06 - kinematic equations – growing mitral valves

04 - kinematic equations

final projects - me337 2010
- mechanically driven skin growth: chris, adrian, xuefeng
- muscle growth: brandon, robyn, esteban, ivan, jenny
- cardiac growth review: manuel
- cardiac growth in response to training: holly, tyler
- cardiac growth in response to heart attack: amit
- cardiac growth in response to medical devices: kyla, andrew
- analytical simulation of arterial growth: andrew
- bone growth in response to medical devices: chinedu
- impact of obesity on osteoarthritis: abhishek, chris
- tumor growth: apoorva
- facial volume aging: jonathan
- idiopathic scoliosis: anusuya
- driving forces for different types of growth: james

homework 01 - due thu in class

where are we???

final projects - me337 2012
- tendon growth: harrison, brandon, mohammed, matthew
- tendon growth and remodeling: peter
- muscle growth: alex
- skin growth and healing: beth, ann, amen
- benign vocal fold nodule and polyp growth: corey
- cerebral aneurysm growth: jina
- growth of swelling gels: hardik, xi, ill
- bone growth in martial arts: kevin, alison, safwan, kamil

homework 01 - due thu in class
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• cerebral aneurysm growth: sheila
• muscle growth: katrina, jaqi
• brain tumor growth: cesare, zhuozhi
• soccer player brain/muscle adaptation: ronaldo, maria, lyndia
• airway wall remodeling: mona
• corneal growth / keratoconus: yanli
• feather growth in birds: jan, david
• neuronal growth in fish: andrew

possible final project – muscle growth

how do our muscles shorten?

\[ F^s = I + (\theta - 1) f_0 \otimes f_t \]

how do brain tumors grow?

increased growth in outer layers
inhomogeneous deformation

how does the airway wall grow during asthma?

hoop stress tensile inside / compressive outside

example – brain tumor growth

final project – airway wall growth

sigg, hrousse, drazen, kamhi [1997], zheng, zhang, su, jiang [2009], jin, cai, suo [2011], melton, gorely [2011], li, cai, feng, gao [2011], cai, li, feng [2012], papastavrou, steinmann, kuhl [2013]
**kinematic equations**

*kinematic equations* [ˈkɪnəˈmætɪk ɪˈkwɪr.əns] describe the motion of objects without the consideration of the masses or forces that bring about the motion. The basis of kinematics is the choice of coordinates. The 1st and 2nd time derivatives of the position coordinates give the velocities and accelerations. The difference in placement between the beginning and the final state of two points in a body expresses the numerical value of strain. Strain expresses itself as a change in size and/or shape.

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**potato - kinematics**

- nonlinear deformation map \( \varphi \)
  
  \[ x = \varphi(X,t) \quad \text{with} \quad \varphi : B_0 \times \mathbb{R} \rightarrow B_t \]

- spatial derivative of \( \varphi \) - deformation gradient
  
  \[ \frac{\text{d}x}{\text{d}X} = F \cdot \text{d}X \quad \text{with} \quad F : TB_0 \rightarrow TB_t \quad F = \frac{\partial \varphi}{\partial X} \bigg|_{t \text{ fixed}} \]

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1. Consider an elastic body \( B_0 \) at time \( t_0 \), unloaded & stressfree

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2. Imagine the body is cut into infinitesimal elements each of which is allowed to undergo volumetric growth
kinematics of finite growth

[1] consider an elastic body \( B_0 \) at time \( t_0 \), unloaded & stressfree
[2] imagine the body is cut into infinitesimal elements each of which is allowed to undergo volumetric growth
[3] after growing the elements, \( B_g \) may be incompatible

potato - kinematics of finite growth

• incompatible growth configuration \( B_g \) & growth tensor \( F_g \)

\[
F = F_c \cdot F_g
\]

rodriguez, hoger & mcculloch [1994]

biologically, the notion of incompatibility implies that subelements of the grown configuration may overlap or have gaps. the implication of incompatibility is the existence of residual stresses necessary to "squeeze" these grown subelements back together. mathematically, the notion of incompatibility implies that unlike the deformation gradient, \( F = \frac{\partial \varphi}{\partial X} \) fixed, the growth tensor cannot be derived as a gradient of a vector field. incompatible configurations are useful in finite strain inelasticity such as viscoelasticity, thermoelasticity, elastoplasticity and growth.
kinematics of finite growth

[3] after growing the elements, $B_g$ may be incompatible

[3a] we then first apply a deformation $F_c$ to squeeze the elements back together to the compatible configuration $B_c$

[3b] and then load the compatible configuration $B_c$ by $F_1$

[4] to generate the compatible current configuration $B_t$
kinematics of finite growth

\[ F = F_1 \cdot F_c \cdot F_g \]

residual stress

The additional deformation of squeezing the grown parts back to a compatible configuration gives rise to residual stresses (see thermal stresses).

concept of residual stress

how can we measure residual stress?

Figure. Compression test to quantify residual stresses in rhubarb. When peeling off the outer epidermal surface, the inner parenchyma core expands as growth-induced residual stresses are released. To quantify the amount of residual stress, the parenchyma core is mounted into the tissue holder \( B \) and loaded by the weight \( P \). The weight \( P \) is calibrated such that the pith recovers its initial length, and is thus a direct measure of the overall residual stress, adopted from Müller [1880].

müller, njc. handbuch der botanik, heidelberg, 1880.

example – growing mitral valves

Kinematics of growing mitral valves

Tsamis, Cheng, Nguyen, Langer, Miller, Kuhl [2012].
mitral regurgitation

- valves regulate unidirectional blood flow
- long-term, progressive leakage, back flow
- mitral regurgitation: 4 mio americans
- mitral valve repair: 300,000 worldwide/year

example – growing mitral valves

can leaflets self repair and grow?

annular geometry

approximation: 16 piecewise cubic splines

\[ c(s,t) = \sum_{i=0}^{3} b_{i,3}(s) \beta_i(t) \]

bernstein polynomials of degree three

\[ b_{0,3} = -s^3 + 3s^2 - 3s + 1 \]
\[ b_{1,3} = 3s^3 - 6s^2 + 3s \]
\[ b_{3,3} = s^3 \]
\[ b_{2,3} = -3s^3 + 3s^2 \]

determined from least square's problem

\[ \sum_{n=1}^{16} ||X_n - c_n(s,t)|| + \lambda \int_a^b ||d^2c(s,t)||^2 ds \rightarrow \min \]

rausch, bothe, kutting, swanson, miller, kuhl [2012]
annular strains

green lagrange annulus strain

\[ E(s, t) = \frac{1}{2} \left( \frac{d_s c(s, t)^2}{d_s c(s, t_0)^2} - 1 \right) \]

length of local tangent vectors

\[ d_s c(s, t) = \sum_{i=0}^{3} d_s b_{i,3}(s) \beta_i(t) \]

first derivative of bernstein coefficients

\[
\begin{align*}
    d_s b_{0,3} &= -3s^2 + 6s - 3 \\
    d_s b_{1,3} &= 9s^2 - 12s + 3 \\
    d_s b_{3,3} &= 3s^2 \\
    d_s b_{2,3} &= -9s^2 + 6s
\end{align*}
\]

annular curvature

\[ \kappa(s, t) = \frac{\|d_s c(s, t) \times d_s^2 c(s, t)\|}{\|d_s c(s, t)\|^3} \]

local second derivative

\[ d_s^2 c(s, t) = \sum_{i=0}^{3} d_s^2 b_{i,3}(s) \beta_i(t) \]

second derivative of bernstein coefficients

\[
\begin{align*}
    d_s^2 b_{0,3} &= -6s + 6 \\
    d_s^2 b_{1,3} &= 18s - 12 \\
    d_s^2 b_{3,3} &= 6s \\
    d_s^2 b_{2,3} &= -18s + 6
\end{align*}
\]
do mitral annuli grow?

rausch, tibayan, ingels, miller, kuhl [2013]

example – growing mitral valves

rausch, tibayan, ingels, miller, kuhl [2013]

example – growing mitral valves

4

how can we prevent growth?

figure 4. Image of the disease-specific annuloplasty ring for ischemic mitral regurgitation. A top view on the left shows the ring’s characteristic features: an asymmetric outline with reduced curvature at the P3 segment and an increased sewing margin. The image on the right shows the same ring from a lateral perspective, with its characteristic dip close to the P3 segment. Photograph courtesy of edwards lifesciences.

rausch, tibayan, ingels, miller, kuhl [2013]

example – growing mitral valves

rausch, tibayan, ingels, miller, kuhl [2013]

example – growing mitral valves

can leaflets self repair and grow?

23 implanted markers

finite element model

rausch, famaei, shultz, bothe, miller, kuhl [2012]

collagen fiber orientation

rausch, famaei, shultz, bothe, miller, kuhl [2012]

example – growing mitral valves

leaflet geometry

reference and current configurations

\[ X(\theta^1, \theta^2) = \sum_{l=1}^{n_{\text{nod}}} N_l(\theta^1, \theta^2) X_l \]

\[ x(\theta^1, \theta^2) = \sum_{l=1}^{n_{\text{nod}}} N_l(\theta^1, \theta^2) x_l \]

covariant base vectors

\[ G_a(\theta^1, \theta^2) = \sum_{l=1}^{n_{\text{nod}}} \frac{\partial N_l}{\partial \theta^a} X_l \]

\[ g_{a\beta} = g_{a} \cdot g_{\beta} \]

covariant surface metrics

\[ g^\alpha = g^a_{\beta} g_{\beta} \]

\[ g^{\alpha\beta} = g^\alpha \cdot g^\beta = [g_{a\beta}]^{-1} \]

covariant base vectors

rausch, bothe, kvitting, goktepe, miller, kuhl [2011]
example – growing mitral valves

leaflet strains

e = e_{αβ} g^{α} \otimes g^{β}

\text{e}_{αβ} = \frac{1}{2} \left[ g_{αβ} - G_{αβ} \right]

circumferential strains and stretches

\varepsilon_{\text{circ}} = n^{\text{circ}} \cdot \varepsilon \cdot n^{\text{circ}}

\lambda_{\text{circ}} = \left[ 1 - 2 \varepsilon_{\text{circ}} \right]^{-1/2}

radial strains and stretches

\varepsilon_{\text{rad}} = n^{\text{rad}} \cdot \varepsilon \cdot n^{\text{rad}}

\lambda_{\text{rad}} = \left[ 1 - 2 \varepsilon_{\text{rad}} \right]^{-1/2}

area strains and stretches

\varepsilon_{\text{area}} = \frac{\text{da} - \text{d}A}{\text{da}}

\lambda_{\text{area}} = \frac{\text{da}}{\text{d}A} = \frac{1}{\left[ 1 - \varepsilon_{\text{area}} \right]}

rausch, bothe, kvitting, goktepe, miller, kuhl [2011]

can leaflets self repair and grow?

rausch, tibayan, miller, kuhl [2012]