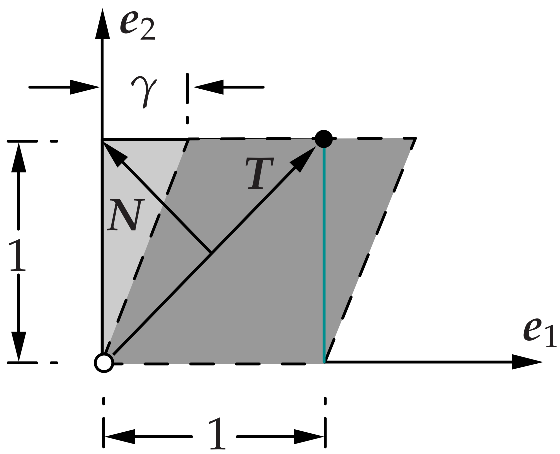
2 ME338A - Homework

Due: May 01, 2008, 4pm

2.1 Kinematics



Consider planar simple shear deformation described by

$$\varphi(\boldsymbol{X}, t) = \begin{cases} x_1 &= X_1 + \gamma(t) X_2 \\ x_2 &= X_2 \\ x_3 &= X_3 \end{cases}$$

with respect to the coordinate system shown above. For this particular problem:

a) Invert the given motion $\varphi(\boldsymbol{X}, t)$ to obtain the inverse motion $\varphi^{-1}(\boldsymbol{x}, t)$ in terms of spatial coordinates $\boldsymbol{x}$ and γ.

b) Determine the material velocity $\boldsymbol{V}(\boldsymbol{X}, t)$ and the material acceleration $\boldsymbol{A}(\boldsymbol{X}, t)$.

c) Obtain the spatial velocity $\boldsymbol{v}(\boldsymbol{x}, t)$ and the spatial acceleration $\boldsymbol{a}(\boldsymbol{x}, t)$ by inserting the inverse motion in the material rates, e.g. $\boldsymbol{a}(\boldsymbol{x}, t) = \boldsymbol{A}(\varphi_t^{-1}(\boldsymbol{x}), t)$.

d) Compute the spatial acceleration $\boldsymbol{a}(\boldsymbol{x}, t)$ through the material time derivative of the spatial velocity $\boldsymbol{v}(\boldsymbol{x}, t)$ and compare with the result obtained in *c*).

e) Derive the explicit expressions for the tangent map $\boldsymbol{F}$, the area map $\mathrm{cof}(\boldsymbol{F})$ and the volume map $\det(\boldsymbol{F})$ in terms of $\gamma(t)$.

f) Consider the material tangent vector $\boldsymbol{T} = \boldsymbol{e}_1 + \boldsymbol{e}_2 \in T_X\mathcal{B}$. Compute the ratio of its deformed length to the reference length $||\boldsymbol{T}||$.

g) Consider the material area normal $\boldsymbol{N} = -\boldsymbol{e}_1 + \boldsymbol{e}_2 \in T_X^*\mathcal{B}$. Compute the ratio of its deformed area to the reference area $||\boldsymbol{N}||$.

h) Calculate the right $\boldsymbol{C}$ and left $\boldsymbol{b}$ Cauchy-Green tensors, the spatial velocity gradient $\boldsymbol{l}$, the rate of deformation tensor $\boldsymbol{d} := \mathrm{sym}(\boldsymbol{l})$ and the spin tensor $\boldsymbol{w} := \mathrm{skew}(\boldsymbol{l})$.

i) Determine the axial vector ω corresponding to the spin tensor $\boldsymbol{w}$.

j) For $\gamma(t) = \sqrt{2}$, carry out the polar decomposition of the deformation gradient $\boldsymbol{F} = \boldsymbol{R} \cdot \boldsymbol{U} = \boldsymbol{V} \cdot \boldsymbol{R}$ into the rotation $\boldsymbol{R}$, the right $\boldsymbol{U}$ and left $\boldsymbol{V}$ stretch tensors. Numerically verify that $\boldsymbol{U} = \boldsymbol{R}^T \cdot \boldsymbol{V} \cdot \boldsymbol{R}$ and $\boldsymbol{V} = \boldsymbol{R} \cdot \boldsymbol{U} \cdot \boldsymbol{R}^T$. In this part *j*), programming packages, such as Matlab can be used provided that the script is also handed in.