16 - everything grows!
midterm summary

day	date		topic
tue	jan	10	motivation - everything grows!
thu	jan	12	basics maths - notation and tensors
tue	jan	17	basic kinematics - large deformation and growth
thu	jan	19	kinematics - growing hearts
tue	jan	24	guest lecture - growing skin
thu	jan	26	guest lecture - growing leaflets
tue	jan	31	basic balance equations - closed and open systems
thu	feb	02	basic constitutive equations - growing tumors
tue	feb	07	volume growth - finite elements for growth
thu	feb	09	volume growth - growing arteries
tue	feb	14	volume growth - growing skin
thu	feb	16	volume growth - growing hearts
tue	feb	21	basic constitutive equations - growing bones
thu	feb	23	density growth - finite elements for growth
tue	feb	28	density growth - growing bones
thu	mar	01	everything grows! - midterm summary
tue	mar	06	midterm
thu	mar	08	remodeling - remodeling arteries and tendons
tue	mar	13	class project - discussion, presentation, evaluation
thu	mar	15	class project - discussion, presentation, evaluation
thu	mar	15	written part of final projects due

me337 - goals

in contrast to traditional engineering structures living structures show the fascinating ability to **grow and adapt their form, shape and microstructure** to a given mechanical environment. this course addresses the phenomenon of growth on a theoretical and computational level and applies the resulting theories to classical biomechanical problems like bone remodeling, hip replacement, wound healing, atherosclerosis or in stent restenosis. this course will illustrate how classical engineering concepts like continuum mechanics, thermodynamics or finite element modeling have to be rephrased in the context of growth. having attended this course, you will be able to develop your own problemspecific finite element based numerical solution techniques and interpret the results of biomechanical simulations with the ultimate goal of improving your **understanding of the complex interplay between form and function**.

me 337 - grading

- 30 % homework - 3 homework assignments, 10% each
- 30 % midterm - closed book, closed notes, one single page cheat sheet
- 20 % final project oral presentations - graded by the class
- 20 % final project essay - graded by instructor

growth, remodeling and morphogenesis

growth [groʊθ] which is defined as **added mass**, can occur through

- hyperplasia / cell division
- hypertrophy / cell enlargement
- secretion of extracellular matrix
- accretion @external or internal surfaces

mass = density x volume

mass changes = density changes x volume changes

taber „biomechanics of growth, remodeling and morphogenesis" [1995]

growth, remodeling and morphogenesis

remodeling [ri mɑd. l.mg] involves **changes in material properties.** these changes, which often are adaptive, may be brought about by alterations in modulus, internal structure, strength, or density. for example, bones, and heart muscle may change their internal structures through reorientation of trabeculae and muscle fibers, respectively.

taber „biomechanics of growth, remodeling and morphogenesis" [1995]

growth, remodeling and morphogenesis

morphogenesis [mɔːr.fo'dʒen.ə.sɪs] is the generation of animal form. usually, the term refers to **embryonic development,** but wound **healing** and organ **regeneration** are also morphogenetic events. morphogenesis contains a complex series of stages, each of which depends on the previous stage. during these stages, genetric and environmental factors guide the spatial-temporal motions and differentiation (specification) of cells. a flaw in any one stage may lead to structural defects.

taber „biomechanics of growth, remodeling and morphogenesis" [1995]

tensor algebra - invariants

- (principal) invariants of second order tensor

$$\begin{aligned} I_A &= \mathrm{tr}\,(\boldsymbol{A}) \\ II_A &= \tfrac{1}{2}\left[\mathrm{tr}^2\,(\boldsymbol{A}) - \mathrm{tr}\,(\boldsymbol{A}^2)\right] \\ III_A &= \det\,(\boldsymbol{A}) \end{aligned}$$

- derivatives of invariants wrt second order tensor

$$\begin{aligned} \partial_{\boldsymbol{A}}\ I_A &= \boldsymbol{I} \\ \partial_{\boldsymbol{A}}\ II_A &= I_A\,\boldsymbol{I} - \boldsymbol{A} \\ \partial_{\boldsymbol{A}}\ III_A &= III_A\,\boldsymbol{A}^{-\mathrm{t}} \end{aligned}$$

constitutive equations are formulated in terms of invariants!

tensor algebra - determinant

- determinant of second order tensor $III_A = \det(\boldsymbol{A})$

$$\begin{aligned}\det(\boldsymbol{A}) &= \det(A_{ij}) = \tfrac{1}{6}\, e_{ijk}\, e_{abc}\, A_{ia}\, A_{jb}\, A_{kc} \\ &= A_{11}A_{22}A_{33} + A_{21}A_{32}A_{13} + A_{31}A_{12}A_{23} \\ &- A_{11}A_{23}A_{32} - A_{22}A_{31}A_{13} - A_{33}A_{12}A_{21}\end{aligned}$$

- properties of determinants of second order tensors

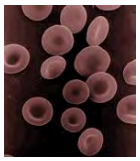
$$\begin{aligned}\det(\boldsymbol{I}) &= 1 \\ \det(\boldsymbol{A}^{\mathrm{t}}) &= \det(\boldsymbol{A}) \\ \det(\alpha\,\boldsymbol{A}) &= \alpha^3 \det(\boldsymbol{A}) \\ \det(\boldsymbol{A}\cdot\boldsymbol{B}) &= \det(\boldsymbol{A})\det(\boldsymbol{B})\end{aligned}$$

the determinant is related to volume changes!

continuum mechanics

continuum hypothesis [kɔnˈtɪn.ju.əm haɪˈpɑːθ.ə.sɪs]
we assume that the characteristic length scale of the microstructure is much smaller than the characteristic length scale of the overall problem, such that the properties at each point can be understood as averages over a characteristic length scale

$$l^{\text{micro}} \ll l^{\text{averg}} \ll l^{\text{conti}}$$



example: biomechanics

$$l^{\text{micro}} = l^{\text{cells}} \approx 10\mu\text{m}$$
$$l^{\text{conti}} = l^{\text{tissue}} \approx 10\text{cm}$$

continuum hypothesis can be applied to analyzing tissues

the potato equations

- kinematic equations - what's strain? $\epsilon = \frac{\Delta l}{l}$
  general equations that characterize the deformation of a physical body without studying its physical cause
- balance equations - what's stress? $\sigma = \frac{F}{A}$
  general equations that characterize the cause of motion of any body
- constitutive equations - how are they related? $\sigma = E\,\epsilon$
  material specific equations that complement the set of governing equations

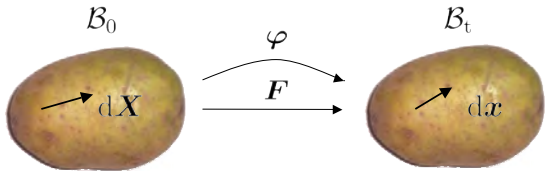
the potato equations

- kinematic equations - why not $\epsilon = \frac{\Delta l}{l}$?
  inhomogeneous deformation » non-constant
  finite deformation » non-linear $\boldsymbol{F} = \nabla_X \boldsymbol{\varphi}$
  inelastic deformation » growth tensor $\boldsymbol{F} = \boldsymbol{F}_{\mathrm{e}} \cdot \boldsymbol{F}_{\mathrm{g}}$
- balance equations - why not $\sigma = \frac{F}{A}$? $\mathrm{Div}(\boldsymbol{P}) + \rho\,\boldsymbol{b}_0 = \boldsymbol{0}$
  equilibrium in deformed configuration » multiple stress measures
- constitutive equations - why not $\sigma = E\,\epsilon$?
  finite deformation » non-linear $\boldsymbol{P} = \boldsymbol{P}(\boldsymbol{F})$
  inelastic deformation » internal variables $\boldsymbol{P} = \boldsymbol{P}(\rho, \boldsymbol{F}, \boldsymbol{F}_{\mathrm{g}})$

kinematic equations

kinematic equations [kɪnəˈmætɪk ɪˈkweɪʒənz] describe the motion of objects without the consideration of the masses or forces that bring about the motion. the basis of kinematics is the choice of coordinates. the 1st and 2nd time derivatives of the position coordinates give the velocities and accelerations. the difference in placement between the beginning and the final state of two points in a body expresses the numerical value of strain. strain expresses itself as a change in size and/or shape.

kinematics equations



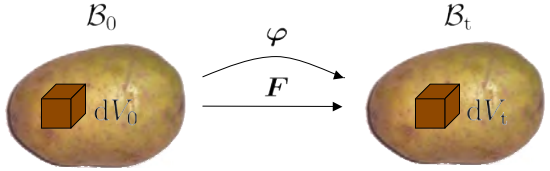
- transformation of line elements - deformation gradient F_{ij}

$\mathrm{d}x_i = F_{ij}\,\mathrm{d}X_j$ with $F_{ij} : T\mathcal{B}_0 \to T\mathcal{B}_t$ $F_{ij} = \left.\frac{\partial \varphi_i}{\partial X_j}\right|_{t \text{ fixed}}$

- uniaxial tension (incompressible), simple shear, rotation

$$F_{ij}^{\text{uni}} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha^{-\frac{1}{2}} & 0 \\ 0 & 0 & \alpha^{-\frac{1}{2}} \end{bmatrix} \quad F_{ij}^{\text{shr}} = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad F_{ij}^{\text{rot}} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

kinematic equations



- transformation of volume elements - determinant of $\boldsymbol{F}$

$$\mathrm{d}V_0 = \mathrm{d}\boldsymbol{X}_1 \cdot [\mathrm{d}\boldsymbol{X}_2 \times \mathrm{d}\boldsymbol{X}_3] \qquad \mathrm{d}V_t = \mathrm{d}\boldsymbol{x}_1 \cdot [\mathrm{d}\boldsymbol{x}_2 \times \mathrm{d}\boldsymbol{x}_3]$$

$$= \det([\mathrm{d}\boldsymbol{x}_1, \mathrm{d}\boldsymbol{x}_2, \mathrm{d}\boldsymbol{x}_3])$$

$$= \det([\mathrm{d}\boldsymbol{X}_1, \mathrm{d}\boldsymbol{X}_2, \mathrm{d}\boldsymbol{X}_3]) \qquad = \det(\boldsymbol{F})\ \det([\mathrm{d}\boldsymbol{X}_1, \mathrm{d}\boldsymbol{X}_2, \mathrm{d}\boldsymbol{X}_3])$$

- changes in volume - determinant of deformation gradient J

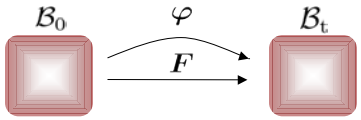
$$\mathrm{d}V_t = J\,\mathrm{d}V_0 \qquad J = \det(\boldsymbol{F})$$

volume growth

volume growth ['vɑl.ju:m groʊθ] is conceptually comparable to thermal expansion. in linear elastic problems, growth stresses (such as thermal stresses) can be superposed on the mechanical stress field. in the nonlinear problems considered here, another approach must be used. the fundamental idea is to refer the strain measures in the constitutive equations of each material element to its current zero-stress configuration, which changes as the element grows.

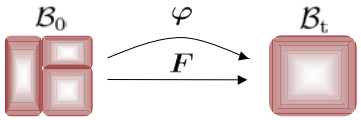
taber "biomechanics of growth, remodeling and morphogenesis" [1995]

kinematics of finite growth



[1] consider an elastic body $\mathcal{B}_0$ at time t_0, unloaded &stressfree

kinematics of finite growth



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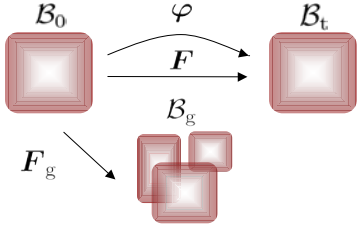
[2] imagine the body is cut into infinitesimal elements each of which is allowed to undergo volumetric growth

kinematics of finite growth



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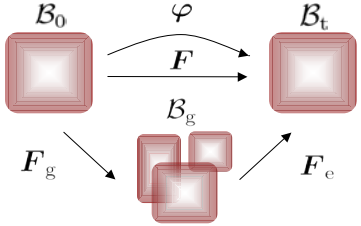
kinematics of finite growth



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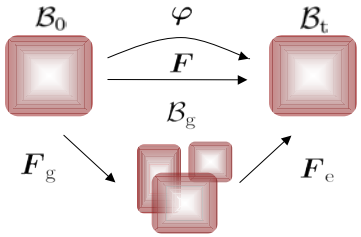
[4] loading generates compatible current configuration $\mathcal{B}_t$

concept of incompatible growth configuration

biologically, the notion of **incompatibility** implies that subelements of the grown configuration may overlap or have gaps. the implication of incompatibility is the existence of residual stresses necessary to `squeeze` these grown subelements back together. mathematically, the notion of **incompatibility** implies that unlike the deformation gradient, $\boldsymbol{F} = \left.\frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{X}}\right|_{t \text{ fixed}}$ the growth tensor cannot be derived as a gradient of a vector field. incompatible configurations are useful in finite strain inelasticity such as viscoelasticity, thermoelasticity, elastoplasticity and growth.

kinematics of finite growth



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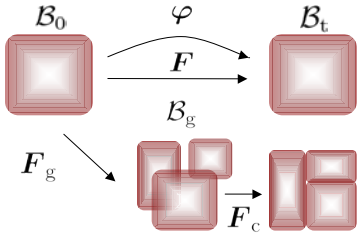
kinematics of finite growth

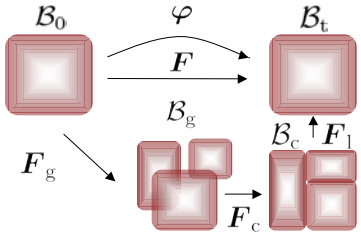


[3] after growing the elements, $\mathcal{B}_g$ may be incompatible

[3a] we then first apply a deformation $\boldsymbol{F}_c$ to squeeze the elements back together to the compatible configuration $\mathcal{B}_c$

[4] to generate the compatible current configuration $\mathcal{B}_t$

kinematics of finite growth



[3] after growing the elements, $\mathcal{B}_g$ may be incompatible

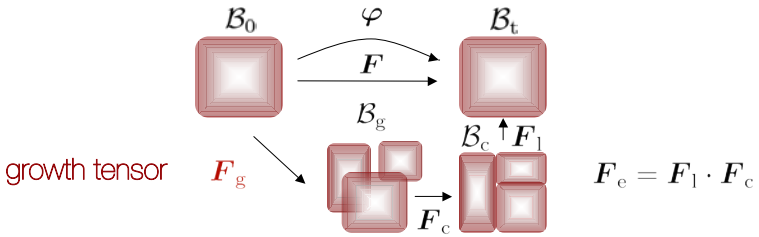
[3a] we then first apply a deformation $\boldsymbol{F}_c$ to squeeze the elements back together to the compatible configuration $\mathcal{B}_c$

[3b] and then load the compatible configuration $\mathcal{B}_c$ by $\boldsymbol{F}_l$

[4] to generate the compatible current configuration $\mathcal{B}_t$

kinematics of finite growth

$$\boldsymbol{F} = \boldsymbol{F}_\mathrm{l} \cdot \boldsymbol{F}_\mathrm{c} \cdot \boldsymbol{F}_\mathrm{g}$$



growth tensor $\boldsymbol{F}_\mathrm{g}$

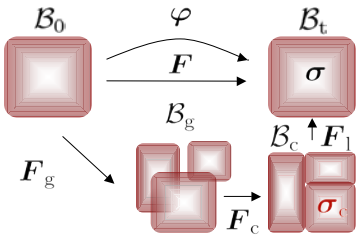
$$\boldsymbol{F}_\mathrm{e} = \boldsymbol{F}_\mathrm{l} \cdot \boldsymbol{F}_\mathrm{c}$$

multiplicative decomposition

lee [1969], simo [1992], rodriguez, hoger & mc culloch [1994], epstein & maugin [2000], humphrey [2002], ambrosi & mollica [2002], himpel, kuhl, menzel & steinmann [2005]

kinematics of finite growth

$$\boldsymbol{F} = \boldsymbol{F}_\mathrm{l} \cdot \boldsymbol{F}_\mathrm{c} \cdot \boldsymbol{F}_\mathrm{g}$$



total stress

residual stress

residual stress

the additional deformation of squeezing the grown parts back to a compatible configuration gives rise to residual stresses (see thermal stresses)

the classical opening angle experiment

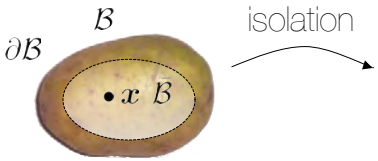
an existence of residual strains in human arteries is well known. it can be observed as an opening up of a circular arterial segment after a radial cut. an opening angle of the arterial segment is used as a measure of the residual strains generally.

fung [1990], horný, chlup, zitný, mackov [2006]

balance equations

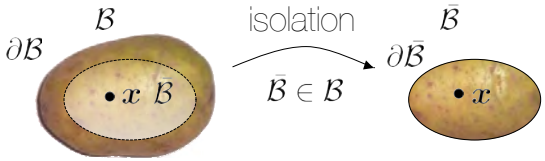
balance equations [ˈbæl.əns ɪˈkweɪ.ʒəns] of mass, momentum, angular momentum and energy, supplemented with an entropy inequality constitute the set of conservation laws. the law of **conservation of mass**/matter states that the **mass of a closed system** of substances will remain **constant**, regardless of the processes acting inside the system. the principle of conservation of momentum states that the total momentum of a closed system of objects is constant.

balance equations



[1] isolation of subset $\bar{\mathcal{B}}$ from $\mathcal{B}$

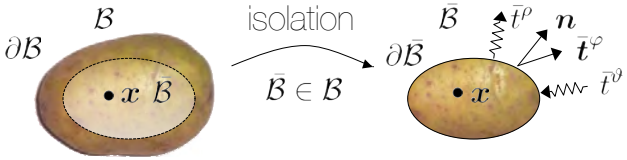
balance equations



[1] isolation of subset $\bar{\mathcal{B}}$ from $\mathcal{B}$

[2] characterization of influence of remaining body through phenomenological quantities - contact fluxes $\bar{t}^{\rho}$, $\bar{\boldsymbol{t}}^{\varphi}$ & $\bar{t}^{\vartheta}$

balance equations



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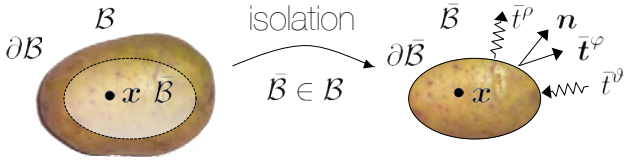
balance equations



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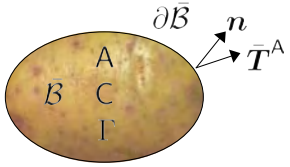
[4] postulation of balance of these quantities

generic balance equation - closed systems

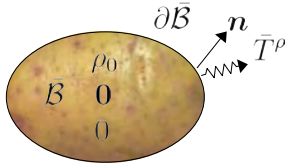


general format

A ... balance quantity
B ... flux $\quad \mathbf{B} \cdot \boldsymbol{n} = \bar{\boldsymbol{T}}^{\mathsf{A}}$
C ... source
Γ ... production

$$\mathrm{D}_t \mathsf{A} = \mathrm{Div}(\mathbf{B}) + \mathsf{C} + \Gamma$$

balance of mass - closed systems



balance of mass

ρ_0 ... density
$\mathbf{0}$ ... no mass flux $\quad \bar{T}^{\rho} = 0$
0 ... no mass source
0 ... no mass production

continuity equation $\quad \mathrm{D}_t \rho_0 = 0$

thermodynamic systems - open systems

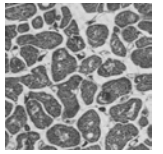
open system ['oʊ.pən 'sɪs.təm] thermodynamic system which is allowed to exchange mechanical work, heat and mass, typically $\boldsymbol{P} = \boldsymbol{P}(\nabla\varphi, ...)$, $\boldsymbol{Q} = \boldsymbol{Q}(\nabla\theta, ...)$ and $\boldsymbol{R} = \boldsymbol{R}(\nabla\rho, ...)$ with its environment. enclosed by a deformable, diathermal, permeable membrane. characterized through its state of deformation φ, temperature θ and density ρ.

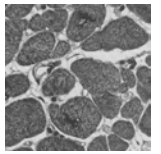
balance of mass - open systems

$$\mathrm{D}_t \rho_0 = \mathrm{Div}(\boldsymbol{R}) + \mathcal{R}_0$$

mass flux $\boldsymbol{R}$
- cell movement (migration)

mass source $\mathcal{R}_0$
- cell growth (proliferation)
- cell division (hyperplasia)
- cell enlargement (hypertrophy)





biological equilbrium

cowin & hegedus [1976], beaupré, orr & carter [1990], harrigan & hamilton [1992], jacobs, levenston, beaupré, simo & carter [1995], huiskes [2000], carter & beaupré [2001]

constitutive equations

constitutive equations [kənˈstɪ.tu.tɪv ɪˈkweɪ.ʒənz] in structural analysis, constitutive relations **connect applied stresses** or forces **to strains** or deformations. the constitutive relations for linear materials are linear. more generally, in physics, a constitutive equation is a relation between two physical quantities (often tensors) that is specific to a material, and does not follow directly from physical law. some constitutive equations are **simply phenomenological;** others are **derived from first principles.**

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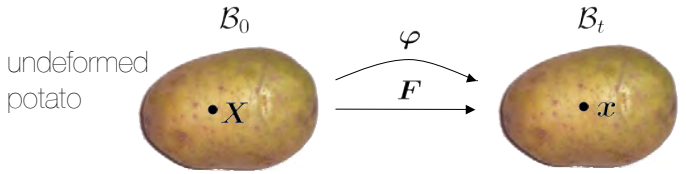
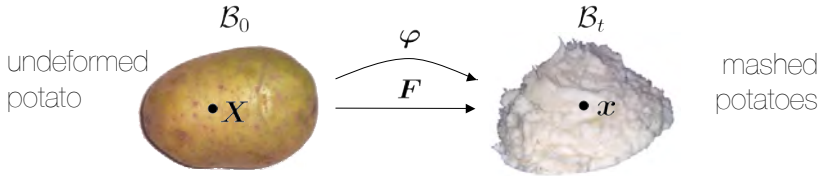
neo hooke'ian elasticity



- free energy $\psi_0^{\text{neo}} = \frac{1}{2}\lambda_0 \ln^2(\det(\boldsymbol{F})) + \frac{1}{2}\mu_0[\,\boldsymbol{F}^{\text{t}}\cdot\boldsymbol{F} : \boldsymbol{I} - n^{\text{dim}} - 2\ln(\det(\boldsymbol{F}))\,]$
- definition of stress

$$\boldsymbol{P}^{\text{neo}} = \mathrm{D}_F\psi_0^{\text{neo}} = \mu_0\,\boldsymbol{F} + [\,\lambda_0\ln(\det(\boldsymbol{F})) - \mu_0\,]\boldsymbol{F}^{-\text{t}}$$

neo hooke'ian elasticity



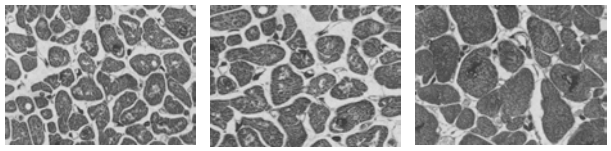
- free energy $\psi^{\text{neo}} = \frac{1}{2}\lambda\ln^2(\det(\boldsymbol{F})) + \frac{1}{2}\mu[\,\boldsymbol{F}^{\text{t}}\cdot\boldsymbol{F} : \boldsymbol{I} - n^{\text{dim}} - 2\ln(\det(\boldsymbol{F}))\,]$
- definition of stress

$$\boldsymbol{P}^{\text{neo}} = \rho_0\mathrm{D}_F\psi = \mu\,\boldsymbol{F} + [\,\lambda\ln(\det(\boldsymbol{F})) - \mu\,]\boldsymbol{F}^{-\text{t}}$$

- remember! mashing potatoes is not an elastic process!

volume growth at constant density

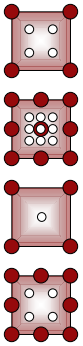
- free energy $\psi_{\text{g}} = \psi_0^{\text{neo}}(\boldsymbol{F}_{\text{e}})$
- stress $\boldsymbol{P}_{\text{e}} = \boldsymbol{P}_{\text{e}}^{\text{neo}}(\boldsymbol{F}_{\text{e}})$
- growth tensor $\boldsymbol{F}_{\text{g}} = \vartheta\,\boldsymbol{I}$ $\quad \mathrm{D}_t\vartheta = k_\vartheta(\vartheta)\,\mathrm{tr}(\boldsymbol{C}_{\text{e}}\cdot\boldsymbol{S}_{\text{e}})$ (growth function, pressure)
- mass source $\mathcal{R}_0 = 3\,\rho_0\vartheta^2\mathrm{D}_t\vartheta$ (increase in mass)



kinematic coupling of growth and deformation

rodriguez, hoger & mc culloch [1994], epstein & maugin [2000], humphrey [2002]

integration point based solution of growth equation



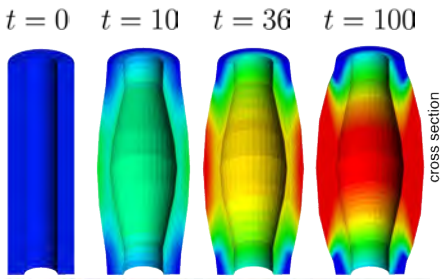
loop over all time steps
- global newton iteration
  - loop over all elements
    - loop over all quadrature points
      - local newton iteration to determine ϑ_{n+1}
      - determine element residual & partial derivative
    - determine global residual and iterational matrix
  - determine φ_{n+1}
- determine state of biological equilibrium

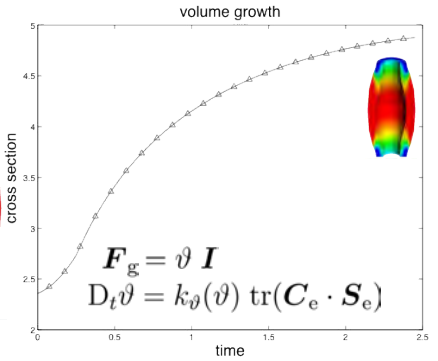
growth multiplier ϑ as internal variable

in-stent restenosis

restenosis is the reoccurrence of stenosis, the narrowing of a blood vessel, leading to restricted blood flow. restenosis usually pertains to a blood vessel that has become narrowed, received treatment, and subsequently became renarrowed. in some cases, surgical procedures to widen blood vessels can cause further narrowing. during balloon angioplasty, the balloon 'smashes' the plaques against the arterial wall to widen the size of the lumen. however, this damages the wall which responds by using physiological mechanisms to repair the damage and the wall thickens.

qualitative simulation of stent implantation

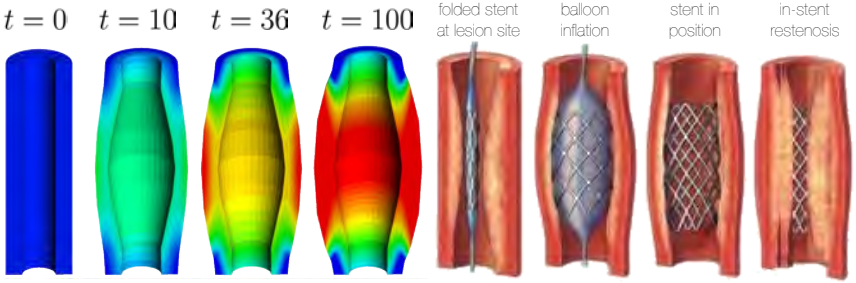




stress-induced volume growth

kuhl, maas, himpel & menzel [2007]

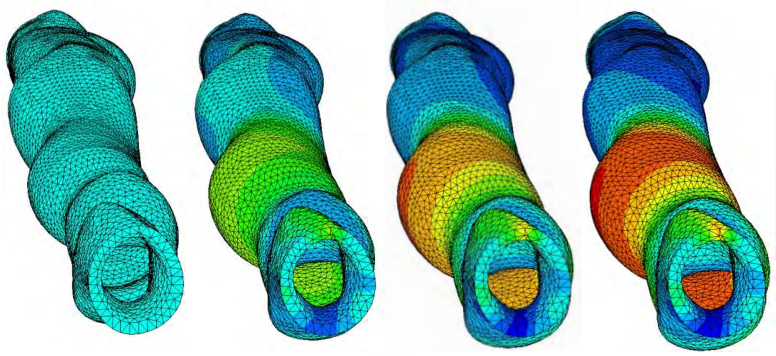
qualitative simulation of stent implantation



stress-induced volume growth

kuhl, maas, himpel & menzel [2007]

virtual stent implantation - patient specific model



tissue growth - response to virtual stent implantation

kuhl, maas, himpel & menzel [2007]

example - stenting and restenosis

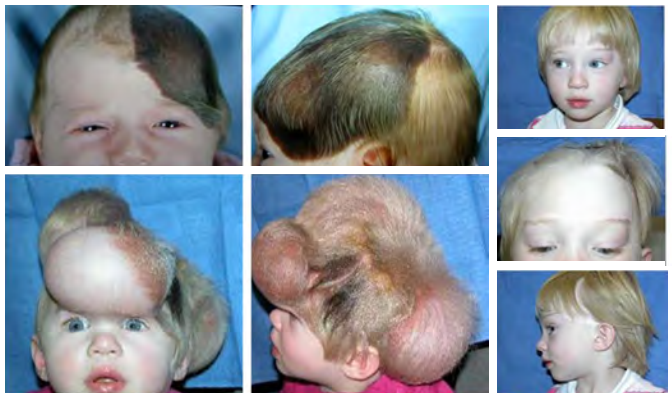
skin expansion

skin expansion is a technique used by plastic and restorative surgeons to cause the body grow additional skin. keeping living tissues under tension causes new cells to form and the amount of tissue to increase. in some cases, this may be accomplished by the implantation of inflatable balloons under the skin. by far the most common method, the surgeon inserts the inflatable expander beneath the skin and periodically, over weeks or months, injects a saline solution to slowly stretch the overlying skin. the growth of tissue is permanent, but will retract to some degree when the expander is removed. within the past 30 years, skin expansion has revolutionized reconstructive surgery. typical applications are breast reconstruction, burn injuries, and pediatric plastic surgery.

WIKIPEDIA
The Free Encyclopedia

example - skin expansion and growth

skin expansion and growth - facial reconstruction

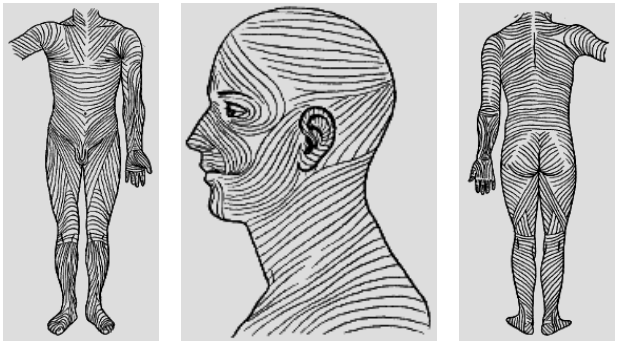


in this study of reconstruction of the forehead in children, the average number of surgical procedures required to complete reconstruction was six, involving an average of three tissue expansion proecures. gosain & cortes [2007]

example - skin expansion and growth

example - skin expansion and growth

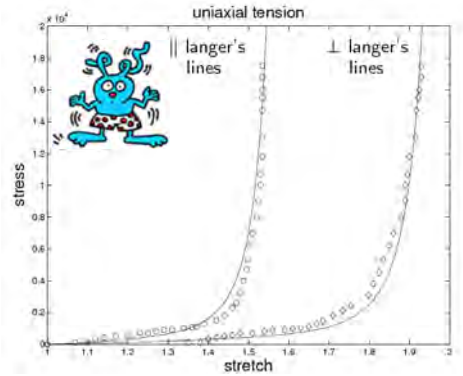
langer's lines - anisotropy of human skin



lines of tension - orientation of collagen fiber bundles

carl ritter von langer [1819-1887]

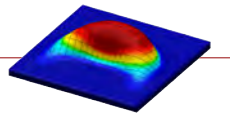
experiment vs simulation - rabbit skin



stiffer || to langer's lines - stress locking @crit stretch

lanir & fung [1974], kuhl, garikipati, arruda & grosh [2005]

volume growth at constant density



- deformation gradient

$$\boldsymbol{F} = \boldsymbol{F}^{\mathrm{e}} \cdot \boldsymbol{F}^{\mathrm{g}} \qquad \text{with} \qquad \boldsymbol{F} = \nabla_{\boldsymbol{X}} \boldsymbol{\varphi}$$

- jacobians ... remember: volume change

$$J = J^{\mathrm{e}}\, J^{\mathrm{g}} \qquad \text{with} \qquad J = \det(\boldsymbol{F})$$
$$J^{\mathrm{e}} = \det(\boldsymbol{F}^{\mathrm{e}}) \text{ and } J^{\mathrm{g}} = \det(\boldsymbol{F}^{\mathrm{g}})$$

- cofactor ... remember: area change

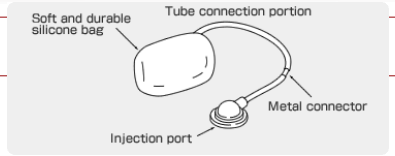
$$\vartheta = \vartheta^{\mathrm{e}}\, \vartheta^{\mathrm{g}} \qquad \text{with} \qquad \vartheta = \| \operatorname{cof}(\boldsymbol{F}) \cdot \boldsymbol{n}_0 \|$$
$$\vartheta^{\mathrm{e}} = \| \operatorname{cof}(\boldsymbol{F}^{\mathrm{e}}) \cdot \boldsymbol{n}_0 \| \text{ and } \vartheta^{\mathrm{g}} = \| \operatorname{cof}(\boldsymbol{F}^{\mathrm{g}}) \cdot \boldsymbol{n}_0 \|$$

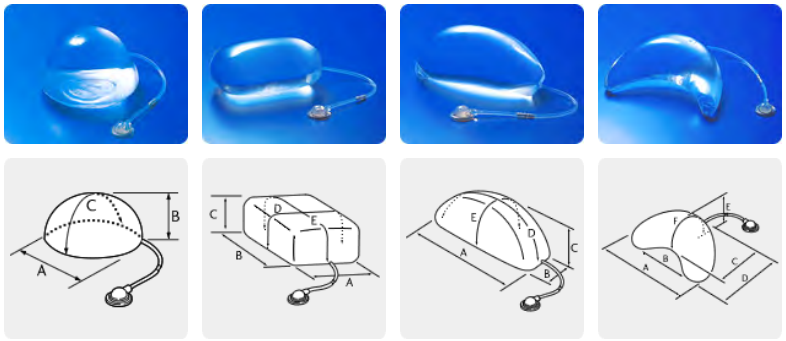
- growth tensor ... growth = area change

$$\boldsymbol{F}^{\mathrm{g}} = \sqrt{\vartheta^{\mathrm{g}}}\, \boldsymbol{I} + [\, 1 - \sqrt{\vartheta^{\mathrm{g}}}\,]\, \boldsymbol{n}_0 \otimes \boldsymbol{n}_0$$

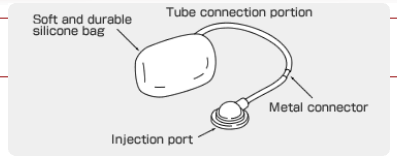
the adrian model [2010]

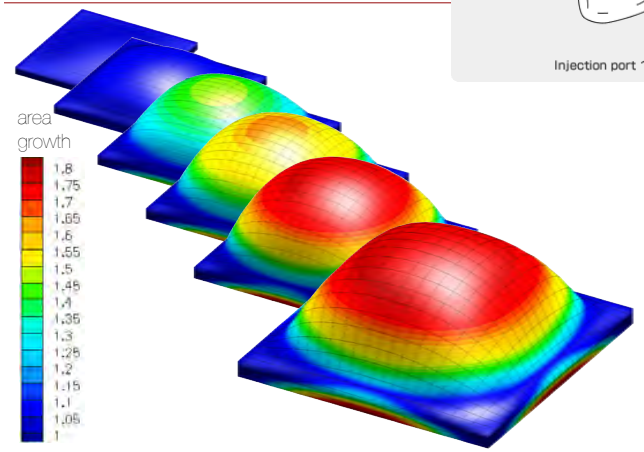
skin expanders

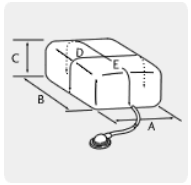




skin expansion and growth

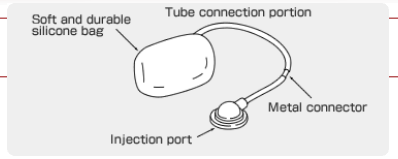


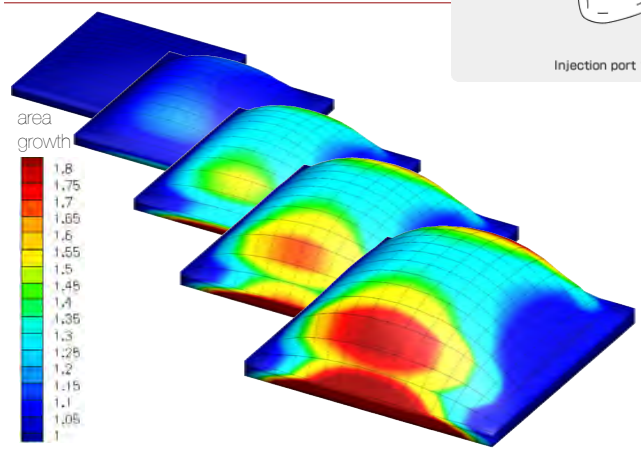


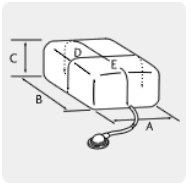


example - skin expansion and growth

skin expansion and growth

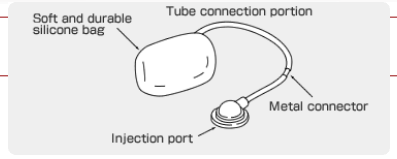


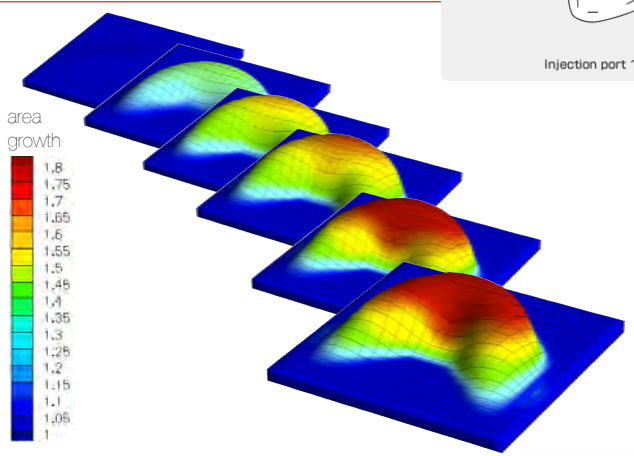


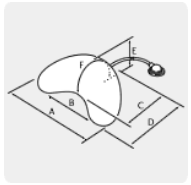


example - skin expansion and growth

skin expansion and growth







example - skin expansion and growth

different forms of cardiac growth

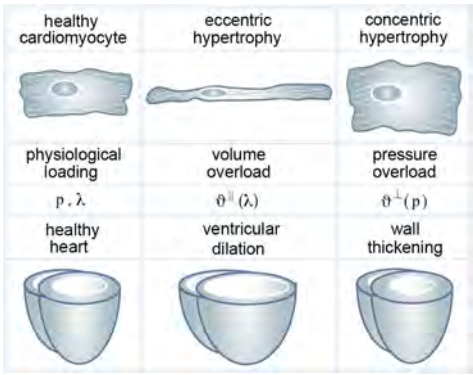
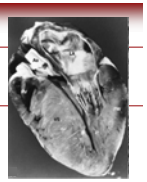


Figure 3. Eccentric and concentric growth on the cellular and organ levels. Compared with the normal heart (left), volume-overload induced eccentric hypertrophy is associated with cell lengthening through the serial deposition of sarcomere units and manifests itself in ventricular dilation in response to volume-overload (center). Pressure-overload induced concentric hypertrophy is associated with cell thickening through the parallel deposition of sarcomere units and manifests itself in ventricular wall thickening in response to pressure-overload (right).

göktepe, abilez, parker, kuhl [2010]

example - cardiac growth

athlete's heart



- multiplicative decomposition
  $\boldsymbol{F} = \boldsymbol{F}^{\mathrm{e}} \cdot \boldsymbol{F}^{\mathrm{g}}$ with $\boldsymbol{F} = \nabla_{\boldsymbol{X}} \boldsymbol{\varphi}$
- growth tensor
  $\boldsymbol{F}^{\mathrm{g}} = \vartheta^{\mathrm{g}}\, \boldsymbol{I}$
- evolution of isotropic growth multiplier
  cardiomyocyte volume increase rate
  $\dot{\vartheta}^{\mathrm{g}} = k^{\mathrm{g}}(\vartheta^{\mathrm{g}})\, \phi^{\mathrm{g}}(\boldsymbol{M}^{\mathrm{e}})$ with $k^{\mathrm{g}}(\vartheta^{\mathrm{g}}) = \frac{1}{\tau} \left[\frac{\vartheta^{\max} - \vartheta^{\mathrm{g}}}{\vartheta^{\max} - 1} \right]^{\gamma}$
- growth criterion
  $\phi^{\mathrm{g}} = \mathrm{tr}(\boldsymbol{M}^{\mathrm{e}}) - M^{\mathrm{e}\,\mathrm{crit}}$

stress-driven isotropic growth

cardiac enlargement through stress-driven isotropic growth

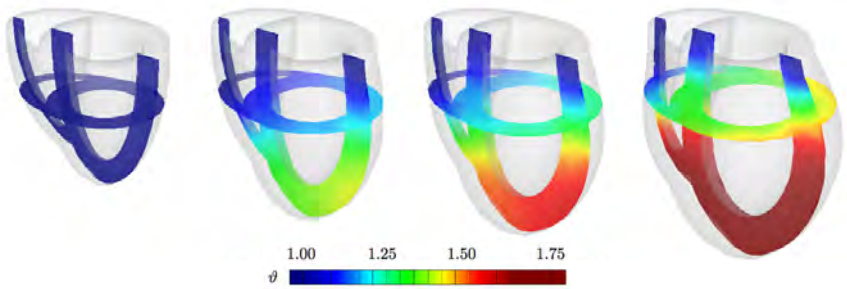


Figure 7. Athlete's heart, stress-driven isotropic eccentric and concentric growth, left ventricular dilation and wall thickening. The isotropic growth multiplier gradually increases from 1.00 to 1.75 as the individual cardiomyocytes grow both eccentrically and concentrically. On the macroscopic scale, the athlete's heart manifests itself in a progressive apical growth with a considerably increase in left ventricular cavity size to enable increased cardiac output during excercise. To withstand higher blood pressure levels during training, the heart muscle grows and the wall thickens.

goktepe, abilez, kuhl [2010]

cardiac dilation

- multiplicative decomposition
  $\boldsymbol{F} = \boldsymbol{F}^{\mathrm{e}} \cdot \boldsymbol{F}^{\mathrm{g}}$ with $\boldsymbol{F} = \nabla_{\boldsymbol{X}} \boldsymbol{\varphi}$
- growth tensor
  $\boldsymbol{F}^{\mathrm{g}} = \boldsymbol{I} + [\lambda^{\mathrm{g}} - 1]\, \boldsymbol{f}_0 \otimes \boldsymbol{f}_0$
- evolution of eccentric growth multiplier
  serial sarcomere deposition rate
  $\dot{\lambda}^{\mathrm{g}} = k^{\mathrm{g}}(\lambda^{\mathrm{g}})\, \phi^{\mathrm{g}}(\lambda^{\mathrm{e}})$ with $k^{\mathrm{g}} = \frac{1}{\tau} \left[\frac{\lambda^{\max} - \lambda^{\mathrm{g}}}{\lambda^{\max} - 1} \right]^{\gamma}$
- growth criterion
  $\phi^{\mathrm{g}} = \lambda^{\mathrm{e}} - \lambda^{\mathrm{crit}} = \frac{\lambda}{\lambda^{\mathrm{g}}} - \lambda^{\mathrm{crit}}$

strain-driven eccentric transversely isotropic growth

cardiac dilation through strain-driven eccentric growth

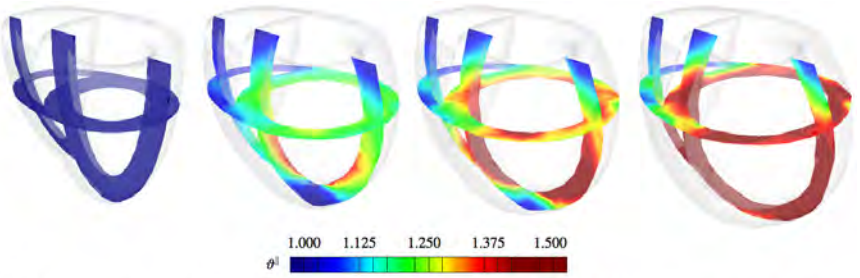
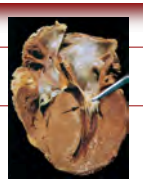


Figure 10. Strain-driven eccentric growth. The eccentric growth multiplier gradually increases from 1.00 to 1.50 as the individual cardiomyocytes grow eccentrically. On the structural level, eccentric growth manifests itself in a progressive dilation of the left ventricle accompanied by a significant increase in cardiac mass, while the thickness of the ventricular wall remains virtually unchanged.

goktepe, abilez, parker, kuhl [2010]

cardiac wall thickening



- multiplicative decomposition

$$\boldsymbol{F} = \boldsymbol{F}^{\mathrm{e}} \cdot \boldsymbol{F}^{\mathrm{g}} \qquad \text{with} \quad \boldsymbol{F} = \nabla_{\boldsymbol{X}} \boldsymbol{\varphi}$$

- growth tensor

$$\boldsymbol{F}^{\mathrm{g}} = \boldsymbol{I} + [\,\vartheta^{\mathrm{g}} - 1\,]\,\boldsymbol{s}_0 \otimes \boldsymbol{s}_0$$

- evolution of concentric growth multiplier
  parallel sarcomere deposition rate

$$\dot{\vartheta}^{\mathrm{g}} = k^{\mathrm{g}}(\vartheta^{\mathrm{g}})\,\phi^{\mathrm{g}}(\boldsymbol{M}^{\mathrm{e}}) \quad \text{with} \quad k^{\mathrm{g}}(\vartheta^{\mathrm{g}}) = \frac{1}{\tau}\left[\frac{\vartheta^{\max} - \vartheta^{\mathrm{g}}}{\vartheta^{\max} - 1}\right]^{\gamma}$$

- growth criterion

$$\phi^{\mathrm{g}} = \mathrm{tr}(\boldsymbol{M}^{\mathrm{e}}) - M^{\mathrm{e\,crit}}$$

stress-driven concentric transversely isotropic growth

cardiac wall thickening through stress-driven concentric growth

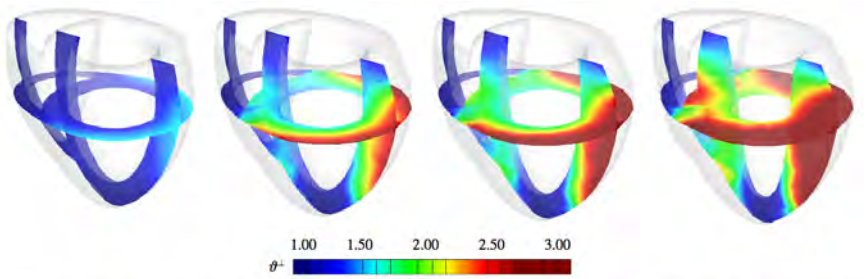


Figure 10. Stress-driven concentric growth. The concentric growth multiplier gradually increases from 1.00 to 3.00 as the individual cardiomyocytes grow concentrically. On the structural level, concentric growth manifests itself in a progressive transmural wall thickening to withstand higher blood pressure levels while the overall size of the heart remains virtually unaffected. Since the septal wall receives structural support through the pressure in the right ventricle, wall thickening is slightly more pronounced in the free wall where the wall stresses are higher.

göktepe, abilez, parker, kuhl [2010]

cardiac wall thickening through stress-driven concentric growth

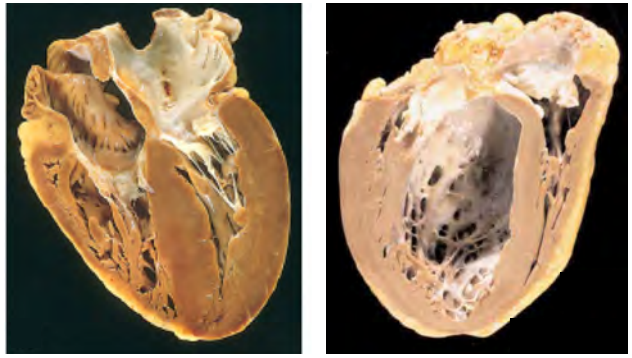
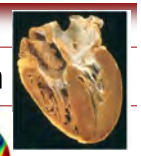
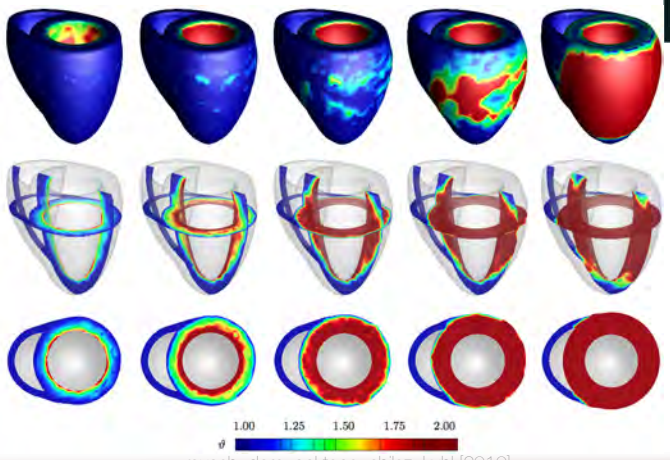


Figure. Stress-driven concentric growth, cardiac wall thickening, and transmural muscle thickening at constant cardiac size. Left ventricular wall thickening in response to systemic hypertension (left) from Kumar, Abbas, Fausto [2005]. Right ventricular wall thickening in response to pulmonary hypertension (right), from Padera.

rausch, dam, göktepe, abilez, kuhl [2010]

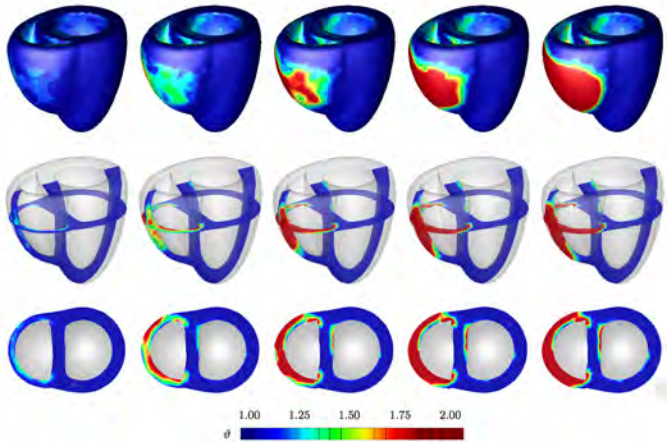
LV wall thickening through systemic hypertension





rausch, dam, göktepe, abilez, kuhl [2010]

RV wall thickening through pulmonary hypertension

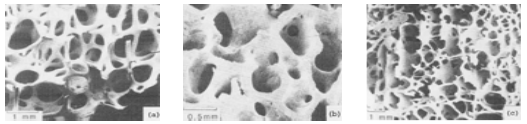


rausch, dam, goktepe, abilez, kuhl [2010]

example - cardiac growth

density growth at constant volume

- free energy $\psi_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^n \psi_0^{\text{neo}}(\boldsymbol{F})$
- stress $\boldsymbol{P} = \left[\frac{\rho_0}{\rho_0^*}\right]^n \boldsymbol{P}^{\text{neo}}(\boldsymbol{F})$
- mass flux $\boldsymbol{R} = R_0 \, \nabla_X \rho_0$
- mass source $\mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m} \psi_0(\boldsymbol{F}) - \psi_0^*$



constitutive coupling of growth and deformation

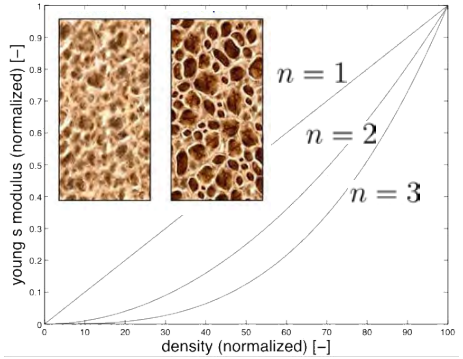
gibson & ashby [1999]

constitutive equations

neo hooke'ian elasticity of cellular materials

free energy $\psi_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^n \psi_0^{\text{neo}}(\boldsymbol{F})$

$\mathrm{E} = 3.790 \; \rho_0^3 \; \mathrm{MPa} \quad \rho_0 \text{ in } \mathrm{g/cm^3}$

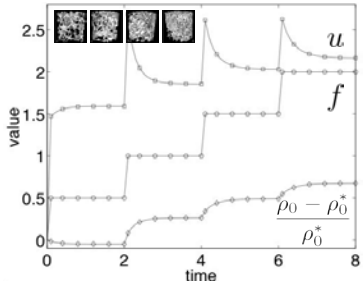


carter & hayes [1977]

constitutive equations

density growth - mass source

$\mathrm{D}_t \rho_0 = \mathcal{R}_0$  $\mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m} \psi_0 - \psi_0^*$



$f = 0.5$ N	$u = 1.5910 \, l$
$f = 1.0$ N	$u = 1.8559 \, l$
$f = 1.5$ N	$u = 2.0310 \, l$
$f = 2.0$ N	$u = 2.1652 \, l$

resorption $-1 < \frac{\rho_0 - \rho_0^*}{\rho_0^*} < 0$

growth $0 < \frac{\rho_0 - \rho_0^*}{\rho_0^*} < \infty$

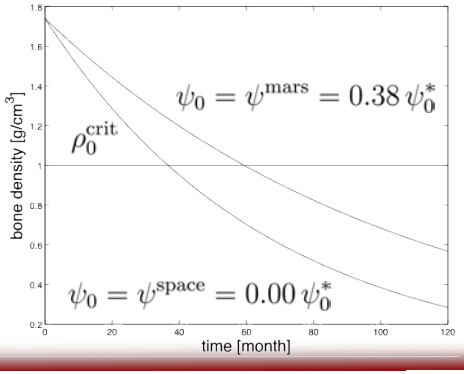
increasing forces causes density increase

constitutive equations

density growth - bone loss in space

$$D_t\rho_0 = c\,\rho_0\left[\frac{\psi_0}{\psi_0^*} - 1\right] \qquad D_t\rho_0 = \frac{1}{\Delta t}\left[\rho_0^{n+1} - \rho_0^n\right]$$

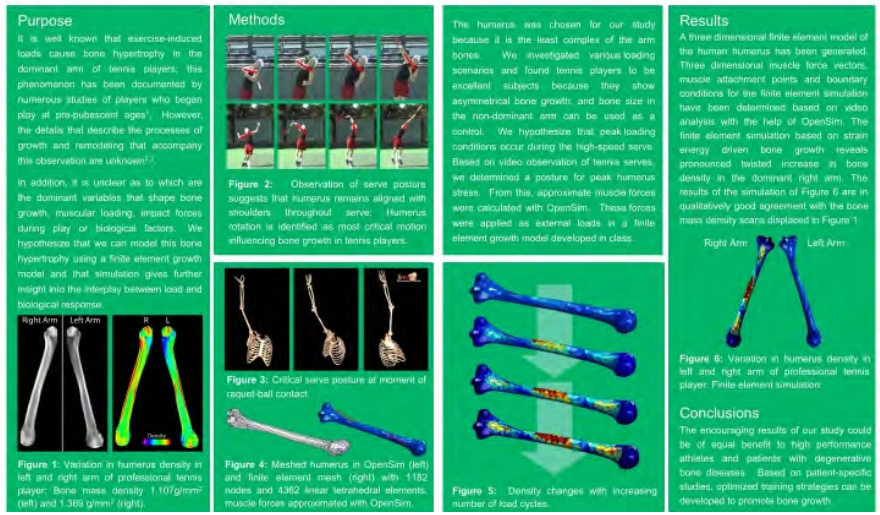
$$\rho_0^{n+1} = \rho_0^n + c\,\rho_0^n\left[\frac{\psi_0}{\psi_0^*} - 1\right]\Delta t \qquad \rho_0(t_0) = 1.79\frac{g}{cm^3}$$



$\rho_0(36) = 1.0098$
$\rho_0(37) = 0.9947$

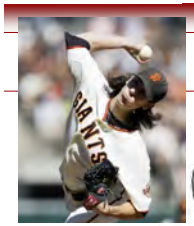
example - bone loss in space 69

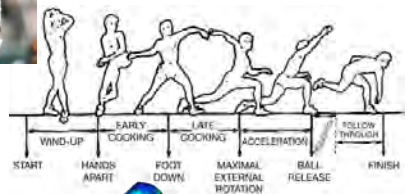
tennis player's arm

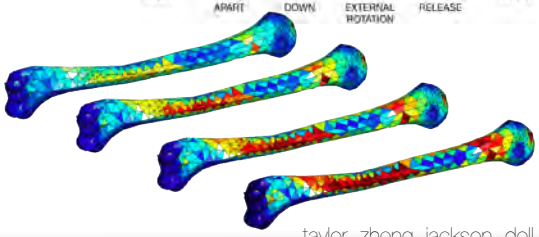


example - tennis player's arm

pitcher's arm







bones in the throwing arm of a baseball pitcher are **denser and thicker** than bones in the other arm.

maximal external shoulder rotation stimulates twisted density growth

taylor, zheng, jackson, doll, chen, holzbaur, besier, kuhl [2009]

example - pitcher's arm 71

staggered solution - integration point based

weinans, huiskes & grootenboer [1992], harrigan & hamilton [1992],, [1994], jacobs, levenston, beaupré,simo & carter [1995]

simultaneous solution - node point based

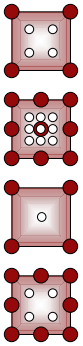
jacobs, levenston, beaupré,simo & carter [1995], fischer, jacobs, levenston & carter [1997], nackenhorst [1997], levenston [1997]]

sequential solution - element based

huiskes, weinans, grootenboer, dalstra, fudala & slooff [1987], carter, orr, fhyrie [1989], beaupré, orr & carter [1990], weinans, huiskes & grootenboer [1992], [1994], jacobs, levenston, beaupré, simo & carter [1995], huiskes [2000], carter & beaupré [2001]

finite element method 72

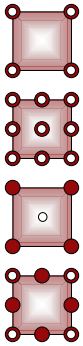
integration point based solution of growth equation



loop over all time steps
- global newton iteration
  - loop over all elements
    - loop over all quadrature points
      - local newton iteration to determine $\rho_{0\,n+1}$
      - determine element residual & partial derivative
    - determine global residual and iterational matrix
  - determine φ_{n+1}
- determine state of biological equilibrium

density ρ_0 as internal variable

node point based solution of growth equation

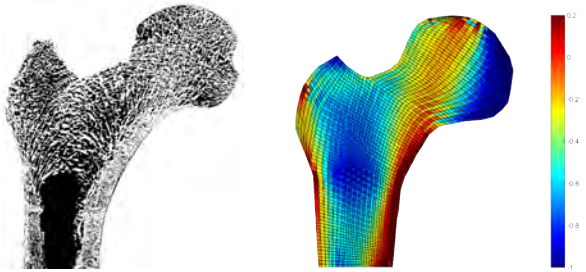


loop over all time steps
- global newton iteration
  - loop over all elements
    - loop over all quadrature points
      - evaulate balance of mass and momentum
      - determine element residuals & partial derivatives
    - determine global residuals and iterational matrices
  - determine $\rho_{0\,n+1}$ and φ_{n+1}
- determine state of biological equilibrium

density ρ_0 as nodal degree of freedom

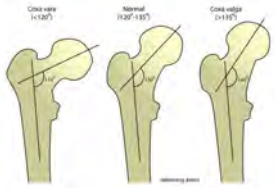
functional adaptation of proxima femur

$$\mathrm{D}_t\rho_0 = \mathcal{R}_0 \qquad \mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m}\psi_0 - \psi_0^*$$



the density develops such that the tissue can just support the given mechanical load

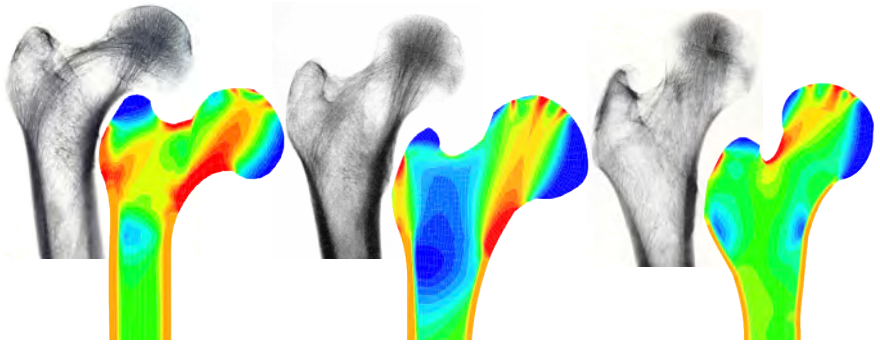
femoral neck deformity



the femoral neck normally forms an angle of 120-135 degrees with the shaft of the bone. this acts as the lever in easing the action of the muscles around the hip joint. an increase or decrease in this angle beyond the normal limits causes improper action of muscles, and interferes with walking. an increase in the angle beyond 135 degrees is called **coxa valga** or outward curvature of the hip joint. a decrease in the angle below 120 degrees is called **coxa vara** or inward curvature of the hip joint.

simulation vs. x-ray scans

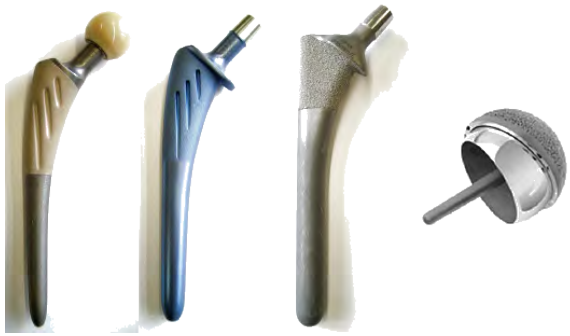
coxa vara | coxa norma | coxa valga



excellent agreement of simulation and x-ray pattern

pauwels [1973], balle [2004], kuhl & balle [2005]

total hip replacement vs hip resurfacing

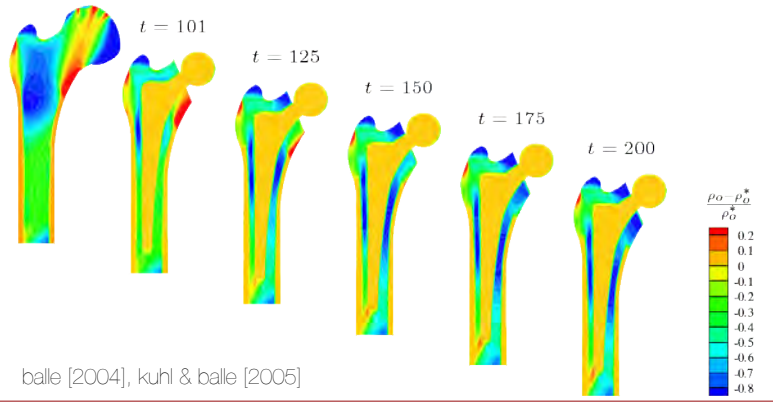


- about 120,000 artificial hip replacements in us per year
- aseptic loosening caused by adaptive bone remodeling
- goal prediction of dredisctribution of bone density

total hip replacement

total hip replacement is a surgical procedure in which the hip joint is replaced by a prostetic implant. a total hip replacement consists of replacing both the acetabulum and the femoral head. hip replacement is currently the most successful and reliable orthopaedic operation. risks and complications include aseptic loosening, dislocation, and pain.in the long term, many problems relate to **bone resorption and subsequent loosening** or fracture often requiring revision surgery.

convential total hip replacement



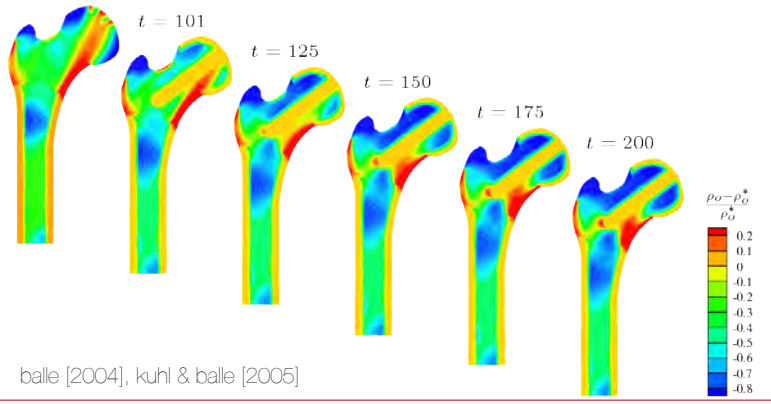
balle [2004], kuhl & balle [2005]

stress shielding • bone resorption • implant loosening

hip resurfacing

hip resurfacing is a surgical procedure which has been developed as an intervention alternative to total hip replacement. the potential advantages of hip resurfacing include **less bone removal**, a potentially lower number of hip dislocations due to a relatively larger femoral head size, and possibly easier revision surgery for a subsequent total hip replacement device. the potential disadvantages are femoral neck fractures, aspectic loosening, and metal wear.

new birmingham hip resurfacing



balle [2004], kuhl & balle [2005]

improved ingrowth • anatomic situation • less resorption