01 - motivation -
challenging problems in continuum mechanics
... what we do ...

kinematic equations for finite growth

\[ F = F_c \cdot F_g \]

balance equations for open systems

\[ D_t \rho_0 = \text{Div}(R) + \mathcal{R}_0 \]
\[ \rho_0 D_t \mathbf{v} = \text{Div}(P) + b_0 \]

constitutive equations for living tissues

\[ P = P(\rho_0, F, F_g) \]

fe analyses for biological structures

continuum- & computational biomechanics

... what we do ...
... why we do what we do ...

kinematic equations for finite growth

\[ F = F_c \cdot F_g \]

balance equations for open systems

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \text{Div}(\mathbf{R}) + \mathbf{R}_0 &= 0 \\
\rho_0 \frac{\partial \mathbf{u}}{\partial t} &= \text{Div}(\mathbf{P}) + \mathbf{b}_0
\end{align*}
\]

constitutive equations for living tissues

\[ \mathbf{P} = \mathbf{P}(\rho_0, F, F_g) \]

fe analyses for biological structures

... because biological structures are ...

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constitutive equations for living tissues

\[ \mathbf{P} = \mathbf{P}(\rho_0, \mathbf{F}, \ldots) \]

... because biological structures are ...

... why we do what we do ...

... what we do ...

kinematic equations for finite growth

\[ \mathbf{F} = \mathbf{F}_c \cdot \mathbf{F}_g \]

highly deformable

living

nonlinear

inelastic
... why we do what we do...

kinematic equations for finite growth
\[ F = F_c \cdot F_g \]

balance equations for open systems
\[ D_{\tau \rho} = \text{Div}(N) + R_0 \]
\[ \rho_0 \frac{\partial v}{\partial t} = \frac{1}{\rho} \text{Div}(N) \]

constitutive equations
\[ P = P(\rho_0, \mathbf{V}, \mathbf{E}) \]

FE analyses for biological structures

... because biological structures are ...

highly deformable
living
anisotropic
nonlinear
inelastic

... what we do ...
... why we do what we do...

kinematic equations for finite growth

\[ F = F_c \cdot F_g \]

balance equations for open systems

\[ D_{t\rho} = \text{Div}(\nabla) + \mathcal{R}_0 \]
\[ \rho_0 D_{\rho} = 0 \]

constitutive equations

\[ P = P(\rho_0, \nabla, \mathcal{R}_0) \]

FE analyses for biological structures

... because biological structures are...

- living
- highly deformable
- nonlinear
- inelastic
- inhomogeneous
- anisotropic

... what we do ...
“...dal che e manifesto, che chi volesse mantenere in un vastissimo gigante le proporzioni, che hanno le membra in un huomo ordinario, bisognerebbe o trouar materia molto più dura, e resistente per formarne l'ossa o vero ammettere, che la robustezza sua fusse a proporzione assai più fiacca, che negli huomini de statura mediocre; altrimente crescendogli a smisurata altezza si vedrebbono dal proprio peso opprimere, e cadere...”

Galileo, “Discorsi e dimostrazioni matematiche”, [1638]
history - 19th century

Culmann & von Meyer „Graphic statics“ [1867]
"...es ist demnach unter dem gesetze der transformation der knochen dasjenige gesetz zu verstehen, nach welchem im gefolge primaerer abaenderungen der form und inanspruchnahme bestimmte umwandlungen der inneren architectur und umwandlungen der aeusseren form sich vollziehen..."

Wolff „Gesetz der Transformation der Knochen“ [1892]
“...whether it be the sweeping eagle in his flight or the open apple-blossom, the toiling work-horse, the blithe swan, the branching oak, the winding stream at its base, the drifting clouds, over all the coursing sun, form ever follows function, and this is the law...”

Sullivan „Form follows function“ [1896]
the system consisting of only the porous structure without its entrained perfusant is open with respect to momentum transfer as well as mass, energy, and entropy transfer. We shall write balance and constitutive equations for only the bone…"

Cowin & Hegedus „Theory of adaptive elasticity“ [1976]
"...the relationship between physical forces and the morphology of living things has piqued the curiosity of every artist, scientist, or philosopher who has contemplated a tree or drawn the human figure. Its importance was a concern of Galileo and later Thompson whose writings remind us that physical causation plays an inescapable role in the development of biological form..."

Beaupré, Carter & Orr, "Theory of bone modeling & remodeling" [1990]
“hypertrophy of the heart: comparison of cross sections of a normal heart (bottom), a heart chronically overloaded by an unusually large blood volume (left) and a heart chronically overloaded by an unusually large diastolic and systolic left ventricular pressure (right)”

Fung „Biomechanics - Motion, flow, stress, and growth“ [1990]
"hypertrophy of the heart: histology of a normal heart (left) and pressure overloaded heart (right) photographed at the same magnification - muscles in the hypertropic heart (right) are much bigger in diameter than those of the normal heart (left)."

Fung „Biomechanics - Motion, flow, stress, and growth“ [1990]
...the process of growth can be seen as an evolution of material point neighbourhoods in a fixed reference configuration. The growth process will cause the development of material inhomogeneities responsible for residual stresses in the body..."

Epstein & Maugin „Theory of volumetric growth“ [2000]

Rodriguez, Holger & McCulloch [1994]
**different load cases**

**Table:**

<table>
<thead>
<tr>
<th>Case</th>
<th>Force (N)</th>
<th>Angle (°)</th>
<th>Force (N)</th>
<th>Angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1] midstance phase of gait</td>
<td>2317</td>
<td>24</td>
<td>703</td>
<td>28</td>
</tr>
<tr>
<td>[2] extreme range of abduction</td>
<td>1158</td>
<td>-15</td>
<td>351</td>
<td>-8</td>
</tr>
<tr>
<td>[3] extreme range of adduction</td>
<td>1548</td>
<td>56</td>
<td>468</td>
<td>35</td>
</tr>
</tbody>
</table>

*Carter & Beaupré [2001]*

**Example - adaptation in bone**
example - adaptation in bone

different load cases

combination of all load cases necessary

Carter & Beaupré [2001]
experiment vs simulation

- dense system of compressive trabaculæ carrying stress into calcar region
- secondary arcuate system, medial joint surface to lateral metaphyseal region
- ward’s triangle, low density region contrasting dense cortical shaft

Carter & Beaupré [2001]

example - adaptation in bone
comparison with x-rays

coxa vara

coxa norma

coxa valga

excellent agreement of simulation and x-ray pattern

example - adaptation in bone
total hip replacement vs hip resurfacing

- about 120,000 artificial hip replacements in US per year
- aseptic loosening caused by adaptive bone remodeling
- goal prediction of redistibution of bone density

example - hip replacement
conventional total hip replacement

stress shielding • bone resorption • implant loosening

example - hip replacement
new birmingham hip resurfacing

improved ingrowth • anatomic situation • less resorption

example - hip replacement
volume growth of aortic wall

normosensitive hypsersensitive severely hypersensitive

wall thickening - thickening of musculoelastic fascicles

Matsumoto & Hayashi [1996], Humphrey [2002]
volume evolution in cylindrical tube

t = 0  t = 10  t = 36  t = 100

stress-induced cross sectional growth

Himpel, Kuhl, Menzel & Steinmann [2006]

example - growth of aortic wall
qualitative simulation of stent implantation

<table>
<thead>
<tr>
<th>initial conditions</th>
<th>adiposis</th>
<th>calcification</th>
<th>growing plaque</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{\text{fat}} = 0.03E_{\text{tissue}} )</td>
<td>( E_{\text{lime}} = 3000E_{\text{tissue}} )</td>
<td>( E_{\text{plaque}} = E_{\text{tissue}} )</td>
<td></td>
</tr>
</tbody>
</table>

early stage
soft plaque pressed into wall

later stage
stiff plaque induces high stresses

overall thickening - thickening of individual fascicles

Holzapfel [2001], Holzapfel & Ogden [2003], Kuhl, Maas, Himpel & Menzel [2007]

text - growth of aortic wall
qualitative simulation of stent implantation

restenotic conditions

re-narrowing of x-section in response to high stress

example - growth of aortic wall
Example - growth of aortic wall

Kuhl, Maas, Himpel & Menzel [2007]
stent implantation - patient specific model

tissue growth-response to virtual stent implantation

Kuhl, Maas, Himpel & Menzel [2007]

example - growth of aortic wall
remodeling of collagen fibers - living tendon

- ex vivo engineered tendon shows characteristics of embryonic tendon
- remodeling of collagen fibers upon mechanical loading
- long term goal mechanically stimulated tissue engineering

Calve, Dennis, Kosnik, Baar, Grosh & Arruda [2004]
remodeling of collagen fibers - living tendon

- finite element simulation of functional adaptation in tendons
- wormlike chain model with initial random anisotropy
- analysis of fiber reorientation in uniaxial tension

Kuhl, Garikipati, Arruda & Grosh [2005]
remodeling of collagen fibers - living tendon

gradual fiber alignment with max principal stress

example - tissue engineering
remodeling of collagen fibers - living tendon

characteristic locking, remodeling & stiffening

example - tissue engineering
tangentially sectioned brain arteries

circularly polarized light micrographs

Finlay [1995]

example - arterial wall
tangentially sectioned brain arteries

example - arterial wall
remodeling of collagen fibers

stress driven functional adaptation

Kuhl & Holzapfel [2007]

example - arterial wall
remodeling of collagen fibers

stress driven functional adaptation

Kuhl & Holzapfel [2007]

example - arterial wall
sensitivity wrt driving force - stress vs strain

\[ \sigma = \lambda^\sigma_i \ n^\sigma_i \otimes n^\sigma_i \]

\[ b = \lambda^b_i \ n^b_i \otimes n^b_i \]

eigenvectors coincide but eigenvalues differ significantly

Kuhl & Holzapfel [2007]

example - arterial wall
sensitivity wrt pressure to stretch ratio

\[ \varepsilon_{zz} = 10\% \]
\[ p = 0 \]

<<< increase of stretch <<<

>>> increase of pressure >>>

\[ \varepsilon_{zz} = 0\% \]
\[ p = p_{\text{blood}} \]

collagen fiber angle governed by pressure\textsuperscript{2}stretch ratio

Kuhl & Holzapfel [2007]

example - arterial wall
heart disease

- primary cause of death in industrialized nations
- affects 80 mio americans, costs more than $430 billion
- damaged cardiac tissue does not self regenerate

can continuum mechanics help us to understand the heart?

mitral regurgitation
kinematic problem

example - mitral regurgitation
mitral regurgitation

- MV regulates unidirectional blood flow from LA to LV
- long-term, progressive leakage, back flow
- mitral regurgitation: 4 mio americans
- mitral valve repair: 300,000 worldwide / year
does annuloplasty ring shape affect leaflet curvature?

**hypothesis**

Leaflet curvature changes indicate repair failure

23 4d marker coordinates

goktepe, bothe, kvitting, swanson, ingels, miller, kuhl [2009]

example - mitral regurgitation
surgical annuloplasty ring implantation

courtesy wolfgang bothe, cardiothoracic surgery, stanford

example - mitral regurgitation
example - mitral regurgitation
videofluoroscopic markers & subdivision surfaces

box spline interpolation on patch
\[ \mathbf{x}(\theta^1, \theta^2) = \sum_{I=1}^{12} N_I(\theta^1, \theta^2) \mathbf{x}_I \]

covariant surface base vectors
\[ a_\alpha = \frac{\partial \mathbf{x}}{\partial \theta^\alpha} = \sum_{I=1}^{\text{nd}} \frac{\partial N_I(\vartheta)}{\partial \theta^\alpha} \mathbf{x}_I \quad a_3 = \frac{a_1 \times a_2}{|a_1 \times a_2|} \]

curvature tensor
\[ K_{\alpha\beta} = \frac{\partial a_\alpha}{\partial \theta^\beta} \cdot a_3 \]

\[ \alpha, \beta = 1, 2 \]

eigenvalue problem
\[ [ \mathbf{K} - \kappa \mathbf{G} ] \cdot \mathbf{n} = 0 \quad \kappa^2 - I_K \kappa + II_K = 0 \]

\text{example - mitral regurgitation}
does ring shape affect leaflet curvature?

goktepe, bothe, kvitting, swanson, ingels, miller, kuhl [2009]

example - mitral regurgitation
Does ring shape affect leaflet curvature?

Maximum opening, end diastole, end systole with ring:

Maximum opening, end diastole, end systole without ring:

Goktepe, Bothe, Kvitting, Swanson, Ingels, Miller, Kuhl [2009]

Example - Mitral regurgitation