(x, t)}{\partial t} |_x \]
- spatial gradient of a spatial field
\[ \text{grad} f(x, t) = \frac{\partial f(x, t)}{\partial x} |_t \]

04 - kinematics

spatial derivatives

- nonlinear deformation map \( \varphi \)
  \[ x = \varphi(X, t) \quad \text{with} \quad \varphi : B_0 \times \mathbb{R} \to B_t \]
- spatial derivative of \( \varphi \) - deformation gradient
  \[ dx = F \cdot dX \quad \text{with} \quad F : \partial B_0 \to \partial B_t \quad F = \frac{\partial \varphi}{\partial X} |_{x \text{ fixed}} \]

deformation gradient

- transformation of line elements - deformation gradient \( F_{ij} \)
  \[ dx = F_{ij} \cdot dX_j \quad \text{with} \quad F_{ij} : \partial B_0 \to \partial B_t \quad F_{ij} = \frac{\partial \varphi_i}{\partial X_j} |_{X \text{ fixed}} \]
- uniaxial tension (incompressible), simple shear, rotation
  \[ F_{\text{uni}} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha^{-\frac{1}{2}} & 0 \\ 0 & 0 & \alpha^{-\frac{1}{2}} \end{bmatrix}, \quad F_{\text{shr}} = \begin{bmatrix} 1 \gamma & 0 \\ 0 & 1 \end{bmatrix}, \quad F_{\text{rot}} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
04 - kinematics

strain tensor

- stretch \( \frac{1}{2} \left[ \lambda^2 - \lambda_0^2 \right] = \frac{1}{2} \left[ n \cdot n - n_0 \cdot n_0 \right] \)
  \[ = \frac{1}{2} \left[ n_0 \cdot F^t \cdot F \cdot n_0 - n_0 \cdot I \cdot n_0 \right] \]
  \[ = n_0 \cdot \frac{1}{2} \left[ F^t \cdot F - I \right] \cdot n_0 = n_0 \cdot E \cdot n_0 \]

- green-lagrange strain tensor
  \( E = \frac{1}{2} \left[ F^t \cdot F - I \right] \) with \( E_{AB} = \frac{1}{2} \left[ F_{aA} F_{bB} - \delta_{AB} \right] \)

04 - kinematics

stretch & deformation tensor

- stretch \( \lambda = |n| \) with \( n = F \cdot n_0 \) and \( |n_0| = 1 \)
  \[ \lambda^2 = n \cdot n = [F \cdot n_0] \cdot [F \cdot n_0] \]
  \[ = n_0 \cdot F^t \cdot F \cdot n_0 = n_0 \cdot C \cdot n_0 \]

- right Cauchy-Green deformation tensor
  \( C = F^t \cdot F \) with \( C_{AB} = F_{aA} F_{bB} \)
  \[ \det(C) = \det^2(F) = J^2 > 0 \]

04 - kinematics

strain tensor

- stretch \( \frac{1}{2} \left[ \lambda^2 - \lambda_0^2 \right] = \frac{1}{2} \left[ n \cdot n - n_0 \cdot n_0 \right] \)
  \[ = \frac{1}{2} \left[ n_0 \cdot F^t \cdot F \cdot n_0 - n_0 \cdot I \cdot n_0 \right] \]
  \[ = n_0 \cdot \frac{1}{2} \left[ F^t \cdot F - I \right] \cdot n_0 = n_0 \cdot E \cdot n_0 \]

- left Cauchy-Green deformation tensor / finger tensor
  \( b = F \cdot F^t \) with \( b_{ab} = F_{aA} F_{bA} \)
  \[ \det(b) = \det^2(F) = J^2 > 0 \]

04 - kinematics
strain tensor

• euler-almansi strain tensor and co-variant push forward

\[ e = \frac{1}{2} [I - F^{-t} \cdot F^{-1}] \quad \text{with} \quad e_{ab} = \frac{1}{2} \left[ \delta_{ab} - F_{Aa} F_{Ab}^{-1} \right] \]

\[ = F^{-t} \cdot \frac{1}{2} \left[ F^t \cdot [I - F^{-t} \cdot F^{-1}] \cdot F \right] \cdot F^{-1} \]

\[ = F^{-t} \cdot \frac{1}{2} \left[ F^t \cdot F - I \right] \cdot F^{-1} = F^{-t} \cdot E \cdot F^{-1} \]

\[ E = \frac{1}{2} [F^t \cdot F - I] \quad \text{with} \quad E_{AB} = \frac{1}{2} [F_{Aa} F_{AB} - \delta_{AB}] \]

04 - kinematics

example 01: interface elements

macroscopic approach: lump failure between elements

1d interface

2d interface

limitation: failure zone must be known a priori

utzinger, bos, fleck, menzel, kuhl, renz, friedrich, schl arb, steinmann [2008]

discontinuous kinematics

example 01: interface elements

thermal impact welded single lap tensile specimen

simulation with interfaces predicts characteristic behavior

utzinger, bos, fleck, menzel, kuhl, renz, friedrich, schl arb, steinmann [2008]

discontinuous kinematics

example 01: interface elements

simulation vs electronic speckle pattern interferometry

utzinger, bos, fleck, menzel, kuhl, renz, friedrich, schl arb, steinmann [2008]
NBC, Feb 27, 2007. A landslide overnight in San Francisco damaged an apartment building and left other structures in precarious situations. A wide swath of hillside came thundering down on a strip club and several apartment buildings in the city’s North Beach district Tuesday. At least 120 residents were displaced and several buildings declared off-limits as engineers tried to figure out how to stabilize the cliff to prevent further damage.
discontinuous kinematics

example 03: discontinuous elements

2d crack element

crack elements evolve dynamically

merghem, kuhl, steinmann, kuhl [2006]

example 03: discontinuous elements

nooru-mohamed test

jager, steinmann, kuhl [2008]
example 03: discontinuous elements

L-shaped specimen test

discontinuous kinematics

example 03: discontinuous elements

peel test & the chewing gum model

discontinuous kinematics

example 03: discontinuous elements

non-symmetric peel test
example 03: discontinuous elements

discontinuous kinematics

symmetric peel test

jager, steinmann, kuhl [2008]

example 03: discontinuous elements

brittle fracture during folding of rocks

jager, schmalholz, schmid, kuhl [2008]

discontinuous kinematics

example 03: discontinuous elements

jager, schmalholz, schmid, kuhl [2008]