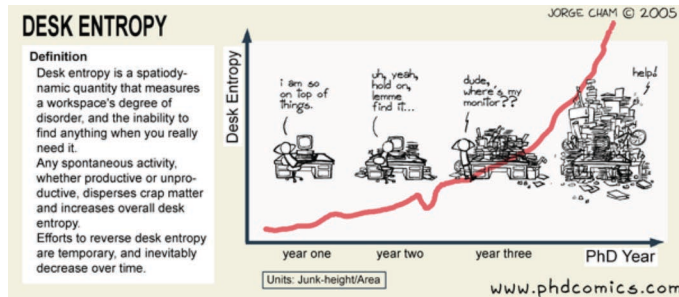


## 07 - basics balance equations - closed and open systems



## 07 - balance equations

1

## final projects

- **tendon growth:** harrison, brandon, mohammed, matthew
- **tendon growth and remodeling:** peter
- **muscle growth:** alex
- **skin growth and healing:** beth, ann, armen
- **benign vocal fold nodule and polyp growth:** corey
- **cerebral aneurysm growth:** jina
- **growth of swelling gels:** hardik, xi, ill
- **bone growth in martial arts:** kevin, alison, safwan, kamil

## where are we???

3

day	date	topic
tue	jan 10	motivation - everything grows!
thu	jan 12	basics maths - notation and tensors
tue	jan 17	basic kinematics - large deformation and growth
thu	jan 19	kinematics - growing hearts
tue	jan 24	guest lecture - growing skin
thu	jan 26	guest lecture - growing leaflets
tue	jan 31	basic balance equations - closed and open systems
thu	feb 02	basic constitutive equations - growing tumors
tue	feb 07	volume growth - finite elements for growth
thu	feb 09	volume growth - growing arteries
tue	feb 14	volume growth - growing skin
thu	feb 16	volume growth - growing hearts
tue	feb 21	basic constitutive equations - growing bones
thu	feb 23	density growth - finite elements for growth
tue	feb 28	density growth - growing bones
thu	mar 01	everything grows! - midterm summary
tue	mar 06	midterm
thu	mar 08	remodeling - remodeling arteries and tendons
tue	mar 13	class project - discussion, presentation, evaluation
thu	mar 15	class project - discussion, presentation, evaluation
thu	mar 15	written part of final projects due

## where are we???

2

## continuum mechanics of growth



kinematic equations for finite growth

$$\mathbf{F} = \mathbf{F}_e \cdot \mathbf{F}_g$$



balance equations for open systems

$$D_t \rho_0 = \text{Div}(\mathbf{R}) + \mathcal{R}_0$$

$$\rho_0 D_t \mathbf{v} = \text{Div}(\mathbf{P}) + \mathbf{b}_0$$



constitutive equations for living tissues

$$\mathbf{P} = \mathbf{P}(\rho_0, \mathbf{F}, \mathbf{F}_g)$$



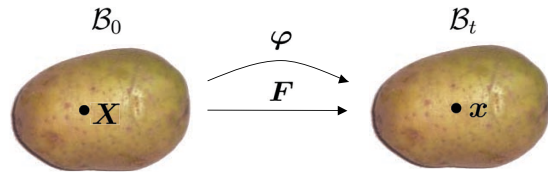
fe analyses for biological structures

continuum- & computational biomechanics

## where are we???

4

## potato - kinematics



- nonlinear deformation map  $\varphi$

$$x = \varphi(X, t) \quad \text{with} \quad \varphi : \mathcal{B}_0 \times \mathbb{R} \rightarrow \mathcal{B}_t$$

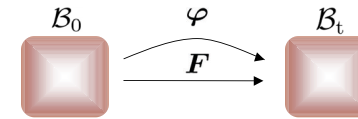
- spatial derivative of  $\varphi$  - deformation gradient

$$dx = F \cdot dX \quad \text{with} \quad F : T\mathcal{B}_0 \rightarrow T\mathcal{B}_t \quad F = \left. \frac{\partial \varphi}{\partial X} \right|_{t \text{ fixed}}$$

## kinematic equations

5

## kinematics of finite growth

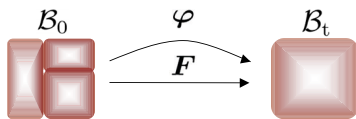


- [1] consider an elastic body  $\mathcal{B}_0$  at time  $t_0$ , unloaded & stressfree

## kinematics of growth

6

## kinematics of finite growth

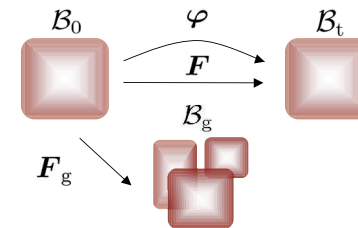


- [1] consider an elastic body  $\mathcal{B}_0$  at time  $t_0$ , unloaded & stressfree
- [2] imagine the body is cut into infinitesimal elements each of which is allowed to undergo volumetric growth

## kinematics of growth

7

## kinematics of finite growth

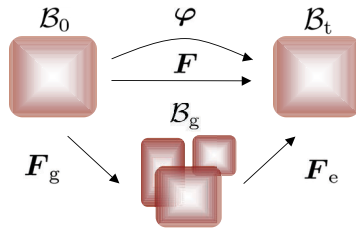


- [1] consider an elastic body  $\mathcal{B}_0$  at time  $t_0$ , unloaded & stressfree
- [2] imagine the body is cut into infinitesimal elements each of which is allowed to undergo volumetric growth
- [3] after growing the elements,  $\mathcal{B}_g$  may be incompatible

## kinematics of growth

8

## kinematics of finite growth

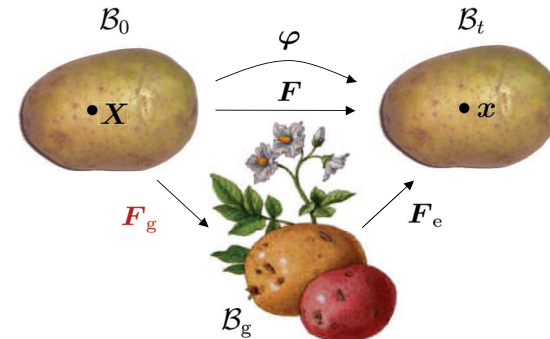


- [1] consider an elastic body  $B_0$  at time  $t_0$ , unloaded & stressfree
- [2] imagine the body is cut into infinitesimal elements each of which is allowed to undergo volumetric growth
- [3] after growing the elements,  $B_g$  may be incompatible
- [4] loading generates compatible current configuration  $B_t$

## kinematics of growth

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## potato - kinematics of finite growth



- incompatible growth configuration  $B_g$  & growth tensor  $F_g$
- $$F = F_e \cdot F_g$$

rodriguez, hoger & mc culloch [1994]

## kinematics of growth

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## balance equations

**balance equations** ['bæl.əns r'kwel.ʒəns] of mass, momentum, angular momentum and energy, supplemented with an entropy inequality constitute the set of conservation laws. the law of **conservation of mass/matter** states that the **mass of a closed system** of substances will remain **constant**, regardless of the processes acting inside the system. the principle of conservation of momentum states that the total momentum of a closed system of objects is constant.



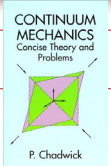
## balance equations

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## balance equations

**balance equations** ['bæl.əns r'kwel.ʒəns] of mass, linear momentum, angular momentum and energy **apply to all material bodies**. each one gives rise to a field equation, holding on the configurations of a body in a sufficiently smooth motion and a jump condition on surfaces of discontinuity. like position, time and body, the concepts of mass, force, heating and internal energy which enter into the formulation of the balance equations are regarded as having primitive status in continuum mechanics.

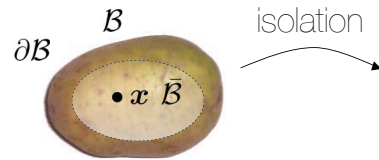
chadwick "continuum mechanics" [1976]



## balance equations

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## potato - balance equations

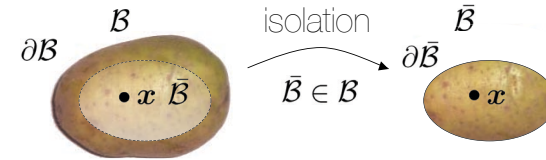


- [1] isolation of subset  $\bar{B}$  from  $B$

## balance equations

13

## potato - balance equations

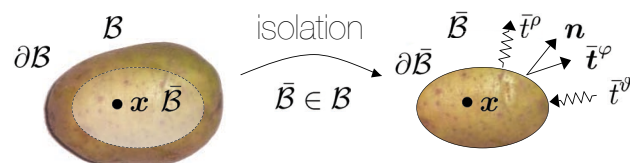


- [1] isolation of subset  $\bar{B}$  from  $B$   
 [2] characterization of influence of remaining body through phenomenological quantities - contact fluxes  $\bar{t}^\rho$ ,  $\bar{t}^\varphi$  &  $\bar{t}^\vartheta$

## balance equations

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## potato - balance equations

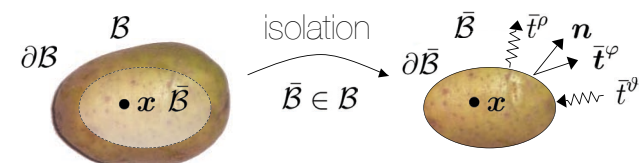


- [1] isolation of subset  $\bar{B}$  from  $B$   
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 [3] definition of basic physical quantities - mass, linear and angular momentum, energy

## balance equations

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## potato - balance equations



- [1] isolation of subset  $\bar{B}$  from  $B$   
 [2] characterization of influence of remaining body through phenomenological quantities - contact fluxes  $\bar{t}^\rho$ ,  $\bar{t}^\varphi$  &  $\bar{t}^\vartheta$   
 [3] definition of basic physical quantities - mass, linear and angular momentum, energy  
 [4] postulation of balance of these quantities

## balance equations

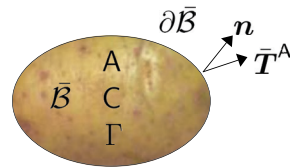
16

## generic balance equation

general format

A ... balance quantity  
**B** ... flux  $\mathbf{B} \cdot \mathbf{n} = \bar{T}^A$   
 C ... source  
 $\Gamma$  ... production

$$D_t A = \text{Div}(\mathbf{B}) + C + \Gamma$$



## balance equations - closed systems

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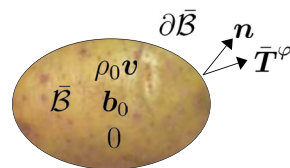
## balance of (linear) momentum

balance of momentum

$\rho_0 \mathbf{v}$  ... linear momentum density  
 $\mathbf{P}$  ... momentum flux - stress  
 $\mathbf{b}_0$  ... momentum source - force  
 0 ... no momentum production

$$\mathbf{P} \cdot \mathbf{n} = \bar{T}^P$$

$$\text{equilibrium equation } D_t(\rho_0 \mathbf{v}) = \text{Div}(\mathbf{P}) + \mathbf{b}_0$$



## balance equations - closed systems

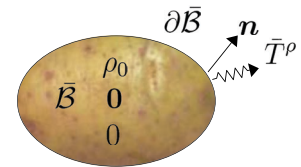
19

## balance of mass

balance of mass

$\rho_0$  ... density  
 0 ... no mass flux  $\bar{T}^P = 0$   
 0 ... no mass source  
 0 ... no mass production

$$\text{continuity equation } D_t \rho_0 = 0$$



## balance equations - closed systems

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## compare



NEWTON'S  
THREE LAWS OF  
GRADUATION

First published in 1679, Isaac Newton's "*Procrastinare Unmaturalis Principia Mathematica*" is often considered one of the most important single works in the history of science. Its Second Law is the most powerful of the three, allowing mathematical calculation of the duration of a doctoral degree.

### SECOND LAW

"The age,  $\mathbf{a}$ , of a doctoral process is directly proportional to the flexibility,  $\mathbf{f}$ , given by the advisor and inversely proportional to the student's motivation,  $\mathbf{m}$ "

Mathematically, this postulate translates to:

$$age_{\text{PhD}} = \frac{\text{flexibility}}{\text{motivation}}$$

$$\mathbf{a} = \mathbf{F} / \mathbf{m}$$

$$\therefore \mathbf{F} = \mathbf{m} \mathbf{a}$$

This Law is a quantitative description of the effect of the forces experienced by a grad student. A highly motivated student may still remain in grad school given enough flexibility. As motivation goes to zero, the duration of the PhD goes to infinity.

PHD.STANFORD.EDU  
JORGE CHAM@THE STANFORD DAILY

$$D_t(\rho_0 \mathbf{v}) = \text{Div}(\mathbf{P}) + \mathbf{b}_0 \quad \text{mass point} \quad m D_t \mathbf{v} = m \mathbf{a} = \mathbf{F}$$

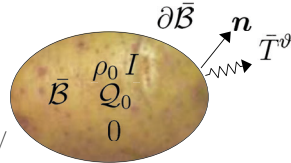
## balance equations - closed systems

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## balance of (internal) energy

balance of internal energy

$\rho_0 I$  ... internal energy density  
 $\mathbf{Q}$  ... heat flux  $-\mathbf{Q} \cdot \mathbf{n} = \bar{T}^\vartheta$   
 $\mathcal{Q}_0$  ... heat source  
 $0$  ... no heat production



energy equation  $D_t(\rho_0 I) = \underbrace{\mathbf{P} : D_t \mathbf{F}}_{\text{internal mechanical power}} - \underbrace{\mathbf{v} \cdot \mathbf{b}_0}_{\text{thermal external power}} + \text{Div}(-\mathbf{Q}) + \mathcal{Q}_0$

internal mechanical power      thermal external power

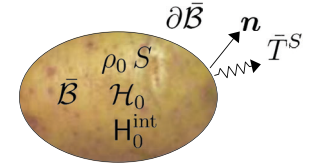
## balance equations - closed systems

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## balance of entropy

balance of entropy

$\rho_0 S$  ... entropy density  
 $\mathbf{H}$  ... entropy flux  $-\mathbf{H} \cdot \mathbf{n} = \bar{T}^S$   
 $\mathcal{H}_0$  ... entropy source  
 $H_0^{\text{int}}$  ... entropy production  $H_0^{\text{int}} \geq 0$



entropy 'inequality'  $D_t(\rho_0 S) = \text{Div}(-\mathbf{H}) + \mathcal{H}_0 + H_0^{\text{int}}$

## balance equations - closed systems

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## dissipation inequality

- dissipation inequality

$$D_0 := \vartheta H_0^{\text{int}} = \vartheta \rho_0 D_t S + \vartheta \text{Div}(\mathbf{H}) - \vartheta \mathcal{H}_0 \geq 0$$

- identification  $\mathbf{H} = \frac{1}{\vartheta} \mathbf{Q}$   $\mathcal{H}_0 = \frac{1}{\vartheta} \mathcal{Q}_0$

- with legendre-fenchel transform  $\psi = I - \vartheta S$

$$D_0 = \mathbf{P} : D_t \mathbf{F} - \rho_0 D_t \psi - \rho_0 S D_t \vartheta + \mathbf{Q} \cdot \nabla_X \ln(\vartheta) \geq 0$$

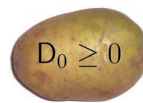
- free energy  $\psi = \psi(\mathbf{F}, \vartheta)$   $D_t \psi = D_F \psi : D_t \mathbf{F} + D_\vartheta \psi D_t \vartheta$

- definition of stress and entropy (e.g., neo hooke's law)

$$\mathbf{P} = \rho_0 D_F \psi \quad S = -\rho_0 D_\vartheta \psi$$

- thermodynamic restriction (e.g., fourier's law)

$$\mathbf{Q} \cdot \nabla_X \ln(\vartheta) \geq 0$$



## balance equations - closed systems

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## thermodynamic systems

**isolated system** ['aɪ.sə.leɪ.tɪd 'sɪs.təm] thermodynamical system which is not allowed to have any interaction with its environment. enclosed by a rigid, adiabatic, impermeable membrane.

## balance equations

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## thermodynamic systems

**adiabatic closed system** ['æ.dɪ.æ.bæ.tɪk klouzd 'sɪs.təm]  
thermodynamic system which is allowed to exchange exclusively mechanical work, typically  $P = P(\nabla\varphi, \dots)$ , with its environment. enclosed by a deformable, adiabatic, impermeable membrane. characterized through its state of deformation  $\varphi$ .

## balance equations

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## thermodynamic systems

**open system** ['ou.pən 'sɪs.təm] thermodynamic system which is allowed to exchange mechanical work, heat and mass, typically  $P = P(\nabla\varphi, \dots)$ ,  $Q = Q(\nabla\theta, \dots)$  and  $R = R(\nabla\rho, \dots)$  with its environment. enclosed by a deformable, diathermal, permeable membrane. characterized through its state of deformation  $\varphi$ , temperature  $\theta$  and density  $\rho$ .

## balance equations

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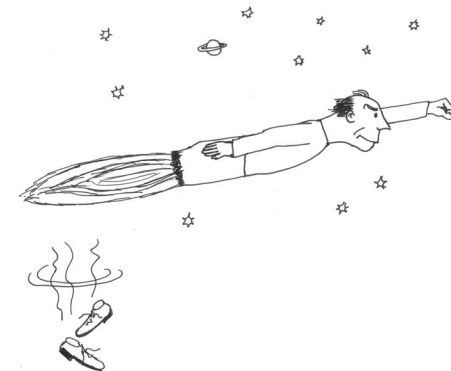
## thermodynamic systems

**closed system** [klouzd 'sɪs.təm] thermodynamic system which is allowed to exchange mechanical work and heat, typically  $P = P(\nabla\varphi, \dots)$  and  $Q = Q(\nabla\theta, \dots)$ , with its environment. enclosed by a deformable, diathermal, impermeable membrane. characterized through its state of deformation  $\varphi$  and temperature  $\theta$ .

## balance equations

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## open system thermodynamics



„...thermodynamics recognizes no special role of the biological...”

bridgman, "the nature of thermodynamics", [1941]

## balance equations - open systems

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## why do we need open systems if we have porous media?

### theory of open systems [ˈoʊ.pən ˈsɪs.təms]

- constituents spatially separated
- overall behavior preliminary determined by one single constituent
- exchange of mass, momentum, energy and entropy with environment

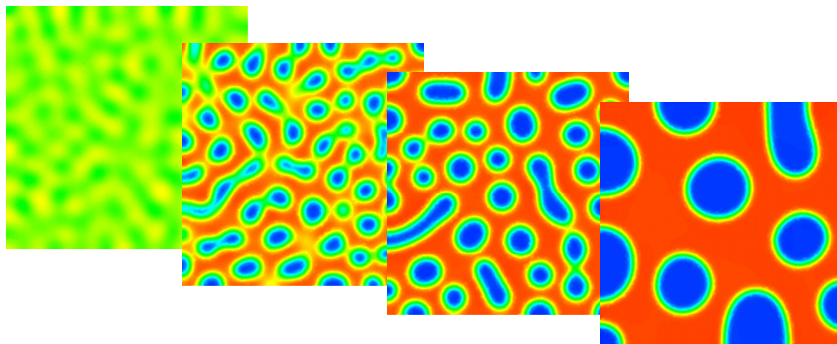
### theory of porous media [ˈpɔɪr.əs ˈmɪdɪ.ə]

- local superposition of constituents
- consideration of mixture of multiple constituents
- exchange of mass, momentum, energy and entropy amongst constituents

## balance equations - open systems

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## balance of mass



## simulation of cell growth - cahn-hilliard equation

kuhl & schmid [2006], wells, kuhl & garikipati [2006]

## balance equations - open systems

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## balance of mass

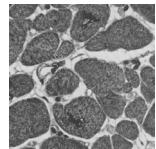
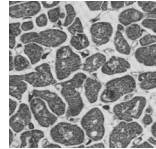
$$D_t \rho_0 = \text{Div}(\mathbf{R}) + \mathcal{R}_0$$

mass flux  $\mathbf{R}$

- cell movement (migration)

mass source  $\mathcal{R}_0$

- cell growth (proliferation)
- cell division (hyperplasia)
- cell enlargement (hypertrophy)



## biological equilibrium

cowin & hegedus [1976], beaupré, orr & carter [1990], harrigan & hamilton [1992], jacobs, levenston, beaupré, simo & carter [1995], huiskes [2000], carter & beaupré [2001]

## balance equations - open systems

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## balance of mass

the model does not take explicitly into account that the **body is growing** due to the **absorption of some other materials**. in absence of some of the vital constituents, no growth is possible. conversely, when part of the material dies, some of the bricks contained in the cellular membrane can be re-used by other cells. in this respect, an approach using mixture theory might be useful.

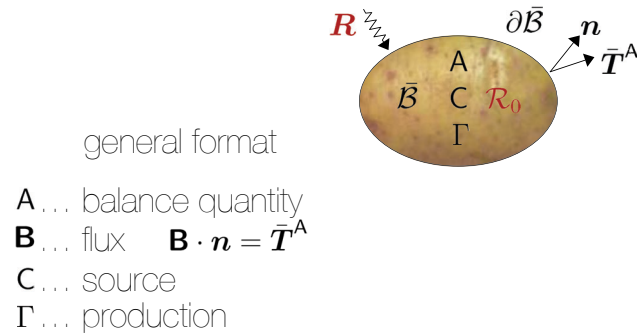
ambrosi & mollica [2002]

## balance equations - open systems

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## generic balance equation



$$D_t(\rho_0 A) = \text{Div}(\mathbf{B} + \mathbf{A} \otimes \mathbf{R}) + [\mathbf{C} + \mathbf{A} \mathcal{R}_0 - \nabla_X \mathbf{A} \cdot \mathbf{R} + \Gamma]$$

## balance equations - open systems

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## balance of (linear) momentum

- volume specific version

$$D_t(\rho_0 \mathbf{v}) = \text{Div}(\mathbf{P} + \mathbf{v} \otimes \mathbf{R}) + [\mathbf{b}_0 + \mathbf{v} \mathcal{R}_0 - \nabla_X \mathbf{v} \cdot \mathbf{R}]$$

- subtract weighted balance of mass

$$\mathbf{v} D_t \rho_0 = \text{Div}(\mathbf{v} \otimes \mathbf{R}) + \mathbf{v} \mathcal{R}_0 - \nabla_X \mathbf{v} \cdot \mathbf{R}$$

- mass specific version

$$\rho_0 D_t \mathbf{v} = \text{Div}(\mathbf{P}) + \mathbf{b}_0$$

## mechanical equilibrium

## balance equations - open systems

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## example of open systems - rocket propulsion



balance of mass

$$D_t m = \mathcal{R} \quad \text{with} \quad \mathcal{R} \leq 0 \quad \text{ejection}$$

balance of momentum - volume specific

$$D_t[m\mathbf{v}] = m D_t \mathbf{v} + D_t m \mathbf{v} = \mathbf{f} + \mathcal{R} \mathbf{v}$$

balance of momentum - mass specific

$$m D_t \mathbf{v} = \mathbf{f} \quad \text{with} \quad \mathbf{f} = \mathbf{f}^{\text{closed}} + \mathbf{f}^{\text{open}}$$

balance of momentum - rocket head-ejection

$$D_t[m\mathbf{v}] - \mathcal{R} \bar{\mathbf{v}} = \mathbf{f}^{\text{closed}}$$

propulsive force

$$\mathbf{f}^{\text{open}} = [\bar{\mathbf{v}} - \mathbf{v}] \mathcal{R} \quad \text{velocity of ejection } \bar{\mathbf{v}}$$

## example - rocket propulsion

35

## example of open systems - rocket propulsion



a saturn v rocket like the one that took men to the moon has a **mass of 2.500.000 kg** at liftoff. it goes straight up vertically and burns fuel at a uniform rate of 16.000 kg/s for a duration of 2 minutes. the exhaust speed of gas from the saturn v is **3.0 km/s**

what is the speed of the rocket immediately after the combustion ceases? you should include the effect of gravity near the surface of the earth, but you can neglect air resistance.

plot the burnout velocity as a function of time over the range of 0 to 120 seconds to see the increase in speed of the rocket with time.

## example - rocket propulsion

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## example of open systems - rocket propulsion



## example - rocket propulsion

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## example of open systems - rocket propulsion



$$m D_t v = f^{\text{closed}} + f^{\text{open}}$$

with  $f^{\text{closed}} = -m g$  gravity

$$f^{\text{open}} = [\bar{v} - v] \mathcal{R} = w D_t m$$

$$m D_t v = -m g - D_t m w \quad || : m$$

$$D_t v = -g - \frac{1}{m} D_t m w$$

integration

$$v(t) = -g t - w \int_{m(0)}^{m(t)} \frac{1}{m} dm$$

$$m(t) = m(0) + \mathcal{R} t$$

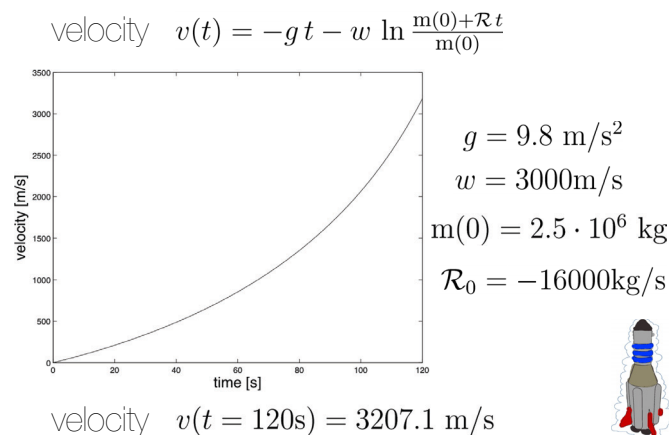
velocity

$$v(t) = -g t - w \ln \frac{m(0) + \mathcal{R} t}{m(0)}$$

## example - rocket propulsion

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## example of open systems - rocket propulsion



## example - rocket propulsion

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## balance of (internal) energy

- volume specific version

$$D_t(\rho_0 I) = \mathbf{P} : D_t \mathbf{F} - \mathbf{v} \cdot \mathbf{b}_0 + \text{Div}(-\mathbf{Q} + I \mathbf{R}) + [\mathcal{Q}_0 + I \mathcal{R}_0 - \nabla_x I \cdot \mathbf{R}]$$

- subtract weighted balance of mass

$$I D_t \rho_0 = \text{Div}(I \mathbf{R}) + I \mathcal{R}_0 - \nabla_x I \cdot \mathbf{R}$$

- mass specific version

$$\rho_0 D_t I = \mathbf{P} : D_t \mathbf{F} - \mathbf{v} \cdot \mathbf{b}_0 + \text{Div}(-\mathbf{Q}) + \mathcal{Q}_0$$

energy equilibrium

## balance equations - open systems

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## balance of entropy

- volume specific version  $H_0^{\text{int}} \geq 0$   
 $D_t(\rho_0 S) = \text{Div}(-\mathbf{H} + \mathbf{S} \mathbf{R}) + [\mathcal{H}_0 + \mathbf{S} \mathcal{R}_0 - \nabla_X \mathbf{S} \cdot \mathbf{R}] + H_0^{\text{int}}$
- subtract weighted balance of mass  
 $S D_t \rho_0 = \text{Div}(\mathbf{S} \mathbf{R}) + \mathbf{S} \mathcal{R}_0 - \nabla_X \mathbf{S} \cdot \mathbf{R}$
- mass specific version  
 $\rho_0 D_t S = \text{Div}(-\mathbf{H}) + \mathcal{H}_0 + H_0^{\text{int}}$

## entropy 'inequality'

## balance equations - open systems

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## dissipation inequality

- dissipation inequality  
 $D_0 := \vartheta H_0^{\text{int}} = \rho_0 \vartheta D_t S + \vartheta \text{Div}(\mathbf{H}) - \vartheta \mathcal{H}_0 \geq 0$
- identification  $\mathbf{H} = \frac{1}{\vartheta} \mathbf{Q} + \mathbf{S} \quad \mathcal{H}_0 = \frac{1}{\vartheta} \mathcal{Q}_0 + \mathcal{S}_0$
- free energy  $\psi = \psi(\rho_0, \mathbf{F}, \vartheta)$
- definition of stress and entropy  
 $\mathbf{P} = \rho_0 D_F \psi \quad S = -D_\vartheta \psi$
- thermodynamic restrictions  
 $\mathcal{S}_0 \leq \rho_0 D_{\rho_0} \psi \frac{1}{\vartheta} \mathcal{R}_0 \quad \mathbf{S} \geq \rho_0 D_{\rho_0} \psi \frac{1}{\vartheta} \mathbf{R} \quad \mathbf{Q} \cdot \nabla_X \ln(\vartheta) \geq 0$

„... a living organism can only keep alive by continuously drawing from its environment negative entropy. It feeds upon negative entropy to compensate the entropy increase it produces by living.“  
 schrödinger "what is life?" [1944]

## balance equations - open systems

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## dissipation inequality

- dissipation inequality  
 $D_0 := \vartheta H_0^{\text{int}} = \rho_0 \vartheta D_t S + \vartheta \text{Div}(\mathbf{H}) - \vartheta \mathcal{H}_0 \geq 0$
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 $\mathbf{P} = \rho_0 D_F \psi \quad S = -D_\vartheta \psi$
- thermodynamic restrictions  
 $\mathcal{S}_0 \leq \rho_0 D_{\rho_0} \psi \frac{1}{\vartheta} \mathcal{R}_0 \quad \mathbf{S} \geq \rho_0 D_{\rho_0} \psi \frac{1}{\vartheta} \mathbf{R} \quad \mathbf{Q} \cdot \nabla_X \ln(\vartheta) \geq 0$

„... a living organism can only keep alive by continuously drawing from its environment negative entropy. It feeds upon negative entropy to compensate the entropy increase it produces by living.“  
 schrödinger "what is life?" [1944]



## balance equations - open systems

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## open systems - dissipation inequality

$$D_0 = \mathbf{P} : D_t \mathbf{F} - \rho_0 D_t \psi - \rho_0 S D_t \vartheta - \mathbf{s}_0 \vartheta + \mathbf{Q} \cdot \nabla_X \ln(\vartheta) \geq 0$$

with definition of arguments of free energy

$$\psi = \psi(\rho_0, \mathbf{F}, \vartheta) \quad D_t \psi = D_{\rho_0} \psi D_t \rho_0 + D_F \psi : D_t \mathbf{F} + D_\vartheta \psi D_t \vartheta$$

evaluation of dissipation inequality

$$D_0 = [\mathbf{P} - \rho_0 D_F \psi] : D_t \mathbf{F} - [\rho_0 S - \rho_0 D_\vartheta \psi] D_t \vartheta + \dots \geq 0$$

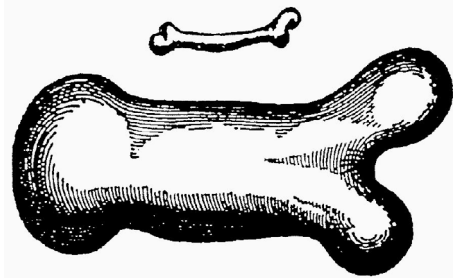
provides guidelines for the appropriate choice of the constitutive equations  $\mathbf{P} = \rho_0 D_F \psi \quad S = -D_\vartheta \psi$

thermodynamically conjugate pairs  $\mathbf{P} \rightleftharpoons \mathbf{F} \quad S \rightleftharpoons \vartheta$

## balance equations - open systems

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## example of open systems - the galileo giant



„...dal che e manifesto, che chi volesse mantener in un vastissimo gigante le proporzioni, che hanno le membra in un huomo ordinario, bisognerebbe o trouar materia molto piu dura, e resistente per formarne l'ossa o vero ammettere, che la robustezza sua fusse a proporzione assai piu fiacca, che negli huomini de statura mediocre; altrimenti crescendo gli a smisurata altezza si vedrebbero dal proprio peso opprimere, e cadere...”

galileo, "discorsi e dimostrazioni matematiche", [1638]

## example - the galileo giant

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## example of open systems - the galileo giant

the tallest man in medical history is robert pershing wadlow. he was born at alton, illinois, on february 22, 1918. he was **2.72m** / 8ft 11.1", tall. his weight was **222.71kg**. his shoe size was 47cm / 18.5", and his hands measured 32.4cm / 12.75". his arm span was 2.88m / 9 ft 5.75", and his peak daily food consumption was 8000 calories.



guinness world records [2010]

## example - the galileo giant

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## example of closed systems - the galileo giant

- balance equation closed systems  $\rho_0 D_t v = \text{Div}(P) + b_0$
- neo hookean free energy  $\psi_0 = \frac{1}{4} E_0 [F^2 - 1 - 2 \ln(F)]$
- stress from dissipation inequality  $P = D_F \psi_0 = \frac{1}{4} E_0 [2F - 2 \frac{1}{F}]$
- quasi-static case  $\rho_0 D_t v = 0$
- constant gravity load  $b_0 = \text{const along the height } h$
- from balance eqn linear stress  $P = [X - h] b_0$   
...linear along the height  $h$
- closed system, constant density  $\rho = \rho_0^* \quad [\rho - \rho_0^*] / \rho_0^* = 0$

## example - the galileo giant

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## example of open systems - the galileo giant

$$[\rho - \rho_0^*] / \rho_0^* = 0$$

$$u = \varphi - X$$

$$[\rho_0 / \rho_0^*]^{-m} \psi_0 = \psi_0$$

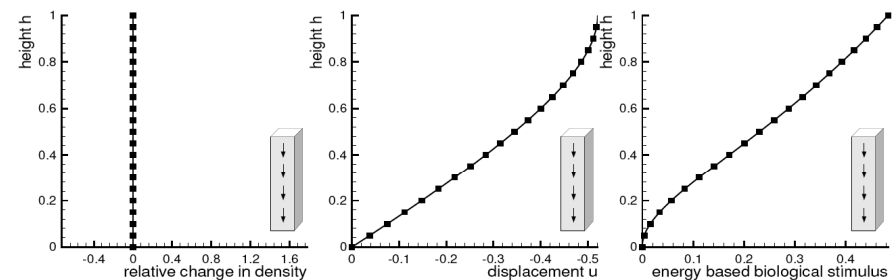


Figure 7.1: One-dimensional model problem - Closed system - Homogeneous density

## example - the galileo giant

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## example of open systems - the galileo giant

- balance equations open systems
 
$$D_t \rho_0 = \text{Div}(R) + \mathcal{R}_0$$

$$\rho_0 D_t v = \text{Div}(P) + b_0$$
- free energy  $\psi_0 = [\rho_0/\rho_0^*]^n \psi_0^{\text{neo}}$ 

$$\psi_0^{\text{neo}} = \frac{1}{4} E_0 [F^2 - 1 - 2 \ln(F)]$$
- stress from dissipation inequality
 
$$P = D_F \psi_0 = \frac{1}{4} E_0 [2F - 2\frac{1}{F}]$$
- quasi-static case
 
$$\rho_0 D_t v = 0$$
- constant gravity load
 
$$b_0 = \text{const along the height } h$$
- from balance eqn linear stress
 
$$P = [X - h] b_0$$

...linear along the height  $h$
- open system, varying density
 
$$D_t \rho_0 = \mathcal{R}_0 = 0$$

$$\mathcal{R}_0 = [\rho_0/\rho_0^*]^{-m} \rho_0 - \rho_0^*$$

## example - the galileo giant

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## example of open systems - the galileo giant

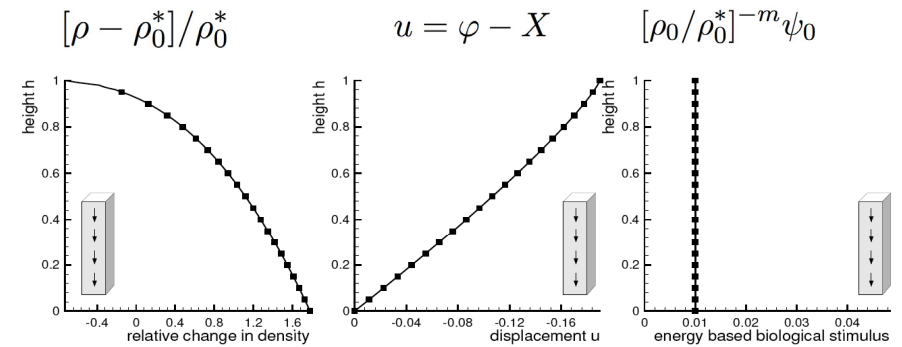


Figure 7.2: One-dimensional model problem - Open system - Homogeneous stimulus

## example - the galileo giant

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