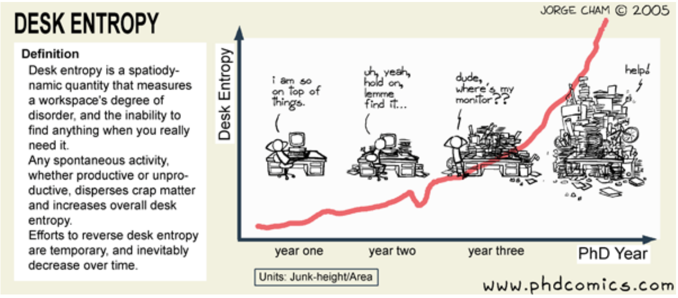
07 - basics balance equations - closed and open systems

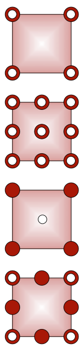


day	date		topic
tue	jan	08	motivation - everything grows!
thu	jan	10	basics maths - notation and tensors
tue	jan	15	basic kinematics - large deformation and growth
thu	jan	17	kinematics - growing hearts
tue	jan	22	guest lecture - growing surfaces
thu	jan	24	kinematics - growing leaflets
tue	jan	29	basic balance equations - closed and open systems
thu	jan	31	basic constitutive equations - growing tumors
tue	feb	05	volume growth - finite elements for growth
thu	feb	07	volume growth - growing arteries
tue	feb	12	volume growth - growing skin
thu	feb	14	volume growth - growing hearts
tue	feb	19	basic constitutive equations - growing bones
thu	feb	21	density growth - finite elements for growth
tue	feb	26	density growth - growing bones
thu	feb	28	everything grows! - midterm summary
tue	mar	05	midterm
thu	mar	07	remodeling - remodeling arteries and tendons
tue	mar	12	class project - discussion, presentation, evaluation
thu	mar	14	class project - discussion, presentation, evaluation
thu	mar	14	written part of final projects due

final projects - me337 2013

- **cerebral aneurysm growth**: sheila
- **muscle growth**: katrina, jaqi
- **brain tumor growth**: cesare, zhuozhi
- **left vs right arms of athletes**: ronaldo, maria, lyndia
- **airway wall remodeling**: mona
- **corneal growth / keratoconus**: yanli
- **bone adaptation in birds**: jan, david
- **neuronal growth in fish**: andrew

continuum mechanics of growth



kinematic equations for finite growth

$$\boldsymbol{F} = \boldsymbol{F}_{\rm e} \cdot \boldsymbol{F}_{\rm g}$$

balance equations for open systems

$$\mathrm{D}_t \rho_0 = \mathrm{Div}(\boldsymbol{R}) + \mathcal{R}_0$$
$$\rho_0 \, \mathrm{D}_t \boldsymbol{v} = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0$$

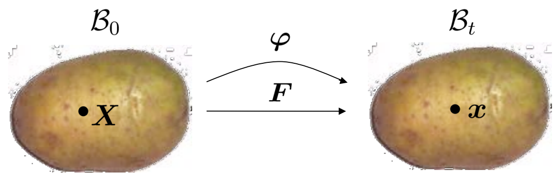
constitutive equations for living tissues

$$\boldsymbol{P} = \boldsymbol{P}(\rho_0, \boldsymbol{F}, \boldsymbol{F}_{\rm g})$$

fe analyses for biological structures

continuum- & computational biomechanics

potato - kinematics



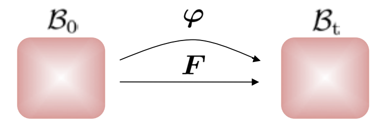
- nonlinear deformation map φ

  $\boldsymbol{x} = \varphi(\boldsymbol{X}, t)$ with $\varphi : \mathcal{B}_0 \times \mathbb{R} \rightarrow \mathcal{B}_t$

- spatial derivative of φ - deformation gradient

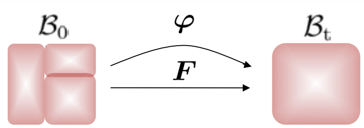
  $\mathrm{d}\boldsymbol{x} = \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{X}$ with $\boldsymbol{F} : T\mathcal{B}_0 \rightarrow T\mathcal{B}_t$ $\quad \boldsymbol{F} = \left.\frac{\partial \varphi}{\partial \boldsymbol{X}}\right|_{t \text{ fixed}}$

kinematics of finite growth



[1] consider an elastic body $\mathcal{B}_0$ at time t_0, unloaded & stressfree

kinematics of finite growth



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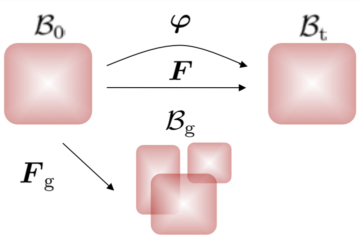
[2] imagine the body is cut into infinitesimal elements each of which is allowed to undergo volumetric growth

kinematics of finite growth



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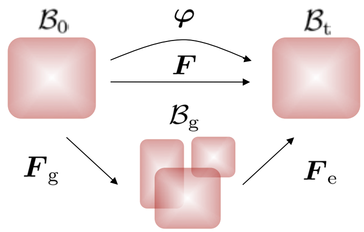
[3] after growing the elements, $\mathcal{B}_g$ may be incompatible

kinematics of finite growth



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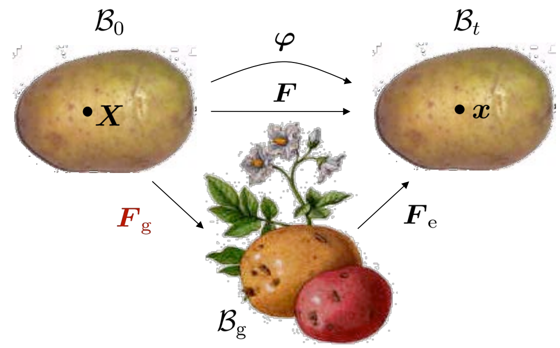
[4] loading generates compatible current configuration $\mathcal{B}_t$

potato - kinematics of finite growth



- incompatible growth configuration $\mathcal{B}_g$ & growth tensor $\boldsymbol{F}_g$

$$\boldsymbol{F} = \boldsymbol{F}_e \cdot \boldsymbol{F}_g$$

rodriguez, hoger & mc culloch [1994]

balance equations

balance equations ['bæl.əns ɪ'kweɪ.ʒəns] of mass, momentum, angular momentum and energy, supplemented with an entropy inequality constitute the set of conservation laws. the law of **conservation of mass**/matter states that the **mass of a closed system** of substances will remain **constant**, regardless of the processes acting inside the system. the principle of conservation of momentum states that the total momentum of a closed system of objects is constant.

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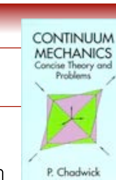
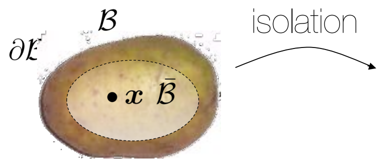
balance equations



balance equations ['bæl.əns ɪ'kweɪ.ʒəns] of mass, linear momentum, angular momentum and energy **apply to all material bodies.** each one gives rise to a field equation, holing on the configurations of a body in a sufficiently smooth motion and a jump condition on surfaces of discontinuity. like position, time and body, the concepts of mass, force, heating and internal energy which enter into the formulation of the balance equations are regarded as having primitive status in continuum mechanics.

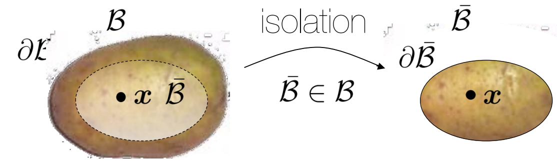
chadwick "continuum mechanics" [1976]

potato - balance equations



[1] isolation of subset $\bar{\mathcal{B}}$ from $\mathcal{B}$

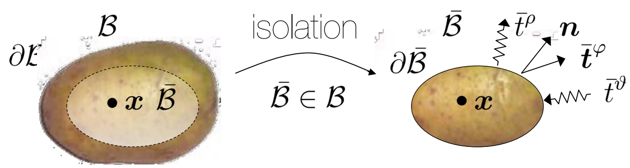
potato - balance equations



[1] isolation of subset $\bar{\mathcal{B}}$ from $\mathcal{B}$

[2] characterization of influence of remaining body through phenomenological quantities - contact fluxes $\bar{t}^\rho$, $\bar{\boldsymbol{t}}^\varphi$ & $\bar{t}^\vartheta$

potato - balance equations



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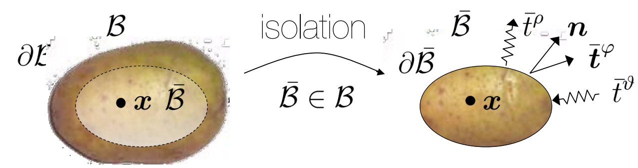
potato - balance equations



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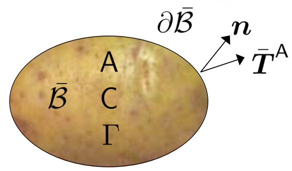
[4] postulation of balance of these quantities

generic balance equation



general format

A ... balance quantity

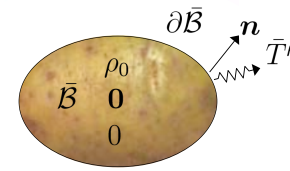
$\mathbf{B}$ ... flux $\quad \mathbf{B}\cdot\boldsymbol{n} = \bar{\boldsymbol{T}}^{\mathsf{A}}$

C ... source

Γ ... production

$$\mathrm{D}_t\mathsf{A} = \mathrm{Div}(\mathbf{B}) + \mathsf{C} + \Gamma$$

balance of mass



balance of mass

ρ_0 ... density

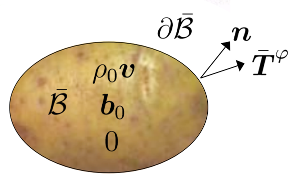
$\mathbf{0}$ ... no mass flux $\quad \bar{T}^{\rho} = 0$

0 ... no mass source

0 ... no mass production

continuity equation $\quad \mathrm{D}_t\rho_0 = 0$

balance of (linear) momentum



balance of momentum

$\rho_0\,\boldsymbol{v}$ ... linear momentum density

$\boldsymbol{P}$ ... momentum flux - stress $\quad \boldsymbol{P}\cdot\boldsymbol{n} = \bar{\boldsymbol{T}}^{\varphi}$

$\boldsymbol{b}_0$ ... momentum source - force

0 ... no momentum production

equilibrium equation $\quad \mathrm{D}_t(\rho_0\boldsymbol{v}) = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0$

compare

NEWTON'S THREE LAWS OF GRADUATION

First published in 1679, Isaac Newton's "**Procrastinare Unnaturalis Principia Mathematica**" is often considered one of the most important single works in the history of science. Its Second Law is the most powerful of the three, allowing mathematical calculation of the duration of a doctoral degree.

SECOND LAW

"The age, a, of a doctoral process is directly proportional to the flexibility, f, given by the advisor and inversely proportional to the student's motivation, m"

Mathematically, this postulate translates to:

$$age_{PhD} = \frac{flexibility}{motivation}$$

$$a = F / m$$

$$\therefore F = m\, a$$

This Law is a quantitative description of the effect of the forces experienced by a grad student. A highly motivated student may still remain in grad school given enough flexibility. As motivation goes to zero, the duration of the PhD goes to infinity.

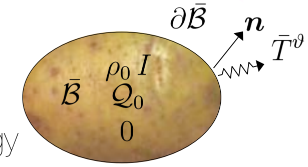
PHD.STANFORD.EDU

JORGE CHAM © THE STANFORD DAILY

$\mathrm{D}_t(\rho_0\boldsymbol{v}) = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0$ mass point $\quad m\,\mathrm{D}_t\boldsymbol{v} = m\,\boldsymbol{a} = \boldsymbol{F}$

balance of (internal) energy

balance of internal energy



$\rho_0\, I$ ... internal energy density

$\boldsymbol{Q}$ ... heat flux $\quad -\boldsymbol{Q}\cdot\boldsymbol{n} = \bar{T}^{\vartheta}$

$\mathcal{Q}_0$ ... heat source

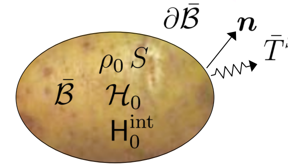
0 ... no heat production

energy equation $\mathrm{D}_t(\rho_0\, I) = \boldsymbol{P} : \mathrm{D}_t\boldsymbol{F} - \boldsymbol{v}\cdot\boldsymbol{b}_0 + \mathrm{Div}(-\boldsymbol{Q}) + \mathcal{Q}_0$

internal mechanical power thermal external power

balance of entropy

balance of entropy



$\rho_0\, S$ ... entropy density

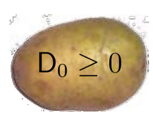
$\boldsymbol{H}$ ... entropy flux $\quad -\boldsymbol{H}\cdot\boldsymbol{n} = \bar{T}^{S}$

$\mathcal{H}_0$ ... entropy source

$\mathrm{H}_0^{\mathrm{int}}$ ... entropy production $\quad \mathrm{H}_0^{\mathrm{int}} \geq 0$

entropy 'inequality' $\mathrm{D}_t(\rho_0 S) = \mathrm{Div}(-\boldsymbol{H}) + \mathcal{H}_0 + \mathrm{H}_0^{\mathrm{int}}$

dissipation inequality



- dissipaltion inequality

$$\mathrm{D}_0 := \vartheta \mathrm{H}_0^{\mathrm{int}} = \vartheta\rho_0\mathrm{D}_t S + \vartheta\mathrm{Div}(\boldsymbol{H}) - \vartheta\mathcal{H}_0 \geq 0$$

- identification $\quad \boldsymbol{H} = \frac{1}{\vartheta}\boldsymbol{Q} \qquad \mathcal{H}_0 = \frac{1}{\vartheta}\mathcal{Q}_0$
- with legendre-fenchel transform $\quad \psi = I - \vartheta\, S$

$$\mathrm{D}_0 = \boldsymbol{P} : \mathrm{D}_t\boldsymbol{F} - \rho_0\mathrm{D}_t\psi - \rho_0\, S\,\mathrm{D}_t\vartheta + \boldsymbol{Q}\cdot\nabla_X \ln(\vartheta) \geq 0$$

- free energy $\quad \psi = \psi(\boldsymbol{F}, \vartheta) \qquad \mathrm{D}_t\psi = \mathrm{D}_{\boldsymbol{F}}\psi : \mathrm{D}_t\boldsymbol{F} + \mathrm{D}_\vartheta\psi\,\mathrm{D}_t\vartheta$

- definition of stress and entropy (e.g., neo hooke's law)

$$\boldsymbol{P} = \rho_0\mathrm{D}_{\boldsymbol{F}}\psi \qquad S = -\rho_0\mathrm{D}_\vartheta\psi$$

- thermodynamic restriction (e.g., fourier's law)

$$\boldsymbol{Q}\cdot\nabla_X\ln(\vartheta) \geq 0$$

thermodynamic systems

isolated system [ˈaɪ.sə.leɪ.tɪd ˈsɪs.təm] thermo-dynamical system which is not allowed to have any interaction with its environment. enclosed by a rigid, adiabatic, imperme-able membrane.

thermodynamic systems

adiabatic closed system [ˈə.dɪ.æ.bæ.tɪk kloʊzd ˈsɪs.təm] thermodynamic system which is allowed to exchange exclusively mechanical work, typically $\boldsymbol{P} = \boldsymbol{P}(\nabla\varphi,...)$, with its environment. enclosed by a deformable, adiabatic, impermeable membrane. characterized through its state of deformation φ .

thermodynamic systems

closed system [kloʊzd ˈsɪs.təm] thermodynamic system which is allowed to exchange mechanical work and heat, typically $\boldsymbol{P} = \boldsymbol{P}(\nabla\varphi,...)$ and $\boldsymbol{Q} = \boldsymbol{Q}(\nabla\theta,...)$, with its environment. enclosed by a deformable, diathermal, impermeable membrane. characterized through its state of deformation φ and temperature θ .

thermodynamic systems

open system [ˈoʊ.pən ˈsɪs.təm] thermodynamic system which is allowed to exchange mechanical work, heat and mass, typically $\boldsymbol{P} = \boldsymbol{P}(\nabla\varphi,...)$, $\boldsymbol{Q} = \boldsymbol{Q}(\nabla\theta,...)$ and $\boldsymbol{R} = \boldsymbol{R}(\nabla\rho,...)$ with its environment. enclosed by a deformable, diathermal, permeable membrane. characterized through its state of deformation φ, temperature θ and density ρ .

open system thermodynamics

"...thermodynamics recognizes no special role of the biological..."

bridgman, 'the nature of thermodynamics', [1941]

why do we need open systems if we have porous media?

theory of open systems
[ˈoʊ.pən ˈsɪs.təms]

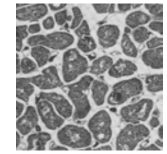
- constituents spatially separated
- overall behavior priliminary deter-mined by one single constituent
- exchange of mass, momentum, energy and entropy with environ-ment

theory of porous media
[ˈpoːr.əs ’miːdiː.ə]

- local superposition of constituents
- consideration of mixture of mul-tiple constituents
- exchange of mass, momentum, energy and entropy amongst constituents

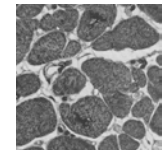
balance of mass

$$D_t\rho_0 = \mathrm{Div}(\boldsymbol{R}) + \mathcal{R}_0$$



mass flux $\boldsymbol{R}$

- cell movement (migration)



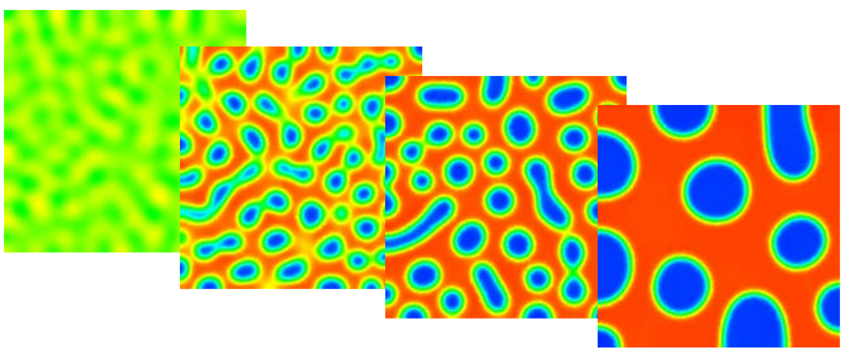
mass source $\mathcal{R}_0$

- cell growth (proliferation)
- cell division (hyperplasia)
- cell enlargement (hypertrophy)

biological equilbrium

cowin & hegedus [1976], beaupré, orr & carter [1990], harrigan & hamilton [1992], jacobs, levenston, beaupré, simo & carter [1995], huiskes [2000], carter & beaupré [2001]

balance of mass



simulation of cell growth - cahn-hilliard equation

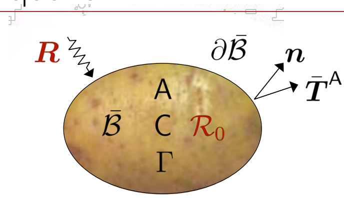
kuhl & schmid [2006], wells, kuhl & garikipati [2006]

balance of mass

the model does not take explicitly into account that the **body is growing** due to the **absorption of some other materials.** in absence of some of the vital constituents, no growth is possible. conversely, when part of the material dies, some of the bricks contained in the cellular membrane can be re-used by other cells. in this respect, an approach using mixture theory might by useful.

ambrosi & mollica [2002]

generic balance equation



general format

A ... balance quantity
B ... flux $\quad \mathbf{B}\cdot\boldsymbol{n} = \bar{\boldsymbol{T}}^{\mathsf{A}}$
C ... source
Γ ... production

$$\mathrm{D}_t(\rho_0 \mathsf{A}) = \mathrm{Div}(\mathbf{B} + \mathsf{A}\otimes\boldsymbol{R}) + [\mathsf{C} + \mathsf{A}\mathcal{R}_0 - \nabla_X \mathsf{A}\cdot\boldsymbol{R} + \Gamma]$$

balance of (linear) momentum

- volume specific version

$$\mathrm{D}_t(\rho_0\boldsymbol{v}) = \mathrm{Div}(\boldsymbol{P} + \boldsymbol{v}\otimes\boldsymbol{R}) + [\boldsymbol{b}_0 + \boldsymbol{v}\mathcal{R}_0 - \nabla_X\boldsymbol{v}\cdot\boldsymbol{R}]$$

- subract weighted balance of mass

$$\boldsymbol{v}\,\mathrm{D}_t\rho_0 = \mathrm{Div}(\boldsymbol{v}\otimes\boldsymbol{R}) + \boldsymbol{v}\mathcal{R}_0 - \nabla_X\boldsymbol{v}\cdot\boldsymbol{R}$$

- mass specific version

$$\rho_0\,\mathrm{D}_t\boldsymbol{v} = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0$$

mechanical equilbrium

example of open systems - rocket propulsion

balance of mass

$\mathrm{D}_t\mathrm{m} = \mathcal{R}$ with $\mathcal{R} \le 0$ ejection

balance of momentum - volume specific

$$\mathrm{D}_t[\mathrm{m}\boldsymbol{v}] = \mathrm{m}\,\mathrm{D}_t\boldsymbol{v} + \mathrm{D}_t\mathrm{m}\,\boldsymbol{v} = \boldsymbol{f} + \mathcal{R}\boldsymbol{v}$$

balance of momentum - mass specific

$\mathrm{m}\,\mathrm{D}_t\boldsymbol{v} = \boldsymbol{f}$ with $\boldsymbol{f} = \boldsymbol{f}^{\mathrm{closed}} + \boldsymbol{f}^{\mathrm{open}}$

balance of momentum - rocket head-ejection

$$\mathrm{D}_t[\mathrm{m}\boldsymbol{v}] - \mathcal{R}\bar{\boldsymbol{v}} = \boldsymbol{f}^{\mathrm{closed}}$$

propulsive force

$\boldsymbol{f}^{\mathrm{open}} = [\bar{\boldsymbol{v}} - \boldsymbol{v}]\mathcal{R}$ velocity of ejection $\bar{\boldsymbol{v}}$

example of open systems - rocket propulsion

a saturn v rocket like the one that took men to the moon has a **mass of 2.500.000 kg** at liftoff. it goes straight up vertically and burns fuel at a uniform **rate of 16.000 kg/s** for a duration of **2 minutes**. the exhaust speed of gas from the saturn **v is 3.0 km/s**

what is the speed of the rocket immediately after the combustion ceases? you should include the effect of gravity near the surface of the earth, but you can neglect air resistance.

plot the **burnout velocity as a function of time** over the range of 0 to 120 seconds to see the increase in speed of the rocket with time.

example of open systems - rocket propulsion

example - rocket propulsion

example of open systems - rocket propulsion

$$\mathrm{m}\mathrm{D}_t v = f^{\text{closed}} + f^{\text{open}}$$

with $f^{\text{closed}} = -\mathrm{m}\, g$ gravity

$$f^{\text{open}} = [\bar{v} - v]\mathcal{R} = w\,\mathrm{D}_t\mathrm{m}$$

$$\mathrm{m}\mathrm{D}_t v = -\mathrm{m}\, g - \mathrm{D}_t m\, w \qquad || : \mathrm{m}$$

$$\mathrm{D}_t v = -g - \tfrac{1}{\mathrm{m}}\mathrm{D}_t\mathrm{m}\, w$$

integration

$$v(t) = -g\,t - w\int_{\mathrm{m}(0)}^{\mathrm{m}(t)} \tfrac{1}{\mathrm{m}}\mathrm{dm}$$

$$\mathrm{m}(t) = \mathrm{m}(0) + \mathcal{R}\,t$$

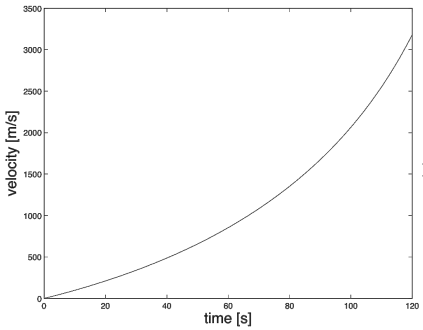
velocity

$$v(t) = -g\,t - w\,\ln\frac{\mathrm{m}(0)+\mathcal{R}\,t}{\mathrm{m}(0)}$$

example - rocket propulsion

example of open systems - rocket propulsion

velocity $v(t) = -g\,t - w\,\ln\frac{\mathrm{m}(0)+\mathcal{R}\,t}{\mathrm{m}(0)}$



$$g = 9.8\ \mathrm{m/s^2}$$

$$w = 3000\mathrm{m/s}$$

$$\mathrm{m}(0) = 2.5\cdot 10^6\ \mathrm{kg}$$

$$\mathcal{R}_0 = -16000\mathrm{kg/s}$$

velocity $v(t = 120\mathrm{s}) = 3207.1\ \mathrm{m/s}$

example - rocket propulsion

balance of (internal) energy

- volume specific version

$$\mathrm{D}_t(\rho_0\, I) = \boldsymbol{P} : \mathrm{D}_t\boldsymbol{F} - \boldsymbol{v}\cdot\boldsymbol{b}_0 + \mathrm{Div}(-\boldsymbol{Q} + I\boldsymbol{R}) + [\mathcal{Q}_0 + I\,\mathcal{R}_0 - \nabla_X I\cdot\boldsymbol{R}]$$

- subract weighted balance of mass

$$I\,\mathrm{D}_t\rho_0 = \mathrm{Div}(I\,\boldsymbol{R}) + I\,\mathcal{R}_0 - \nabla_X I\cdot\boldsymbol{R}$$

- mass specific version

$$\rho_0\,\mathrm{D}_t I = \boldsymbol{P} : \mathrm{D}_t\boldsymbol{F} - \boldsymbol{v}\cdot\boldsymbol{b}_0 + \mathrm{Div}(-\boldsymbol{Q}) + \mathcal{Q}_0$$

energy equilbrium

balance equations - open systems

balance of entropy

- volume specific version $\mathrm{H}_0^{\rm int} \ge 0$

$$\mathrm{D}_t(\rho_0 S) = \mathrm{Div}(-\boldsymbol{H} + S\,\boldsymbol{R}) + [\mathcal{H}_0 + S\,\mathcal{R}_0 - \nabla_X S \cdot \boldsymbol{R}] + \mathrm{H}_0^{\rm int}$$

- subract weighted balance of mass

$$S\,\mathrm{D}_t\rho_0 = \mathrm{Div}(S\,\boldsymbol{R}) + S\,\mathcal{R}_0 - \nabla_X S \cdot \boldsymbol{R}$$

- mass specific version

$$\rho_0 \mathrm{D}_t S = \mathrm{Div}(-\boldsymbol{H}) + \mathcal{H}_0 + \mathrm{H}_0^{\rm int}$$

entropy 'inequality'

dissipation inequality

- dissipaltion inequality

$$\mathrm{D}_0 := \vartheta \mathrm{H}_0^{\rm int} = \rho_0 \vartheta \mathrm{D}_t S + \vartheta \mathrm{Div}(\boldsymbol{H}) - \vartheta \mathcal{H}_0 \ge 0$$

- identification $\boldsymbol{H} = \frac{1}{\vartheta}\boldsymbol{Q} + \boldsymbol{S}$ $\quad \mathcal{H}_0 = \frac{1}{\vartheta}\mathcal{Q}_0 + \mathcal{S}_0$
- free energy $\psi = \psi(\rho_0, \boldsymbol{F}, \vartheta)$
- definition of stress and entropy

$$\boldsymbol{P} = \rho_0\,\mathrm{D}_F\psi \qquad S = -\,\mathrm{D}_\vartheta\psi$$

- thermodynamic restrictions

$$\mathcal{S}_0 \le \rho_0 \mathrm{D}_{\rho_0}\psi \tfrac{1}{\vartheta}\mathcal{R}_0 \qquad \boldsymbol{S} \ge \rho_0 \mathrm{D}_{\rho_0}\psi \tfrac{1}{\vartheta}\boldsymbol{R} \qquad \boldsymbol{Q}\cdot\nabla_X \ln(\vartheta) \ge 0$$

"... a living organism can only keep alive by continuously drawing from its environment negative entropy. it feeds upon negative entropy to compensate the entropy increase it produces by living."

schrödinger "what is life?" [1944]

open systems - dissipation inequality

$$\mathrm{D}_0 = \boldsymbol{P} : \mathrm{D}_t\boldsymbol{F} - \rho_0 \mathrm{D}_t\psi - \rho_0\,S\,\mathrm{D}_t\vartheta - \mathrm{s}_0\,\vartheta + \boldsymbol{Q}\cdot\nabla_X \ln(\vartheta) \ge 0$$

with definition of arguments of free energy

$$\psi = \psi(\rho_0, \boldsymbol{F}, \vartheta) \qquad \mathrm{D}_t\psi = \mathrm{D}_\rho\psi\,\mathrm{D}_t\rho + \mathrm{D}_F\psi : \mathrm{D}_t\boldsymbol{F} + \mathrm{D}_\vartheta\psi\,\mathrm{D}_t\vartheta$$

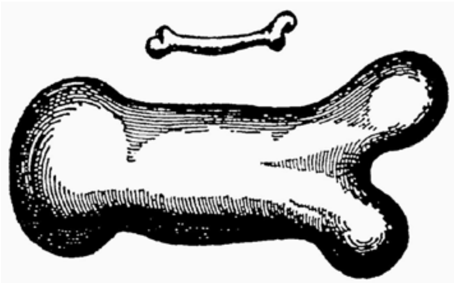
evaluation of dissipation inequality

$$\mathrm{D}_0 = [\,\boldsymbol{P} - \rho_0\mathrm{D}_F\psi\,] : \mathrm{D}_t\boldsymbol{F} - [\,\rho_0\,S - \rho_0\,\mathrm{D}_\vartheta\psi\,]\,\mathrm{D}_t\vartheta + \ldots \ge 0$$

provides guidelines for the appropriate choice of the constitutive equations $\boldsymbol{P} = \rho_0\,\mathrm{D}_F\psi$ $\quad S = -\,\mathrm{D}_\vartheta\psi$

thermodynamically conjugate pairs $\boldsymbol{P} \rightleftharpoons \boldsymbol{F}$ $\quad S \rightleftharpoons \vartheta$

example of open systems - the galileo giant



„...dal che e manifesto, che chi volesse mantener in un vastissimo gigante le proporzioni, che hanno le membra in un huomo ordinario, bisognerebbe o trouar materia molto piu dura, e resistente per formarne l'ossa o vero ammettere, che la robustezza sua fusse a proporzione assai piu fiacca, che negli huomini de statura mediocre; altrimente crescendogli a smisurata altezza si vedrebbono dal proprio peso opprimere, e cadere..."

galileo,"discorsi e dimostrazioni matematiche", [1638]

example of open systems - the galileo giant

the tallest man in medical history is robert pershing wadlow. he was born at alton, illinois, on february 22, 1918. he was **2.72m** / 8ft 11.1", tall.

his weight was **222.71kg**. his shoe size was 47cm / 18.5", and his hands measured 32.4cm / 12.75". his arm span was 2.88m / 9 ft 5.75", and his peak daily food consumption was 8000 calories.

guiness world records [2010]

example of open systems - the galileo giant

consider and compare the two cases:

calculale the vertical displacement and the energy for a giant with a **constant bone mineral density along the height**

calculate the vertical displacement and the bone mineral density for a giant with a **constant energy along the height**

guiness world records [2010]

example of closed systems - the galileo giant

- balance equation closed systems $\rho_0 \, \mathrm{D}_t v = \mathrm{Div}(P) + b_0$
- neo hookean free energy $\psi_0 = \frac{1}{4} \, E_0 [F^2 - 1 - 2 \, \ln(F)]$
- stress from dissipation inequality $P = \mathrm{D}_F \psi_0 = \frac{1}{4} E_0 [\, 2F - 2 \frac{1}{F} \,]$
- quasi-static case $\rho_0 \, \mathrm{D}_t v = 0$
- constant gravity load $b_0 = \text{const along the height } h$
- from balance eqn linear stress $P = [X - h] \, b_0$ ...linear along the height h
- closed system, constant density $\rho = \rho_0^* \qquad [\rho - \rho_0^*]/\rho_0^* = 0$

example of open systems - the galileo giant

density $[\rho - \rho_0^*]/\rho_0^* = 0$

displacement $u = \varphi - X$

energy $[\rho_0/\rho_0^*]^{-m} \psi_0 = \psi_0$

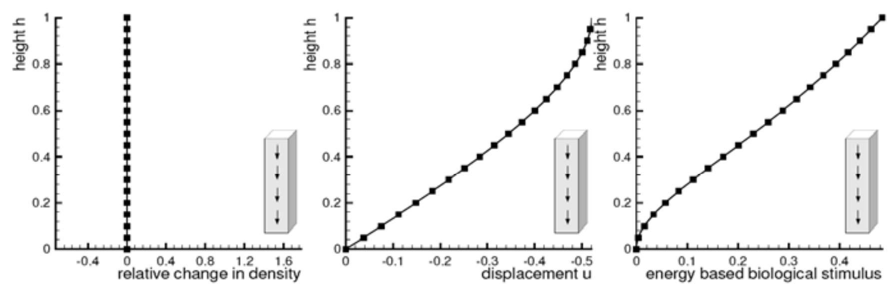


Figure 7.1: One–dimensional model problem - Closed system - Homogeneous density

example of open systems - the galileo giant

- balance equations open systems
$$\begin{aligned} \mathrm{D}_t \rho_0 &= \mathrm{Div}(R) + \mathcal{R}_0 \\ \rho_0 \; \mathrm{D}_t v &= \mathrm{Div}(P) + b_0 \end{aligned}$$
- free energy $\psi_0 = [\rho_0/\rho_0^*]^n \, \psi_0^{\mathrm{neo}}$ $\quad \psi_0^{\mathrm{neo}} = \frac{1}{4} \, E_0 [F^2 - 1 - 2 \ln(F)]$
- stress from dissipation inequality $\quad P = \mathrm{D}_F \psi_0 = \frac{1}{4} E_0 [\, 2F - 2\frac{1}{F} \,]$
- quasi-static case $\quad \rho_0 \, \mathrm{D}_t v = 0$
- constant gravity load $\quad b_0 = \text{const along the height } h$
- from balance eqn linear stress $\quad P = [X - h] \, b_0$ ...linear along the height h
- open system, varying density $\quad \mathrm{D}_t \rho_0 = \mathcal{R}_0 = 0$
$$\mathcal{R}_0 = [\rho_0/\rho_0^*]^{-m} \rho_0 - \rho_0^*$$

example of open systems - the galileo giant

density $[\rho - \rho_0^*]/\rho_0^*$ | displacement $u = \varphi - X$ | energy $[\rho_0/\rho_0^*]^{-m} \psi_0$

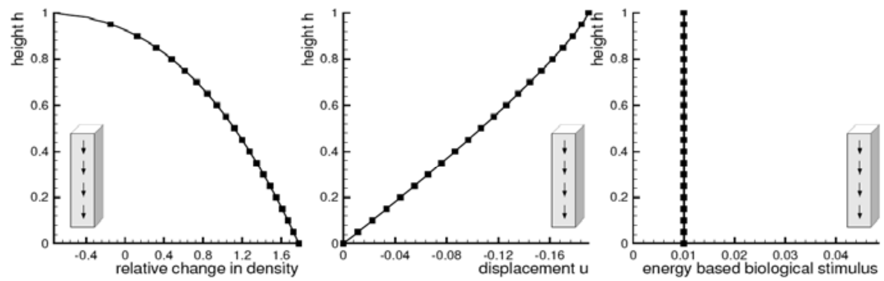


Figure 7.2: One–dimensional model problem - Open system - Homogeneous stimulus