1 ME338A - Homework

due 01/21/10, 12:15pm, 530-127

late homework can be dropped in a box in front of durand 217, please mark clearly with date and time @drop off, we will take off 1/10 of points for each 24 hours late

1.1 Index vs tensor notation

Write the following expressions

1.1 $t_i = \sigma_{ji}\, n_j$

1.2 $W = \frac{1}{2}\, \sigma_{ij}\, \varepsilon_{ij}$

1.3 $p = \sigma_{ii}$

1.4 $\sigma_{ij}^{\text{dev}} = 2\, \mu\, \varepsilon_{ij}^{\text{dev}}$

1.5 $\bar{\epsilon}_{ij}^{\text{dev}} = Q_{ki}\, \epsilon_{kl}\, Q_{lj}$

1.6 $S_{ij} = \frac{1}{2}[\, A_{ij} + A_{ji}\,]$

1.7 $\dfrac{\partial \sigma_{ij}}{\partial x_j} + \rho\, b_i = \rho\, \ddot{x}_i$

(i) in long form for each component $i, j = 1, 2, 3$ and
(ii) in compact tensor notation, and
(iii) try to find out what they mean.

1.2 Skalar and dyadic products

Given the second order tensor $\boldsymbol{A}$ and the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$,

$$
\begin{aligned}
\boldsymbol{A} &= A_{ij}\,\boldsymbol{e}_i \otimes \boldsymbol{e}_j & A_{ij} &= \begin{bmatrix} 4 & 1 & 0 \\ 0 & 3 & 1 \\ 2 & -1 & 5 \end{bmatrix} \\
\boldsymbol{u} &= u_i\,\boldsymbol{e}_i & u_i &= [\,4,\ 1,\ 2\,]^{\mathrm{t}} \\
\boldsymbol{v} &= v_i\,\boldsymbol{e}_i & v_i &= [\,1,\ 3,\ 1\,]^{\mathrm{t}}
\end{aligned}
$$

2.1 determine the expression $\boldsymbol{u} \cdot \boldsymbol{A} \cdot \boldsymbol{v}$

2.2 determine the expression $\operatorname{tr}([\boldsymbol{A} \cdot \boldsymbol{v}] \otimes \boldsymbol{u})$

2.3 determine the expression $\boldsymbol{v} \cdot \boldsymbol{A}^{\mathrm{t}} \cdot \boldsymbol{u}$

2.4 determine the expression $[\boldsymbol{v} \otimes \boldsymbol{u}] : \boldsymbol{A}^{\mathrm{t}}$

2.5 determine the expression $\boldsymbol{A} : [\boldsymbol{u} \otimes \boldsymbol{v}]$

2.6 what is special about the second order tensor $[\boldsymbol{u} \otimes \boldsymbol{v}]$ and what does that imply for its inverse $[\boldsymbol{u} \otimes \boldsymbol{v}]^{-1}$

2.7 find at least one more expression for $\boldsymbol{u} \cdot \boldsymbol{A} \cdot \boldsymbol{v}$

2.8 use the index notation to show that the expressions [2.1] to [2.5] are identical for arbitrary $\boldsymbol{A}$, $\boldsymbol{u}$, and $\boldsymbol{v}$

1.3 Volumetric-deviatoric decomposition

Given the second order tensor $\boldsymbol{A}$,

$$\boldsymbol{A} = A_{ij}\boldsymbol{e}_i \otimes \boldsymbol{e}_j \qquad [A]_{ij} = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 2 & -1 \\ 2 & -1 & 4 \end{bmatrix}$$

determine

3.1 the first invariant $I_A = \text{tr}(\boldsymbol{A})$

3.2 the second invariant $II_A = \frac{1}{2}\,[\,\text{tr}^2(\boldsymbol{A}) - \text{tr}(\boldsymbol{A}^2)\,]$

3.3 the third invariant $III_A = \det(\boldsymbol{A})$

3.4 the volumetric part $\boldsymbol{A}^{\text{vol}}$

3.5 the deviatoric part $\boldsymbol{A}^{\text{dev}}$

1.4 Scalar products

Given the second order tensor

$$\sigma_{ij} = 2\,\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\,\delta_{ij}$$

4.1 show that $W = \frac{1}{2}\,\sigma_{ij}\varepsilon_{ij} = \mu\,\varepsilon_{ij}\,\varepsilon_{ij} + \frac{1}{2}\,\lambda\,\varepsilon_{kk}^2$

4.2 show that $p = \sigma_{ij}\,\sigma_{ij} = 4\,\mu^2\,\varepsilon_{ij}\,\varepsilon_{ij} + \varepsilon_{kk}^2\,[4\,\mu\,\lambda + 3\,\lambda^2]$

4.3 rewrite [4.1] and [4.2] in compact tensor notation

4.4 try to find a physical interpretation for σ, ε and W

1.5 Symmetric-skew-symmetric decomposition

Given the second order tensor $\boldsymbol{A}$,

$$\boldsymbol{A} = A_{ij}\boldsymbol{e}_i \otimes \boldsymbol{e}_j \qquad [A]_{ij} = \begin{bmatrix} 5 & 6 & 2 \\ 2 & 5 & -3 \\ -2 & 3 & 4 \end{bmatrix}$$

5.1 determine its symmetric part $\boldsymbol{S} = \boldsymbol{A}^{\text{sym}}$

5.2 determine its skew symmetric part $\boldsymbol{W} = \boldsymbol{A}^{\text{skw}}$

5.3 show that $\boldsymbol{S} : \boldsymbol{W} = 0$

5.4 determine the axial vector $\boldsymbol{w}$ of its skew symmetric part $\boldsymbol{W}$

5.5 determine the square $\boldsymbol{S}^2$ of its symmetric part $\boldsymbol{S}$

5.6 determine the square root $\sqrt{\boldsymbol{S}}$ of its symmetric part $\boldsymbol{S}$ (hint: one of the eigenvalues of $\boldsymbol{S}$ is $\lambda_{S1} = 1$)

5.7 control your results with $(\sqrt{\boldsymbol{S}})^2 = \boldsymbol{S}$

you may use a program like matlab to control your results, but the derivations must be clearly documented on paper