

BIOMECH PREPRINT SERIES

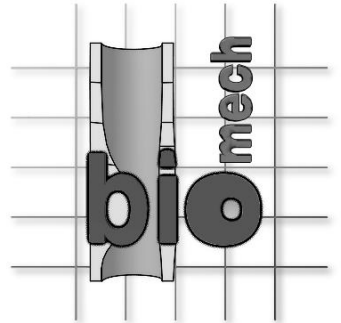
Paper No. 7

November 2000

Biomechanics of Soft Tissue

G.A. Holzapfel

START-Project Y74-TEC



**COMPUTATIONAL
BIOMECHANICS**

Schiesstattgasse 14B

A - 8010 Graz, Austria

<http://www.cis.tu-graz.ac.at/biomech>

Institute for
Structural Analysis

Graz University
of Technology

Biomechanics of Soft Tissue¹

Gerhard A. Holzapfel

Graz University of Technology
Institute for Structural Analysis – Computational Biomechanics
Schießstattgasse 14-B
A-8010 Graz, Austria

Appeared by Academic Press in the

HANDBOOK OF MATERIAL BEHAVIOR
Nonlinear Models and Properties

edited by Jean Lemaitre, LMT-Cachan, France

TABLE OF CONTENTS

1. Validity
2. Background on the structure of soft tissues – collagen and elastin
3. General mechanical characteristic of soft tissues
4. Description of the model
5. Representative example: A model for the artery
6. Identification of the material parameters
7. How to use it
8. Table of parameters
9. References

¹Financial support for this research was provided by the *Austrian Science Foundation (FWF)* under *START-Award Y74-TEC*.

Biomechanics of Soft Tissue

by Gerhard A. Holzapfel
Institute for Structural Analysis
Computational Biomechanics
Graz University of Technology
8010 GRAZ – AUSTRIA

1. Validity

An efficient constitutive formulation approximates all types of soft tissues with a reasonable accuracy over a large strain range. We request a simple constitutive equation with only a few material parameters involved that allow an ‘explanation’ of the material response of tissues in terms of their structure. In addition, we request that the constitutive formulation is fully three-dimensional and consistent with both mechanical and mathematical requirements, applicable for arbitrary geometries and suitable for use within the context of finite element methods in order to solve complex initial boundary-value problems.

The presented general model is a fully three-dimensional material description of soft tissues for which nonlinear continuum mechanics is used as the fundamental basis [10], [18]. It has the special feature that it is based partly on histological information (i.e. the microscopic structure of organs and tissues). The general model describes the highly nonlinear and anisotropic behavior of soft tissues as composites reinforced by two families of collagen fibers. The constitutive framework is based on the theory of the mechanics of fiber-reinforced composites [26] and is suitable to describe a wide variety of physical phenomena of soft tissues. The performance and the physical mechanism of the model is presented in [11]. As a representative example, the general model for soft tissues is specified to predict the mechanical response of *healthy* and *young* arteries under physiological loading conditions [12]. The model neglects active components, i.e. contracting elements with biochemical energy supply which are controlled by biological mechanisms, and is concerned with the description of the *passive* state of arteries.

The models are suitable for predicting the anisotropic *elastic* response of soft tissues in the large strain domain. A suitable constitutive and numerical model, which is general enough to describe the finite *viscoelastic* domain, is documented in [11]. The presented models do not consider acute and long-term changes in geometry and/or the mechanical response of tissues due to, for example, drugs, ageing and disease. When soft tissues are subjected to loads that are beyond the physiological range the load-carrying fibers of the tissue slip relative to each other. In clinical procedures tissues may undergo irreversible (plastic) deformations [12] which are of medical importance. Constitutive equations for describing plastic deformations of, for example, arteries are proposed in [27], [8].

2. Background on the structure of soft tissues – collagen and elastin

What do we mean by soft tissues? A primary group of tissue which binds, supports and protects our human body and structures such as organs is *soft connective tissue*. In contrary to other tissues, it is a wide-ranging biological material in which the cells are separated by extracellular material.

Connective tissues may be distinguished from hard (mineralized) tissues such as bones for their high flexibility and their soft mechanical properties. In this article we are mainly concerned to say something about it from the points of view of material science, biomechanics and structural engineering (for more details see, for example, [6], Chapter 7).

Examples for soft tissues are *tendons*, *ligaments*, *blood vessels*, *skins* or *articular cartilages* among many others. Tendons are muscle-to-bone linkages to stabilize the bony skeleton (or to produce motion), while ligaments are bone-to-bone linkages to restrict relative motion. Blood vessels are prominent organs composed of soft tissues which have to distend in response to pulse waves. The skin is the largest single organ (16% of the human adult weight). It supports internal organs and protects our body. Articular cartilages form the surface of body joints (which is a layer of connective tissue with a thickness of 1-5 mm) and distribute loads across joints and minimize contact stresses and friction.

Soft connective tissues of our body are complex fiber-reinforced composite structures. Their mechanical behavior is strongly influenced by the concentration and structural arrangement of constituents such as *collagen* and *elastin*, the hydrated matrix of *proteoglycans*, and the topographical site and respective function in the organism.

Collagen. Collagen is a protein which is a major constituent of the extracellular matrix of connective tissue. It is the main load carrying element in a wide variety of soft tissues and is very important to human physiology (for example, the collagen content of (human) achilles tendon is about 20 times that of elastin).

Collagen is a macromolecule with length of about 280 nm. Collagen molecules are linked to each other by covalent bonds building collagen fibrils. Depending on the primary function and the requirement of strength of the tissue the diameter of collagen fibrils varies (the order of magnitude is 1.5 nm [17]). In the structure of tendons and ligaments, for example, collagen appears as parallel oriented fibers [1], while many other tissues have an intricate disordered network of collagen fibers embedded in a gelatinous matrix of proteoglycans.

More than 12 types of collagen have been identified [17]. The most common collagen is type I, which can be isolated from any tissue. It is the major constituent in blood vessels. The rod-like shape of the collagen molecule comes from three polypeptide chains which are composed in a right-handed triple-helical conformation. Most of the collagen molecule consists of three amino acids; glycine (33%), which enhances the stability of the molecule, proline (15%) and hydroxyproline (15%) [23].

The intramolecular crosslinks of collagen gives the connective tissues the strength which varies with age, pathology, etc. (for a correlation between the collagen content in the tissue, % dry weight, and its ultimate tensile strength see Table 1). The function and integrity of organs are maintained by the *tension* in collagen fibers. They shrink upon heating due to breakdown of the crystalline structure (at 65°C, for example, mammalian collagen shrinks to about one-third of its initial length, [6], p. 263).

Material	Ultimate tensile strength [Mpa]	Ultimate tensile strain [%]	Collagen (% dry weight)	Elastin (% dry weight)
Tendon	50-100	10-15	75-85	< 3
Ligament	50-100	10-15	70-80	10-15
Aorta	0.3-0.8	50-100	25-35	40-50
Skin	1-20	30-70	60-80	5-10
Articular Cartilage	9-40	60-120	40-70	-

Table 1: Mechanical properties [25], [6], [15] and associated biochemical data [30] of some representative organs mainly consisting of soft connective tissues.

Elastin. Elastin, like collagen, is a protein which is a major constituent of the extracellular matrix of connective tissue. It is present as thin strands in soft tissues such as skin, lung, ligamenta flava of the spine and ligamentum nuchae (the elastin content of the latter is about 5 times that of collagen).

The long flexible elastin molecules build up a three-dimensional (rubber-like) network, which may be stretched to about 2.5 of the initial length of the unloaded configuration. In contrast to collagen fibers, this network does not exhibit a pronounced hierarchical organization. As for collagen, 33% of the total amino acids of elastin consists of glycine. However, the proline and hydroxyproline contents are much lower than in collagen molecules.

The mechanical behavior of elastin may be explained within the concept of entropic elasticity. As for rubber, the random molecular conformations, and hence the entropy, change with deformation. Elasticity arises through entropic straightening of the chains, i.e. a decrease of entropy, or an increase of internal energy (see, for example, [9], [10], Chapter 7.1). Elastin is essentially a linearly elastic material (tested for the ligamentum nuchae of cattle). It displays very small relaxation effects (they are larger for collagen).

3. General mechanical characteristic of soft tissues

Before describing a model for soft tissues it is beneficial and instructive to give some insight in their general mechanical characteristic. Soft tissues behave anisotropically because of their fibers which tend to have preferred directions. In a microscopic sense they are non-homogeneous materials because of their composition. The tensile response of soft tissue is nonlinear stiffening and tensile strength depends on the strain rate. In contrast to hard tissues, soft tissues may undergo large deformations. Some soft tissues show viscoelastic behavior (relaxation and/or creep), which has been associated with the shear interaction of collagen with the matrix of proteoglycans [16] (the matrix provides a viscous lubrication between collagen fibrils).

In a simplified way we explain here the tensile stress-strain behavior for skin, an organ consisting mainly of connective tissues, which is representative of the mechanical behavior of many (collagenous) soft connective tissues. For the connective tissue parts of the skin the three-dimensional network of fibers appears to have preferred directions parallel to the surface. However, in order to prevent out-of-plane shearing, some fiber orientations also have components out-of-plane.

Figure 1 shows a schematic diagram of a typical J-shaped (tensile) stress-strain curve for skin.

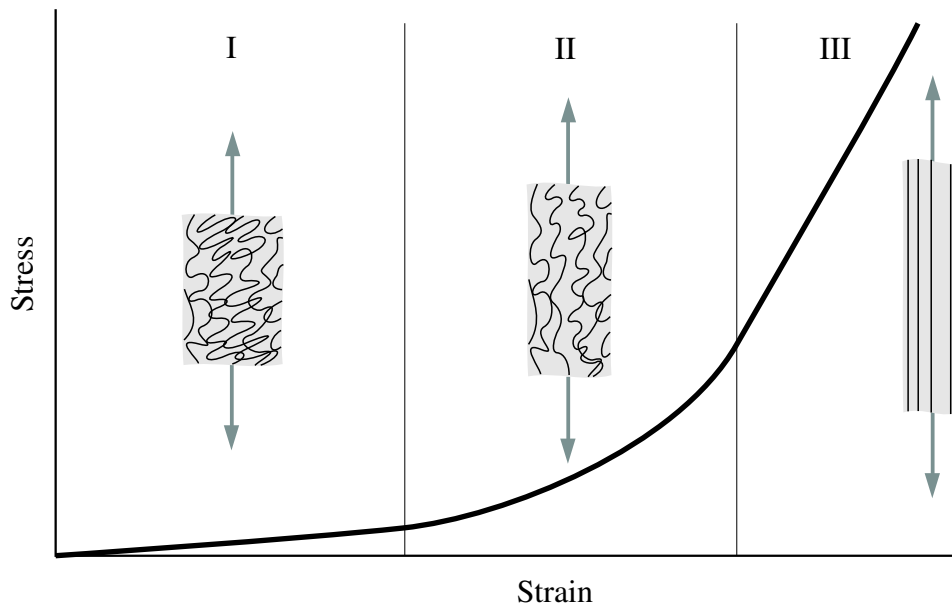


Figure 1: Schematic diagram of a typical (tensile) stress-strain curve for skin showing the associated collagen fiber morphology.

This form, representative for many soft tissues, differs significantly from stress-strain curves of hard tissues or from other types of (engineering) materials. In addition, Figure 1 shows how the collagen fibers straighten with increasing stress.

The deformation behavior for skin may be studied in three phases I, II and III:

Phase I. In the absence of load the collagen fibers, which are woven into rhombic-shaped pattern, are in relaxed conditions and appear wavy and crimped. Unstretched skin behaves approximately isotropically. Initially low stress is required to achieve large deformations of the individual collagen fibers without requiring stretch of the fibers. In phase I the tissue behaves like a very soft (isotropic) rubber sheet, and the elastin fibers (which keep the skin smooth) are mainly responsible for the stretching mechanism. The stress-strain relation is approximately linear, the elastic modulus of skin in phase I is low (0.1-2 MPa).

Phase II. In phase II, as the load is increased, the collagen fibers tend to line up with the load direction and bear loads. The crimped collagen fibers gradually elongate and they interact with the hydrated matrix. With deformation the crimp angle in collagen fibrils leads to a sequential uncrimping of fibrils. Note, that the skin is normally under tension *in vivo*.

Phase III. In phase III, at high tensile stresses, the crimp patterns disappear and the collagen fibers become straighter. They are primarily aligned with one another in the direction in which the load is applied. The straightened collagen fibers resist the load strongly and the tissue becomes stiff at higher stresses. The stress-strain relation becomes linear again. Beyond the third phase the ultimate tensile strength is reached and fibers begin to break.

The mechanical properties of soft tissues depend strongly on the topography, risk factors, age, species, physical and chemical environmental factors such as temperature, osmotic pressure, pH, and on the strain rate. The material properties are strongly related to the quality and completeness of experimental data, which come from *in vivo* or *in vitro* tests having the aim of mimicking real

loading conditions. Therefore, to present specific values for the ultimate tensile strength and strain of a specific tissue is a difficult task. Nevertheless, Table 1 attempts to present ranges of values of mechanical properties and collagen/elastin contents (% dry weight) in some representative organs mainly consisting of soft connective tissues.

4. Description of the model

At any referential position \mathbf{X} of the tissue we postulate the existence of a Helmholtz free-energy function Ψ . We assume the *decoupled* form

$$\Psi = U(\mathbf{X}; J) + \bar{\Psi}(\mathbf{X}; \bar{\mathbf{C}}, \mathbf{A}_1, \mathbf{A}_2), \quad (1)$$

where U is a purely *volumetric* (dilatational) contribution and $\bar{\Psi}$ is a purely *isochoric* (volume-preserving) contribution to the free energy Ψ . Here $\bar{\mathbf{C}} = \bar{\mathbf{F}}^T \bar{\mathbf{F}}$ denotes the modified right Cauchy-Green tensor and $\bar{\mathbf{F}} = J^{-1/3} \mathbf{F}$ is the unimodular (distortional) part of the deformation gradient \mathbf{F} , with $J = \det \mathbf{F} > 0$ denoting the local volume ratio. In addition, in eq. (1), $\{\mathbf{A}_1, \mathbf{A}_2\}$ is a set of two (second-order) tensors which characterize the anisotropic properties of the tissue at any \mathbf{X} . The structure tensors \mathbf{A}_1 and \mathbf{A}_2 are defined as the tensor products $\mathbf{a}_{0i} \otimes \mathbf{a}_{0i}$, where \mathbf{a}_{0i} , $i = 1, 2$, are two unit vectors characterizing the orientations of the families of collagen fibers in the (undeformed) reference configuration of the tissue (see Figure 2).

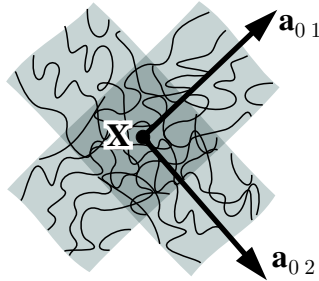


Figure 2: Arrangement of collagen fibers in the reference configuration characterized by two unit vectors \mathbf{a}_{01} , \mathbf{a}_{02} at position \mathbf{X} .

Since most types of soft tissues are regarded as incompressible (for example, arteries do not change their volume within the physiological range of deformation [2]) we now focus attention on the description of their isochoric deformation behavior characterized by the energy function $\bar{\Psi}$. We suggest the simple additive split

$$\bar{\Psi} = \bar{\Psi}_{\text{iso}}(\mathbf{X}; \bar{I}_1) + \bar{\Psi}_{\text{aniso}}(\mathbf{X}; \bar{I}_4, \bar{I}_6) \quad (2)$$

of $\bar{\Psi}$ into a part $\bar{\Psi}_{\text{iso}}$ associated with *isotropic* deformations and a part $\bar{\Psi}_{\text{aniso}}$ associated with *anisotropic* deformations. This is sufficiently general to capture the salient mechanical feature of soft tissue elasticity as described in Section 3 (a more general constitutive framework is presented

in [8], [11], [12]). In relation (2) we used $\bar{I}_1 = \bar{\mathbf{C}} : \mathbf{I}$ for the first invariant of tensor $\bar{\mathbf{C}}$ (\mathbf{I} is the second-order unit tensor), and the definitions

$$\bar{I}_4(\bar{\mathbf{C}}, \mathbf{a}_{01}) = \bar{\mathbf{C}} : \mathbf{A}_1, \quad \bar{I}_6(\bar{\mathbf{C}}, \mathbf{a}_{02}) = \bar{\mathbf{C}} : \mathbf{A}_2 \quad (3)$$

of the invariants, which are stretch measures for the two families of collagen fibers (see, for example, [26], [10]). The invariants \bar{I}_4 and \bar{I}_6 are squares of the stretches in the directions of \mathbf{a}_{01} and \mathbf{a}_{02} , respectively. Isotropy is described through the invariant \bar{I}_1 and anisotropy through \bar{I}_4 and \bar{I}_6 .

Since the (wavy) collagenous structure of tissues is not active at low stresses (it does not store strain energy) we associate $\bar{\Psi}_{\text{iso}}$ with the mechanical response of the non-collagenous matrix of the material (which is less stiff than its elastin fiber constituent). To determine the non-collagenous matrix response we propose to use the isotropic neo-Hookean model according to

$$\bar{\Psi}_{\text{iso}} = \frac{c}{2}(\bar{I}_1 - 3), \quad (4)$$

where $c > 0$ is a stress-like material parameter. However, to model the (isotropic) non-collagenous matrix material any Ogden-type elastic material may be applied [18].

According to morphological findings at highly-loaded tissues the families of collagen fibers become straighter and the resistance to stretch is almost entirely due to collagen fibers (the tissue becomes stiff). Hence, the strain energy stored in the collagen fibers is taken to be governed by the polyconvex (anisotropic) function

$$\bar{\Psi}_{\text{aniso}} = \frac{k_1}{2k_2} \{ \exp[k_2(\bar{I}_4 - 1)^2] - 1 \} + \frac{k_3}{2k_4} \{ \exp[k_4(\bar{I}_6 - 1)^2] - 1 \}, \quad (5)$$

where $k_1 > 0, k_3 > 0$ are stress-like material parameters and $k_2 > 0, k_4 > 0$ are dimensionless parameters. According to relations (2), (4), (5), the collagen fibers do not influence the mechanical response of the tissue in the low stress domain. Due to the crimp structure of collagen fibers we assume that they do not support compressive stresses which implies that they are inactive in compression. Hence the relevant part of the anisotropic function (5) is omitted for this case. If, for example, $\bar{I}_4 \leq 1$ and $\bar{I}_6 \leq 1$, then the soft tissue responds similarly to a rubber-like (purely isotropic) material described by the energy function (4). However, in extension, that is when $\bar{I}_4 > 1$ or $\bar{I}_6 > 1$, the collagen fibers are active and energy is stored in the fibers.

Function (1) enables the Cauchy stress tensor, denoted $\boldsymbol{\sigma}$, to be derived in the *decoupled* form

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_{\text{vol}} + \bar{\boldsymbol{\sigma}} \quad \text{with} \quad \boldsymbol{\sigma}_{\text{vol}} = p\mathbf{I}, \quad \bar{\boldsymbol{\sigma}} = 2J^{-1} \text{dev} \left(\bar{\mathbf{F}} \frac{\partial \bar{\Psi}}{\partial \bar{\mathbf{C}}} \bar{\mathbf{F}}^T \right), \quad (6)$$

with the *volumetric* contribution $\boldsymbol{\sigma}_{\text{vol}}$ and the *isochoric* contribution $\bar{\boldsymbol{\sigma}}$ to the Cauchy stresses. In the stress relation (6), $p = dU/dJ$ denotes the hydrostatic pressure and $\text{dev}(\bullet)$ furnishes the deviatoric operator in the Eulerian description. The operator is defined as $\text{dev}(\bullet) = (\bullet) - \frac{1}{3}[(\bullet) : \mathbf{I}]\mathbf{I}$, so that $\text{dev}(\bullet) : \mathbf{I} = 0$.

Using the additive split (2) and particularizations (4), (5), we get with (6)₃ an explicit constitutive expression for the isochoric behavior of soft connective tissues in the Eulerian description, i.e.

$$\bar{\boldsymbol{\sigma}} = c \text{dev} \bar{\mathbf{b}} + \sum_{i=4,6} 2\bar{\Psi}_i \text{dev}(\mathbf{a}_i \otimes \mathbf{a}_i), \quad (7)$$

where $\bar{\mathbf{b}} = \overline{\mathbf{F}\mathbf{F}^T}$ denotes the modified left Cauchy-Green tensor,

$$\bar{\Psi}_4 = \frac{\partial \bar{\Psi}_{\text{aniso}}}{\partial \bar{I}_4} = k_1(\bar{I}_4 - 1) \{ \exp[k_2(\bar{I}_4 - 1)^2] - 1 \}, \quad (8)$$

$$\bar{\Psi}_6 = \frac{\partial \bar{\Psi}_{\text{aniso}}}{\partial \bar{I}_6} = k_3(\bar{I}_6 - 1) \{ \exp[k_4(\bar{I}_6 - 1)^2] - 1 \} \quad (9)$$

are (scalar) response functions and $\mathbf{a}_i = \bar{\mathbf{F}}\mathbf{a}_{0i}$, $i = 1, 2$, are the Eulerian counterparts of the unit vectors \mathbf{a}_{0i} .

The specific form of the proposed constitutive equation (7) requires the *five* material parameters c, k_1, k_2, k_3, k_4 whose interpretations can be partly based on the underlying histological structure, i.e. matrix and collagen of the tissue. Note that in (7), orthotropic ($k_1 = k_3, k_2 = k_4$), transversely isotropic ($k_1 = 0$ or $k_3 = 0$) and isotropic hyperelastic descriptions ($k_1 = k_3 = 0$) at finite strains are included as special cases.

5. Representative example: A model for the artery

In this section we describe a model for the *passive* state of the *healthy* and *young* artery (no pathological changes in the intima, which is the innermost arterial layer frequently affected by atherosclerosis) suitable for predicting three-dimensional distributions of stresses and strains under physiological loading conditions with reasonable accuracy. It is a specification of the constitutive framework for soft tissues stated in previous section. For an adequate model of arteries incorporating the active state (contraction of smooth muscles) see [22]. For a detailed study of the mechanics of arterial walls see the extensive review [13].

Experimental tests show that the elastic properties of the media (middle layer of the artery) and adventitia (outermost layer of the artery) are significantly different [31]. The media is much stiffer than the adventitia. In particular, in the unloaded configuration the mean value of Young's modulus for the media, for several pig thoracic aortas, is about an order of magnitude higher than that of the adventitia [32]. In addition, the arterial layers have different physiological tasks, and hence the artery is modeled as a thick-walled elastic circular tube consisting of two layers corresponding to the media and adventitia. In a young non-diseased artery the intima (innermost layer of the artery) exhibits negligible wall-thickness and mechanical strength.

Each tissue layer is treated as a composite reinforced by two families of collagen fibers which are symmetrically disposed with respect to the cylinder axis. Hence, each tissue layer is considered as *cylindrically orthotropic* (already postulated in the early work [20]) so that a tissue layer behaves like a so-called balanced angle-ply laminate. We use the *same* forms of strain-energy functions (4), (5) for each tissue layer (each layer responds with similar mechanical characteristics) but use a *different* set of material parameters. Hence, eq. (2) takes on the specified form

$$\bar{\Psi}_M = \frac{c_M}{2}(\bar{I}_1 - 3) + \frac{k_{1M}}{2k_{2M}} \sum_{i=4,6} \{ \exp[k_{2M}(\bar{I}_i - 1)^2] - 1 \}, \quad (10)$$

$$\bar{\Psi}_A = \frac{c_A}{2}(\bar{I}_1 - 3) + \frac{k_{1A}}{2k_{2A}} \sum_{i=4,6} \{ \exp[k_{2A}(\bar{I}_i - 1)^2] - 1 \}. \quad (11)$$

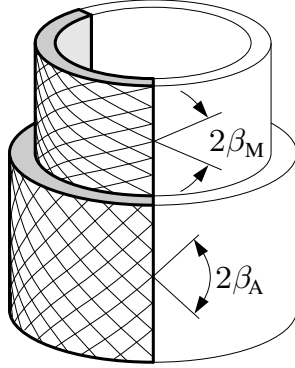


Figure 3: Load-free configuration of an idealized artery modeled as a thick-walled circular tube consisting of two layers, i.e. the media and adventitia.

We end up with a two-layer model incorporating *six* material parameters, three for the media M, i.e. c_M, k_{1M}, k_{2M} , and three for the adventitia A, i.e. c_A, k_{1A}, k_{2A} .

The invariants, associated with the anisotropic parts of the two tissue layers are defined by $\bar{I}_{4j} = \bar{\mathbf{C}} : \mathbf{A}_{1j}$ and $\bar{I}_{6j} = \bar{\mathbf{C}} : \mathbf{A}_{2j}$, $j = M, A$. The structure tensors $\mathbf{A}_{1j}, \mathbf{A}_{2j}$ are given by

$$\mathbf{A}_{1j} = \mathbf{a}_{01j} \otimes \mathbf{a}_{01j}, \quad \mathbf{A}_{2j} = \mathbf{a}_{02j} \otimes \mathbf{a}_{02j}, \quad j = M, A. \quad (12)$$

Employing a cylindrical coordinate system, the components of the unit (direction) vectors \mathbf{a}_{01j} and \mathbf{a}_{02j} read in matrix notation

$$[\mathbf{a}_{01j}] = \begin{bmatrix} 0 \\ \cos \beta_j \\ \sin \beta_j \end{bmatrix}, \quad [\mathbf{a}_{02j}] = \begin{bmatrix} 0 \\ \cos \beta_j \\ -\sin \beta_j \end{bmatrix}, \quad j = M, A, \quad (13)$$

and β_j , $j = M, A$, are the angles between the collagen fibers and the circumferential direction in the media and adventitia (see Figure 3). Small components of the (collagen) fiber orientation in the radial direction, as, for example, reported for human brain arteries [5], are neglected.

Residual stresses. It has been known for some years that arteries which are excised from the body and not subjected to any loads are *not* stress-free (or strain-free) [28]. If, for example, the media and adventitia are separated and cut in a radial direction the two arterial layers will spring open to form open (stress-free) sectors, which, in general, have different opening angles (see, for example, the experimental studies [29] for bovine specimens). In general, the residual stress-state is very complex, and residual stresses (strains) in the axial direction may also occur. Note that residual stresses result from growth and remodelling mechanisms [24], [21].

By considering the arterial layers as circular cylindrical tubes we may characterize the reference (stress-free) configuration of one arterial layer as a circular sector, as shown in Figure 4. For each

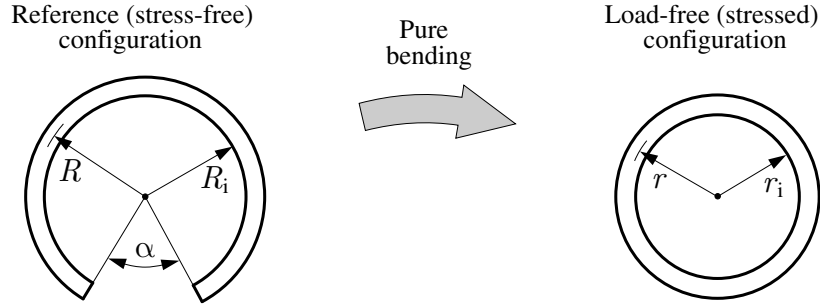


Figure 4: Cross-sectional representation of one arterial layer at the reference (stress-free) and load-free (stressed) configurations.

arterial layer of the blood vessel a certain opening angle α can be found by experimental methods. The importance of incorporating residual stresses associated with the load-free (but stressed) configuration into the computation has been emphasized in, for example, [4], [12]. Considerations of residual strains has a strong influence on the global pressure/radius response of arteries and also on the stress and strain distributions across the deformed arterial wall. For analytical studies of residual stresses see, for example, the works [14], [22], which contain further references.

Therefore, it is essential to incorporate the residual stresses inherent in many biologic tissues. One possible approach to consideration of the influence of residual stresses on the overall three-dimensional stress behavior is to measure the strain energy from the load-free (stressed) configuration and to include the residual stresses [19]. Another approach is to start with the energy function relative to the stress-free (and fixed) configuration, as assumed in the presented models, and determine the deformation required to reach the load-free (stressed) configuration. Figure 4 shows the cross-sectional representation of one arterial layer at the load-free configuration obtained from the reference configuration by pure bending.

With the condition of incompressibility, the radius r of an arterial layer in the load-free configuration may be computed from the radius R of the associated reference configuration as [12]

$$r = \sqrt{\frac{R^2 - R_i^2}{k\lambda_z} + r_i^2}, \quad k = \frac{2\pi}{2\pi - \alpha}, \quad (14)$$

where r_i , R_i are the internal radii associated with the two configurations. The (constant) axial stretch is denoted by λ_z and the parameter k is a convenient measure of the tube opening angle in the stress-free configuration.

6. Identification of the material parameters

Preferred directions in soft tissues are well specified by the orientation of prolate cell nuclei. They can be identified in microphotographs of appropriately stained histological sections. By visual inspection there exists a high directional correlation between smooth muscle cells and collagen

fibers. Hence, the (bell-shaped distribution of) collagen fiber orientations may be obtained from an image processing analysis of stained histological sections. The angle β (and thus the unit vectors $\mathbf{a}_{01}, \mathbf{a}_{02}$) may be identified as the mean value of the corresponding statistical distribution.

Values of the material parameters associated with the model for soft tissues are then obtained by fitting the equations to the experimental data of the soft tissue of interest by using standard nonlinear fitting algorithms, such as the Levenberg-Marquardt algorithm. For the case that the mean values of the orientation of cell nuclei (collagen fiber) may not be identified experimentally, it is suggested to treat the collagen fiber orientations as additional (phenomenological) ‘material’ parameters.

7. How to use it

The energy functions are well-suited for use in nonlinear finite element software, which enables complex boundary-value problems to be solved. Aspects of finite element implementation and numerical analysis of the model are presented in [11]. Furthermore, computations may be carried out with some of the commercially available mathematical software-packages such as *Mathematica* or *Maple*, which allow symbolic computation. Based on *Mathematica*, in [12] a numerical technique for solving the bending, axial extension, inflation and torsion problem of an artery is described.

8. Table of parameters

Values of the parameters correspond to the functions (10), (11) and are given for a representative carotid artery from a rabbit (experiment no. 71 in [7]). The material parameters c, k_1, k_2 and the angles of collagen fibers β are summarized in Table 2.

Media	Adventitia
$c_M = 3.0000[\text{kPa}]$	$c_A = 0.3000[\text{kPa}]$
$k_{1M} = 2.3632[\text{kPa}]$	$k_{1A} = 0.5620[\text{kPa}]$
$k_{2M} = 0.8393[-]$	$k_{2A} = 0.7112[-]$
$\beta_M = 29.0^\circ$	$\beta_A = 62.0^\circ$

Table 2: Table of parameters for a carotid artery from a rabbit (experiment no. 71 in [7]) in respect of eqs. (10), (11).

In the adventitia many collagen fibers run closer to the axial direction of the artery, while in the media the collagen fibers tend to run around the circumference. The fiber angles β are meant to be associated with the reference (stress-free) configuration. Note that the change of the through-thickness mean value of the angle due to bending to the load-free (stressed) configuration (see Figure 4) is small so that it has a negligible influence on the stress-strain analysis of arteries.

By using a wall thickness of 0.39 mm (adopted from [3]), and making the assumption that the media occupies 2/3 of the arterial wall thickness, the parameters in Table 2 predict the characteristic orthotropic behavior of a carotid artery under combined bending, inflation, axial extension and torsion, as documented in [12].

References

- [1] D. F. Betsch and E. Baer. Structure and mechanical properties of rat tail tendon. *Biorheology*, 17:83–94, 1980.
- [2] T. E. Carew, R. N. Vaishnav, and D. J. Patel. Compressibility of the arterial wall. *Circ. Res.*, 23:61–68, 1968.
- [3] C. J. Chuong and Y. C. Fung. Three-dimensional stress distribution in arteries. *ASME J. Biomech. Engr.*, 105:268–274, 1983.
- [4] C. J. Chuong and Y. C. Fung. Residual stress in arteries. In G. W. Schmid-Schönbein, S. L.-Y. Woo, and B. W. Zweifach, editors, *Frontiers in Biomechanics*, pages 117–129. Springer-Verlag, New York, 1986.
- [5] H. M. Finlay, L. McCullough, and P. B. Canham. Three-dimensional collagen organization of human brain arteries at different transmural pressures. *J. Vasc. Res.*, 32:301–312, 1995.
- [6] Y. C. Fung. *Biomechanics. Mechanical Properties of Living Tissues*. Springer-Verlag, New York, 2nd edition, 1993.
- [7] Y. C. Fung, K. Fronek, and P. Patitucci. Pseudoelasticity of arteries and the choice of its mathematical expression. *Am. Physiological Soc.*, 237:H620–H631, 1979.
- [8] T. C. Gasser and G. A. Holzapfel. Rate-independent elastoplastic constitutive modeling of biological soft tissues: Part I. Continuum basis, algorithmic formulation and finite element implementation. *Int. J. Solids and Structures*, 2000. in press.
- [9] C. A. J. Hoeve and P. J. Flory. The elastic properties of elastin. *J. Am. Chem. Soc.*, 80:6523–6526, 1958.
- [10] G. A. Holzapfel. *Nonlinear Solid Mechanics. A Continuum Approach for Engineering*. John Wiley & Sons, Chichester, 2000.
- [11] G. A. Holzapfel and T. C. Gasser. A viscoelastic model for fiber-reinforced composites at finite strains: Continuum basis, computational aspects and applications. *Comput. Methods Appl. Mech. Engr.*, 2000. in press.
- [12] G. A. Holzapfel, T. C. Gasser, and R. W. Ogden. A new constitutive framework for arterial wall mechanics and a comparative study of material models. *J. Elasticity*, 2000. in press.
- [13] J. D. Humphrey. Mechanics of the arterial wall: Review and directions. *Critical Reviews in Biomed. Engr.*, 23:1–162, 1995.
- [14] B. E. Johnson and A. Hoger. The use of strain energy to quantify the effect of residual stress on mechanical behaviour. *Math. Mech. of Solids*, 3:447–470, 1998.
- [15] R. B. Martin, D. B. Burr, and N. A. Sharkey. *Skeletal Tissue Mechanics*. Springer-Verlag, New York, 1998.

- [16] R. J. Minns, P. D. Soden, and D. S. Jackson. The role of the fibrous components and ground substance in the mechanical properties of biological tissues: A preliminary investigation. *J. Biomech.*, 6:153–165, 1973.
- [17] M. E. Nimni and R. D. Harkness. Molecular structure and functions of collagen. In M. E. Nimni, editor, *Collagen*, pages 3–35. CRC Press, Boca Raton, FL, 1988.
- [18] R. W. Ogden. *Non-linear Elastic Deformations*. Dover, New York, 1997.
- [19] R. W. Ogden and C. A. J. Schulze-Bauer. Phenomenological and structural aspects of the mechanical response of arteries. *ASME, Mechanics in Biology Symposium, Orlando, Florida*, 2000. in press.
- [20] D. J. Patel and D. L. Fry. The elastic symmetry of arterial segments in dogs. *Circ. Res.*, 24:1–8, 1969.
- [21] A. Rachev. Theoretical study of the effect of stress-dependent remodeling on arterial geometry under hypertensive conditions. *J. Biomech.*, 30:819–827, 1997.
- [22] A. Rachev and K. Hayashi. Theoretical study of the effects of vascular smooth muscle contraction on strain and stress distributions in arteries. *Ann. Biomed. Engr.*, 27:459–468, 1999.
- [23] G. N. Ramachandran. Chemistry of collagen. In G. N. Ramachandran, editor, *Treatise on Collagen*, pages 103–183. Academic Press, New York, 1967.
- [24] E. K. Rodriguez, A. Hoger, and A. D. McCulloch. Stress-dependent finite growth in soft elastic tissues. *J. Biomech.*, 27:455–467, 1994.
- [25] F. H. Silver, D. L. Christiansen, and C. M. Buntin. Mechanical properties of the aorta: A review. *Critical Reviews in Biomed. Engr.*, 17:323–358, 1989.
- [26] A. J. M. Spencer. Constitutive theory for strongly anisotropic solids. In A. J. M. Spencer, editor, *Continuum Theory of the Mechanics of Fibre-Reinforced Composites*, pages 1–32. Springer-Verlag, Wien, 1984. CISM Courses and Lectures No. 282, International Centre for Mechanical Sciences.
- [27] E. Tanaka, H. Yamada, and S. Murakami. Inelastic constitutive modeling of arterial and ventricular walls. In *Computational Biomechanics*, K. Hayashi and H. Ishikawa (Eds), Vienna, 1996. Springer-Verlag. 137-163.
- [28] R. N. Vaishnav and J. Vossoughi. Estimation of residual strains in aortic segments. In *Biomedical Engineering II: Recent Developments*, C. W. Hall (Ed.), New York, 1983. Pergamon Press. 330-333.
- [29] J. Vossoughi, Z. Hedjazi, and F. S. Borris. Intimal residual stress and strain in large arteries. In *Bed-Vol. 24, 1993 Bioengineering Conference ASME*, pages 434–437, 1993.
- [30] S. L. Y. Woo, M. A. Gomez, and W. H. Akeson. Mechanical behaviors of soft tissues: Measurements, modifications, injuries and treatment. In *The Biomechanics of Trauma*, A. M. Nahum and J. Melvin (Eds), Norwalk, 1985. Appleton Crofts. 107-133.
- [31] J. Xie, J. Zhou, and Y. C. Fung. Bending of blood vessel wall: Stress-strain laws of the intima-media and adventitia layers. *J. Biomech. Engr.*, 117:136–145, 1995.
- [32] Q. Yu, J. Zhou, and Y. C. Fung. Neutral axis location in bending and Young’s modulus of different layers of arterial wall. *Am. J. Physiol.*, 265:H52–H60, 1993.

Preprints in this series

Pdf-files of the listed preprints may be obtained from our website at <http://www.cis.tu-graz.ac.at/biomech/> or by sending e-mail to gh@biomech.tu-graz.ac.at

1. **G.A. Holzapfel, M. Stadler and R.W. Ogden**, Aspects of stress softening in filled rubbers incorporating residual strains, Graz, October 1999.
2. **G.A. Holzapfel and T.C. Gasser**, A viscoelastic model for fiber-reinforced composites at finite strains: Continuum basis, computational aspects and applications, Graz, July 2000.
3. **G.A. Holzapfel, C.A.J. Schulze-Bauer and M. Stadler**, Mechanics of angioplasty: Wall, balloon and stent, Graz, August 2000.
4. **R.W. Ogden and C.A.J. Schulze-Bauer**, Phenomenological and structural aspects of the mechanical response of arteries, Graz, August 2000.
5. **C.A.J. Schulze-Bauer**, Biomechanical properties of human iliac arteries, Graz, October 2000.
6. **R. Eberlein, G.A. Holzapfel and C.A.J. Schulze-Bauer**, An anisotropic constitutive model for annulus tissue and enhanced finite element analyses of intact lumbar disc bodies, Graz, October 2000.
7. **G.A. Holzapfel**, Biomechanics of Soft Tissue, Graz, November 2000.