ME 338A – Continuum Mechanics

Tue/Thu 11:00-12:15, 530-127

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Although the basic concepts of continuum mechanics have been established more than five decades ago, the 21st century faces many new and exciting potential applications of continuum mechanics that go way beyond the standard classical theory. When applying continuum mechanics to these challenging new phenomena, it is important to understand the main three ingredients of continuum mechanics: the kinematic equations, the balance equations and the constitutive equations. After a brief repetition of the relevant equations in tensor algebra and analysis, this class will introduce the basic concepts of small strain kinematics. We will then discuss the balance equations for mass, momentum, moment of momentum, energy and entropy. While all these equations are general and valid for any kind of material, the last set of equations, the constitutive equations, specifies particular subclasses of materials. Towards the end of the quarter, we will illustrate how the three sets of equations will change in nonlinear continuum mechanics. Throughout this class, we will discuss the mechanics of the heart with its strains, fiber stretches, stresses, forces, and constitutive equations, to illustrate how continuum mechanics can be applied to relevant clinical problems.

Grading

Homework	30 %	three homework assignments, 10% each
Midterm	40 %	closed book, closed notes, one single page cheat sheet
Final Project	30 %	written evaluation of a manuscript and its discussion in class

Tue 01/05 **Tensor calculus I – Vector algebra**
Scalar product
Vector product
Scalar triple product

Thu 01/07 **Tensor calculus II – Tensor algebra**
Scalar product
Dyadic product
Scalar triple product

Tue 01/12 **Tensor calculus III – Tensor algebra**
Invariants, trace, determinant, inverse
Spectral decomposition
Symmetric – skew-symmetric decomposition
Volumetric – deviatoric decomposition
Orthogonal tensors

Thu 01/14 **Tensor calculus IV – Tensor analysis**
Frechet and Gateaux derivatives
Gradient, Divergence, Laplace operator
Integral transformations – Green theorem, Gauss theorem

Tue 01/19 **Kinematics I – Motion**
Motion, rates and gradients of kinematic quantities
Symmetric – skew-symmetric decomposition
Strain and rotation

Thu 01/21 **Kinematics II – Strain**
Volumetric – deviatoric decomposition
Strain vector
Normal-shear decomposition
Principal strains – stretch
Compatibility
Plane strain

Tue 01/26 **Balance equations I – Contact fluxes**
Concept of mass and heat flux
Concept of stress
Cauchy's postulate, lemma, and theorem
Stress tensor

Thu 01/28 **Balance equations II – Concept of stress**
Volumetric – deviatoric decomposition
Normal – shear decomposition
Principal stresses
Plane stress

Tue 02/02 **Balance eqations III – Mass and momentum**
Global and local balance of mass
Global and local balance of momentum

Thu 02/04 **Balance equations IV – Angular momentum and energy**
Global and local balance of angular momentum
Global and local balance of energy
First law of thermodynamics

Tue 02/09 **Balance equations V - Entropy**
Global and local balance of entropy
Second law of thermodynamics
Global and local generic balance equation
Thermodynamic potentials

Thu 02/11 **Constitutive equations I – Linear equations**
Mass flux – Fick's law
Momentum flux – Hooke's law
Heat flux – Fourier's law

Tue 02/16 **Constitutive equations II - Hyperelasticity**
Specific stored energy
Specific complementary energy
Isotropic hyperelasticity

Thu 02/18 **Constitutive equations III – Isotropic hyperelasticity**
Specific stored energy
Volumetric – deviatoric decomposition
Specific complementary energy
Volumetric – deviatoric decomposition
Elastic constants

Tue 02/23 **Prep midterm / Problems**
Kinematics
Balance equations
Constitutive equations

Thu 02/25 **Midterm**

Tue 03/02 **Intro to nonlinear continuum mechanics I – Finite strain kinematics**
Motion, configurations
Velocity, acceleration
Geometric maps
Push forward and pull back

Thu 03/04 **Intro to nonlinear continuum mechanics II – Stress measures**
First and second Piola Kirchhoff, Cauchy, and Kirchhoff stresses
Closure problem
Frame invariance and material symmetry

Tue 03/09 **Intro to nonlinear continuum mechanics III – Constitutive equations**
Isotropic hyperelasticity
Incompressible isotropic elasticity, example: Ogden model
Transversely isotropic elasticity

Thu 03/11 **Journal club – Final project**
Final presentations / discussion
Written projects due