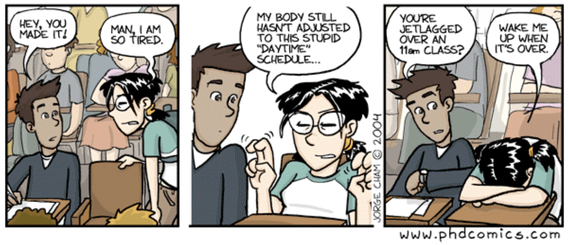
13 - basic constitutive equations - growing bones



day	date		topic	slides
tue	sep	21	motivation - everything grows!	s01
thu	sep	23	basics maths - notation and tensors	s02
tue	sep	28	guest lecture: class project - growing tennis player arms	s03
thu	sep	30	guided reading - no class	s04
tue	oct	05	basics kinematics - large deformation and growth	s05
thu	oct	07	guest lecture: growing arteries	s06
tue	oct	12	basic balance equations - closed and open systems	s07
thu	oct	14	basic constitutive equations - growing tumors	s08
tue	oct	19	volume growth - finite elements for growth	s09
thu	oct	21	volume growth - growing arteries	s10
tue	oct	26	volume growth - growing skin	s11
thu	oct	28	volume growth - growing hearts	s12
tue	nov	02	basic constitutive equations - growing bones	s13
thu	nov	04	density growth - finite elements for growth	s14
tue	nov	09	density growth - growing bones	s15
thu	nov	11	density growth - finite elements for growth	s16
tue	nov	16	midterm	
thu	nov	18	remodeling - remodeling arteries and tendons	s18
tue	nov	30	class project - discussion, presentation, evaluation	
thu	dec	02	class project - discussion, presentation, evaluation	
thu	dec	02	written part of final projects due	

CENTER FOR TEACHING AND LEARNING
STANFORD UNIVERSITY
OFFICE OF THE VICE PROVOST FOR UNDERGRADUATE EDUCATION

Student Small Group Evaluation for
Professor Ellen Kuhl
ME 337 "Mechanics of Growth"
Thursday, October 28, 2010 at 11:55am

The instructor was particularly interested in feedback about the diversity of academic background represented by the students (ranging from early undergraduates to advanced graduate students). Students did not raise any related issues in small groups and, when probed, responded that all seemed to have a common starting point for the material and felt there were sufficient external resources (e.g. office hours) for specific aspects that needed more attention.

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What is going well in this class and contributes to your learning?

- (3) Real life applications throughout the course are great; class examples are interesting. [100% of class agreed]
- (2) The class project is very motivating, gets students involved, interested in learning. [100% of class agreed] Starting the project early is good.
- (2) Course topics are flexible and based on student interests [~95% of class agreed]
- (2) Homework helps reinforce the main concepts (and focuses ideas for the project). [~55% of class agreed]

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What could use improvement? If not already obvious, what specific suggestions would you make for change?

- (3) It can be hard to stay engaged with only the PowerPoint. It is very helpful when you also use the board (e.g. the finite element example). [~95% agreed]
- (1) We would understand the equations better if more time was spent defining the variables. [~95% of class agreed]
- (1) We'd like to know more about what is expected on the midterm. [~95% of class agreed]
- (1) It would help in reviewing if the slides were more detailed; can be hard to look back without remembering what the professor said.[~80% of class agreed]
- (1) More representative homework, especially to help with mathematics. [~55% of class agreed]

homework III - revise your final project

due 11/11/10, 11:11am, 420-040

Late homework can be dropped off in a box in front of Durand 217. Please mark clearly with date and time @drop off. We will take off 1/10 of points for each 24 hours late, every 12pm after due date. This homework will count 10% towards your final grade.

problem 1 - growth tensors

We have introduced different growth tensors $\boldsymbol{F}^g$ in class. Discuss the following growth tensors.

1.1 $\boldsymbol{F}^g = \vartheta\, \boldsymbol{I}$

1.2 $\boldsymbol{F}^g = \boldsymbol{I} + [\vartheta - 1]\, \boldsymbol{f}_0 \otimes \boldsymbol{f}_0$

1.3 $\boldsymbol{F}^g = \boldsymbol{I} + [\vartheta - 1]\, \boldsymbol{s}_0 \otimes \boldsymbol{s}_0$

1.4 $\boldsymbol{F}^g = \sqrt{\vartheta}\, \boldsymbol{I} + [1 - \sqrt{\vartheta}]\, \boldsymbol{n}_0 \otimes \boldsymbol{n}_0$

Herein, $\boldsymbol{f}_0$ denotes a distinct fiber direction, $\boldsymbol{s}_0$ is a sheet direction, $\boldsymbol{n}_0$ is the normal to a characteristic microstructural plane, and ϑ is a scalar-valued growth multiplier.

For each growth tensor, focus on: (i) its mechanical interpretation, e.g., isotropic, transversely isotropic, orthotropic, generally anisotropic; (ii) its microstructural interpretation, e.g., spherical growth, fiber lengthening, fiber thickening, area growth; (iii) its biological application, e.g., which type of tissue growth does it characterize, why does this ansatz make sense.

homework III - revise your final project

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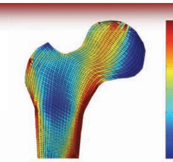
problem 2 - growth tensors

Assume the following microstructural vectors, $\boldsymbol{f}_0 = [1,0,0]^t$, $\boldsymbol{s}_0 = [0,1,0]^t$, and $\boldsymbol{n}_0 = [0,0,1]^t$ aligned with the cartesian coordinates, and a growth multiplier of $\vartheta = 2$.

2.1 Calculate the four growth tensors $\boldsymbol{F}^g$ from 1.1 to 1.4.

2.2 Calculate the volume change of a cube of unit length for all four growth tensors $\boldsymbol{F}^g$ from 1.1 to 1.4 using the Jacobian $J^g = \det(\boldsymbol{F}^g)$.

2.3 Draw a cubic block of tissue of unit length in a three-dimensional coordinate system. Add the unit vectors $\mathrm{d}X_1 = [1,0,0]^t$, $\mathrm{d}X_2 = [0,1,0]^t$, and $\mathrm{d}X_3 = [0,0,1]^t$. For each of the growth tensors $\boldsymbol{F}^g$ in 1.1 to 1.4, calculate and illustrate the deformed vectors $\mathrm{d}x_1$, $\mathrm{d}x_2$ and $\mathrm{d}x_3$ using $\mathrm{d}x = \boldsymbol{F}^g \cdot \mathrm{d}X$. Illustrate the grown block.



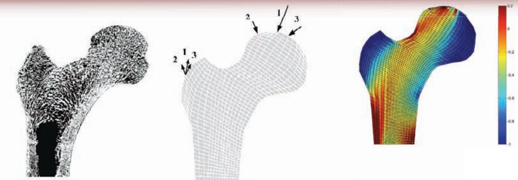
problem 3 - growing bones in matlab

3.1 Download the matlab file package from the coursework website or from the website `http://biomechanics.stanford.edu`.

3.2 To run the femur example, open the main file `nlin_fem.m`, and make sure that all input file readings are commented out by a % sign in the first column of lines 6 through 19. The only active input line should be line 12 reading `ex_femur`.

3.3 In the command window, call the main file by typing `nlin_fem` and wait for the mesh to be generated.

3.4 Run the density evolution algorithm for 5 time steps by typing `step,,5`. Describe what you see in the command window and in the graphics window. How many iterations does a typical load step take to find the equilibrium of the nonlinear problem? Focus on load step 5. Report the residuals, i.e., the errors in solution, to demonstrate quadratic convergence of the Newton Raphson scheme. Then, run the algorithm for an additional 25 time steps by typing `step,,25` in the command window. Quit the algorithmic environment by typing `quit`.



[1]	midstance phase of gait	2317 N	24°	703 N	28°
[2]	extreme range of abduction	1158 N	-15°	351 N	-8°
[3]	extreme range of adduction	1548 N	56°	468 N	35°

3.7 You have now optimized the density of the proxima femur based on the three most significant load cases as shown in the figure above. Now, change the loading. Open the input file `ex_femur.m`. In lines 58 through 62, you can see how the load is generated. Manipulate the direction of the compressive load of load case 1 on node 723 which is currently `F_ext(723*2-1)=-0.9424`, i.e., -2317N sin(24^0) in the horizontal direction and `F_ext(723*2)=-2.1167`, i.e., -2317N cos(24^0) in the vertical direction. Rotate the compressive force of load case 1 by 21^0 in the clockwise direction, i.e., change its line of action to -2.317 sin(45^0) in the horizontal and -2.317 cos(45^0) in the vertical direction by modifying line 62 in the code accordingly. Save your input file `ex_femur.m`, and rerun the simulation `nlin_fem` for `step`,,30 time steps and quit your algorithmic environment by typing `quit`. Then, type `caxis([-1.0 0.2])` to change the color axis, turn off the shadows by `lighting none`, and plot your final figure with `print('-depsc','-r300','figure02.eps')`.

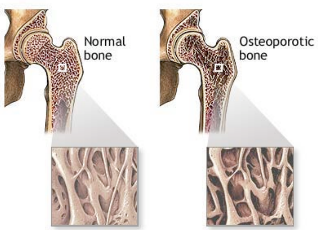
problem 4 - revise your final project

4.1 Download the style file `me337_project_sample.doc` from the coursework website and paste in your title, outline, opening sentence, introduction, schematic drawing, and references from homework II.

4.2 Expand the reference section to at least three key references and seven additional references. Make sure your citations all have the same style.

4.3 Revise your introduction and make sure that all your references are cited. The introduction should: (i) contain your catchy opening sentence with citation, (ii) motivate your work, and (iii) give an overview of the current state of the field. It should be one to two columns long.

4.4 Draft an outline of all the figures you would like to include in your manuscript. This is the most important step of drafting your paper, since most scientific papers are written around figures. For each figure, create a place holder or the figure itself. Create meaningful figure captions. The figure captions in the sample file are actually not a good example. In the biological literature, captions are usually more detailed and can be several lines long. When you adopt figures from the literature, cite your source. Remember that in a real journal paper, you cannot use other authors' figures without copyright agreement.

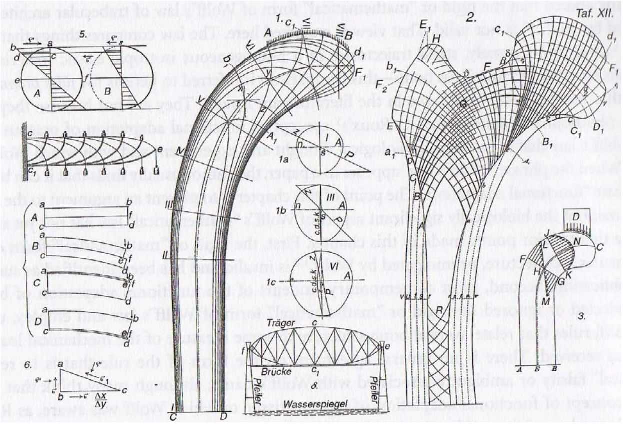


osteoporosis

osteoporosis is a disease of bones that leads to an increased risk of fracture. osteoporosis literally means porous bones. in osteoporosis the bone mineral density is reduced, bone microarchitecture is disrupted, and the amount of variety of proteins in bone is altered. the diagnosis of osteoporosis can be made using conventional radiography. bone mineral density can be measured by dual energy x-ray absorptiometry, dxa or dexa. osteoporosis can be prevented with lifestyle changes and sometimes medication. lifestyle changes include exercise and preventing falls as well as reducing protein intake which may cause calcium to be taken from the bones.

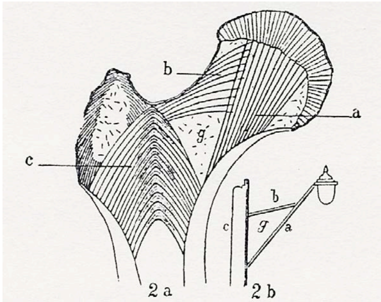
WIKIPEDIA The Free Encyclopedia

history - 19th century



culmann & von meyer "graphic statics" [1867]

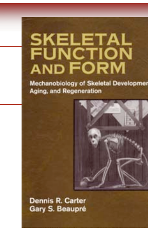
history - 19th century



„…es ist demnach unter dem gesetze der transformation der knochen dasjenige gesetz zu verstehen, nach welchem im gefolge primaerer abaenderungen der form und inanspruchnahme bestimmte umwandlungen der inneren architectur und umwandlungen der aeusseren form sich vollziehen…"

wolff "gesetz der transformation der knochen" [1892]

different load cases



[1]	midstance phase of gait	2317 N	24°	703 N	28°
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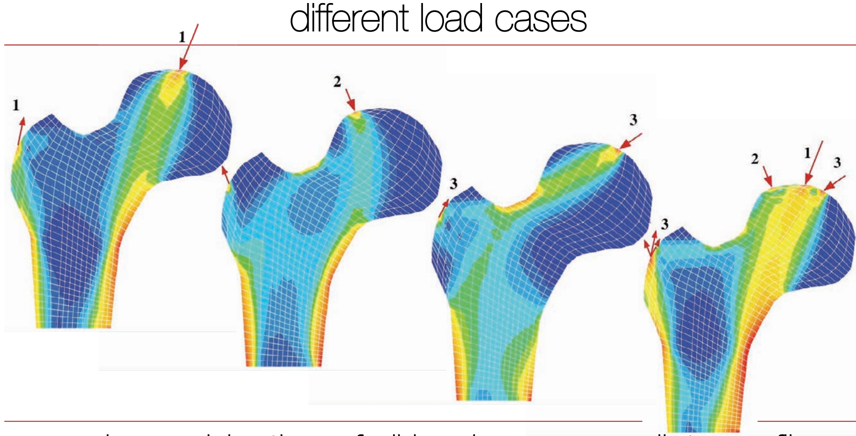
carter & beaupré [2001]

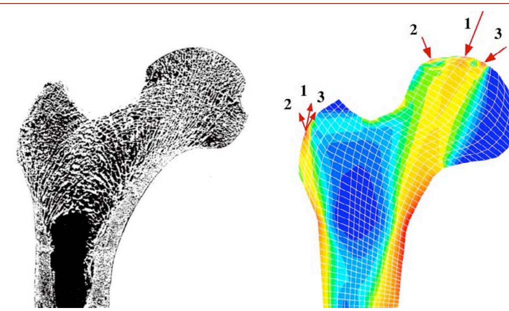
different load cases



only combination of all load cases predicts profile

carter & beaupré [2001]

experiment vs simulation



- dense system of compressive trabaculae carrying stress into calcar region
- secondary arcuate system, medial joint surface to lateral metaphyseal region
- ward's triangle, low density region contrasting dense cortical shaft

carter & beaupré [2001]

pitcher's arm

a physical-conditioning program for pitchers is geared to striking a balance between muscle strength and endurance, tendon/ligament strength and flexibility, and optimal cartilage and **bone density**. bone hypertrophy occurs in response to physical activity. the bones in the throwing arm of a baseball pitcher are **denser and thicker** than those of the other arm. bone hypertrophy is **stimulated by the magnitude of loading** rather than by the frequency.

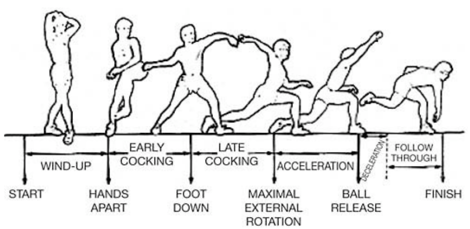
pitcher's arm

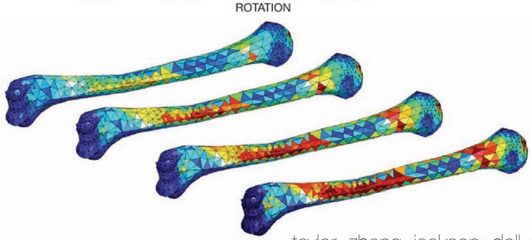
the real secret to tim lincecum's overpowering velocity is all **stored within his pitching mechanics**. it has little to do with his size and strength. six things in tim lincecum's pitching delivery create his amazing arm speed:

- move fast from back leg to front leg
- use back leg to move out very low to ground
- get throwing arm up very late in delivery
- stride length of over 100% of pitcher's height
- brace front leg to increase upper body speed
- land in a straight line toward plate

tim lincecum's pitching mechanics, because he moves fast into a long stride and stays low, forces his body to **put as many muscles on stretch as quickly as possible** which helps develop **maximum elastic energy** so that **his body acts like a huge rubber band** stretching to it's maximum length ready to be let go and whip the arm through.

pitcher's arm

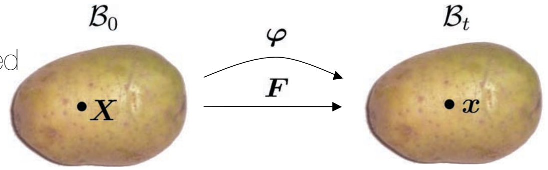




maximal external shoulder rotation stimulates twisted density growth

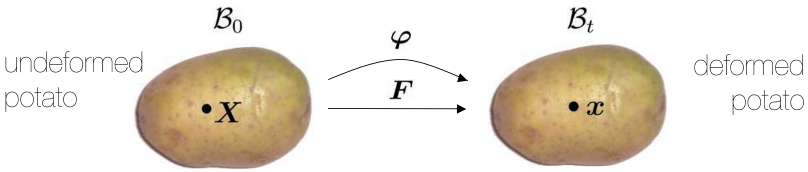
taylor, zheng, jackson, doll, chen, holzbaur, besier, kuhl [2009]

neo hooke'ian elasticity of solid materials



- free energy $$\psi_0^{\text{neo}} = \tfrac{1}{2}\,\lambda_0 \ln^2(\det(\boldsymbol{F})) + \tfrac{1}{2}\,\mu_0[\,\boldsymbol{F}^{\text{t}}\cdot\boldsymbol{F}:\boldsymbol{I} - n^{\text{dim}} - 2\,\ln(\det(\boldsymbol{F}))\,]$$
- definition of stress $$\boldsymbol{P}^{\text{neo}} = \mathrm{D}_{\boldsymbol{F}}\psi_0^{\text{neo}} = \mu_0\,\boldsymbol{F} + [\,\lambda_0 \ln(\det(\boldsymbol{F})) - \mu_0\,]\,\boldsymbol{F}^{\text{-t}}$$

neo hooke'ian elasticity of solid materials



- free energy $\quad \psi_0^{\text{neo}} = \frac{1}{2}\,\lambda_0 \ln^2(\det(\boldsymbol{F})) + \frac{1}{2}\,\mu_0[\,\boldsymbol{F}^{\text{t}}\cdot\boldsymbol{F} : \boldsymbol{I} - n^{\text{dim}} - 2\,\ln(\det(\boldsymbol{F}))\,]$
- large strain - lamé parameters and bulk modulus
  $$\lambda = \frac{E\,\nu}{[1+\nu][1-2\,\nu]} \qquad \mu = \frac{E}{2\,[1+\nu]} \qquad \kappa = \frac{E}{3\,[1-2\,\nu]}$$
- small strain - young's modulus and poisson's ratio
  $$E = 3\,\kappa\,[1-2\,\nu] \qquad \nu = \frac{3\,\kappa - 2\,\mu}{2\,[3\,\kappa+\mu]}$$

neo hooke'ian elasticity of solid materials

- free energy $\quad \psi_0^{\text{neo}} = \frac{1}{2}\,\lambda_0 \ln^2(\det(\boldsymbol{F})) + \frac{1}{2}\,\mu_0[\,\boldsymbol{F}^{\text{t}}\cdot\boldsymbol{F} : \boldsymbol{I} - n^{\text{dim}} - 2\,\ln(\det(\boldsymbol{F}))\,]$
- definition of stress
  $$\begin{aligned}\boldsymbol{P}^{\text{neo}} &= \mathrm{D}_{F}\psi_0^{\text{neo}}\\ &= \tfrac{1}{2}\,\lambda_0\,2\,\ln(\det \boldsymbol{F})\boldsymbol{F}^{\text{-t}} + \tfrac{1}{2}\mu_0\,2\,\boldsymbol{F} - \mu_0\boldsymbol{F}^{\text{-t}}\\ &= \mu_0\,\boldsymbol{F} + [\,\lambda_0 \ln(\det(\boldsymbol{F})) - \mu_0\,]\boldsymbol{F}^{\text{-t}}\end{aligned}$$
- definition of tangent operator
  $$\begin{aligned}\mathsf{A}^{\text{neo}} &= \mathrm{D}_{FF}\psi_0^{\text{neo}} = \mathrm{D}_{F}\boldsymbol{P}^{\text{neo}}\\ &= \lambda_0\boldsymbol{F}^{\text{-t}}\otimes\boldsymbol{F}^{\text{-t}} + \mu_0\boldsymbol{I}\,\overline{\otimes}\,\boldsymbol{I}\\ &\quad+[\mu_0 - \lambda_0\ln(\det(\boldsymbol{F}))]\;\boldsymbol{F}^{\text{-t}}\,\underline{\otimes}\,\boldsymbol{F}^{\text{-1}}\end{aligned}$$

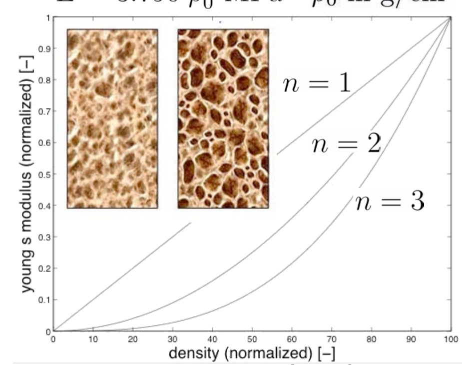
neo hooke'ian elasticity of solid materials

- free energy $\quad \psi_0^{\text{neo}} = \frac{1}{2}\,\lambda_0 \ln^2(\det(F_{ij})) + \frac{1}{2}\,\mu_0[\,F_{ij}F_{ij} - n^{\text{dim}} - 2\,\ln(\det(F_{ij}))\,]$
- definition of stress
  $$\begin{aligned}P_{ij}^{\text{neo}} &= \mathrm{D}_{F_{ij}}\psi_0^{\text{neo}}\\ &= \tfrac{1}{2}\,\lambda_0\,2\,\ln(\det F_{ij})F_{ji}^{\text{-1}} + \tfrac{1}{2}\mu_0\,2\,F_{ij} - \mu_0 F_{ji}^{\text{-1}}\\ &= \mu_0\,F_{ij} + [\,\lambda_0 \ln(\det(F_{ij})) - \mu_0\,]F_{ji}^{\text{-1}}\end{aligned}$$
- definition of tangent operator
  $$\begin{aligned}\mathsf{A}_{ijkl}^{\text{neo}} &= \mathrm{D}_{F_{ij}F_{kl}}\psi_0^{\text{neo}} = \mathrm{D}_{F_{kl}}P_{ij}^{\text{neo}}\\ &= \lambda_0 F_{ji}^{\text{-1}}\;F_{lk}^{\text{-1}} + \mu_0 I_{ik}\;I_{jl}\\ &\quad+[\mu_0 - \lambda_0\ln(\det(F_{ij}))]\;F_{li}^{\text{-1}}\;F_{jk}^{\text{-1}}\end{aligned}$$

neo hooke'ian elasticity of cellular materials

free energy $\quad \psi_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^n \psi_0^{\text{neo}}(\boldsymbol{F})$

$\mathrm{E} = 3.790\;\rho_0^3\;\mathrm{MPa} \qquad \rho_0 \text{ in g/cm}^3$



carter & hayes [1977]

open systems - balance of mass

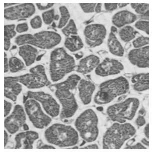
$$\mathrm{D}_t \rho_0 = \mathrm{Div}(\boldsymbol{R}) + \mathcal{R}_0$$

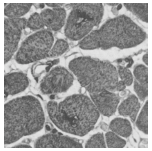
mass flux $\boldsymbol{R}$

- cell movement (migration)

mass source $\mathcal{R}_0$

- cell growth (proliferation)
- cell division (hyperplasia)
- cell enlargement (hypertrophy)





biological equilibrium

cowin & hegedus [1976], beaupré, orr & carter [1990], harrigan & hamilton [1992], jacobs,levenston,beaupré,simo & carter [1995], huiskes [2000], carter & beaupré [2001]

open systems - balance of momentum

- volume specific version

$$\mathrm{D}_t(\rho_0 \boldsymbol{v}) = \mathrm{Div}(\boldsymbol{P} + \boldsymbol{v} \otimes \boldsymbol{R}) + [\boldsymbol{b}_0 + \boldsymbol{v}\mathcal{R}_0 - \nabla_X \boldsymbol{v} \cdot \boldsymbol{R}]$$

- subraction of weighted balance of mass

$$\boldsymbol{v}\, \mathrm{D}_t \rho_0 = \mathrm{Div}(\boldsymbol{v} \otimes \boldsymbol{R}) + \boldsymbol{v}\mathcal{R}_0 - \nabla_X \boldsymbol{v} \cdot \boldsymbol{R}$$

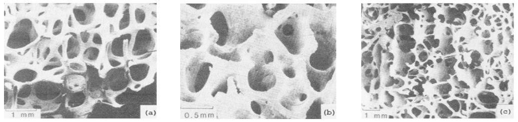
- mass specific version

$$\rho_0\, \mathrm{D}_t \boldsymbol{v} = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0$$

mechanical equilibrium

density growth at constant volume

- free energy $\quad \psi_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^n \psi_0^{\mathrm{neo}}(\boldsymbol{F})$
- stress $\quad \boldsymbol{P} = \left[\frac{\rho_0}{\rho_0^*}\right]^n \boldsymbol{P}^{\mathrm{neo}}(\boldsymbol{F})$
- mass flux $\quad \boldsymbol{R} = R_0\, \nabla_X \rho_0$
- mass source $\quad \mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m} \psi_0(\boldsymbol{F}) - \psi_0^*$



constitutive coupling of growth and deformation

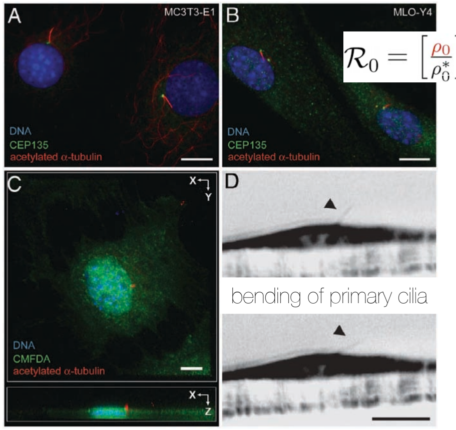
gibson & ashby [1999]

driving force for density increase - why does bone grow

$$\mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m} \psi_0(\boldsymbol{F}) - \psi_0^*$$

energy driven

mechanotransduction



bending of primary cilia

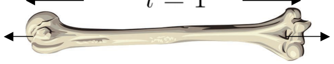
shear stress driven

malone, anderson, tummala, kwon, johnston, stearns, jacobs [2007]

density growth - mass source

$\mathrm{D}_t \rho_0 = \mathcal{R}_0$ 1d model problem $\mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m} \psi_0 - \psi_0^*$

$l = 1$

f, u  f, u

$n = 2$
$m = 3$

$E = 1 \quad \nu = 0 \quad \rho_0^* = 1 \quad \psi_0^* = 1$

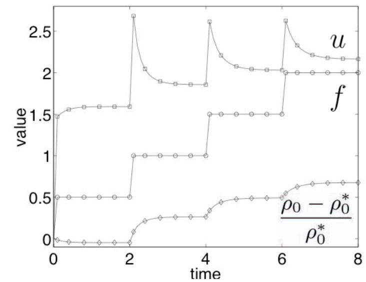
gradually increased workout

$0 < t \leq 2 : f = 0.5\,\mathrm{N}$
$2 < t \leq 4 : f = 1.0\,\mathrm{N}$
$4 < t \leq 6 : f = 1.5\,\mathrm{N}$
$6 < t \leq 8 : f = 2.0\,\mathrm{N}$

how does the mass source control growth?

density growth - mass source

$\mathrm{D}_t \rho_0 = \mathcal{R}_0$ f  f $\mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m} \psi_0 - \psi_0^*$



$f = 0.5\,\mathrm{N} \quad u = 1.5910\,l$
$f = 1.0\,\mathrm{N} \quad u = 1.8559\,l$
$f = 1.5\,\mathrm{N} \quad u = 2.0310\,l$
$f = 2.0\,\mathrm{N} \quad u = 2.1652\,l$

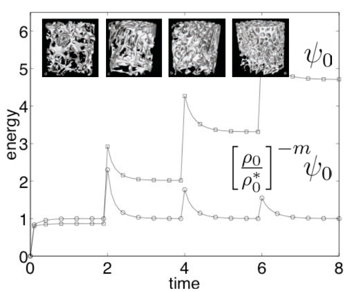
resorption $-1 < \frac{\rho_0 - \rho_0^*}{\rho_0^*} < 0$

growth $0 < \frac{\rho_0 - \rho_0^*}{\rho_0^*} < \infty$

increasing forces causes density increase

density growth - mass source

$\mathrm{D}_t \rho_0 = \mathcal{R}_0$ f  f $\mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m} \psi_0 - \psi_0^*$



first time interval $f = 0.5\mathrm{N}$

$u = 1.5910 l \quad \varphi = X + u = 2.5910 l$

$F = \frac{\partial \varphi}{\partial X} = \frac{\partial\, 2.5910\, l}{\partial l} = 2.5910$ homog.

$E = 1 \quad \lambda_0 = \frac{E\,\nu}{[1+\nu][1-2\,\nu]} = 0$

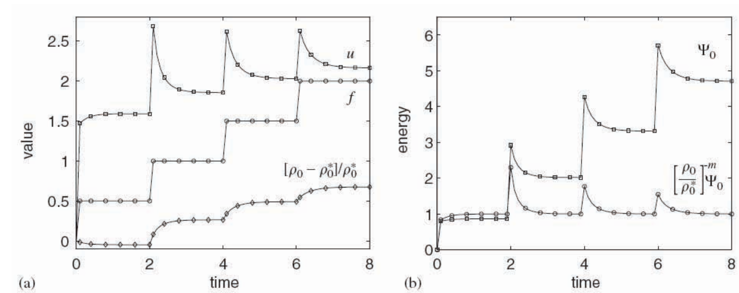
$\nu = 0 \quad \mu_0 = \frac{E}{2\,[1+\nu]} = 0.5$

$\psi_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^n \frac{1}{2}\,\mu_0 [\, F^2 - 1 - 2\,\ln(F)\,]$

increasing force causes energy increase

density growth - mass source

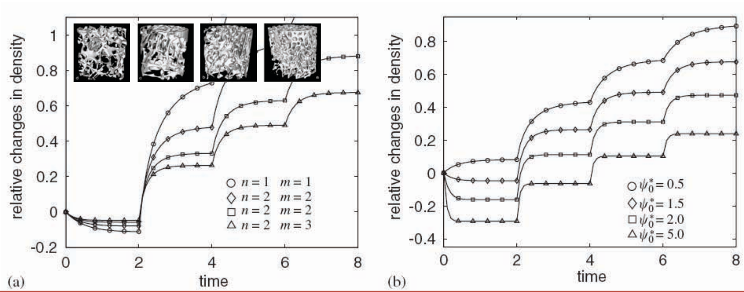
$\mathrm{D}_t \rho_0 = \mathcal{R}_0$ f  f $\mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m} \psi_0 - \psi_0^*$



convergence towards biological equilibrium $\mathrm{D}_t \rho_0 = 0$

density growth - mass source

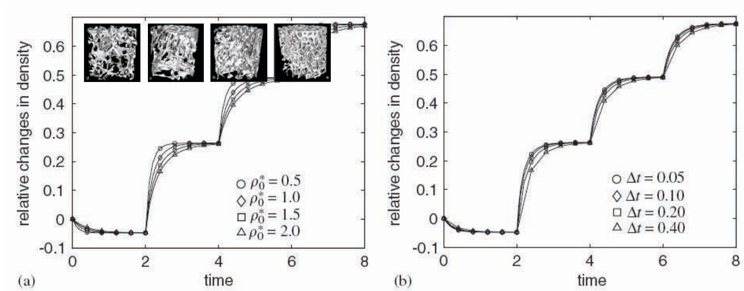
$$\mathrm{D}_t\rho_0 = \mathcal{R}_0 \qquad \mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m}\psi_0 - \psi_0^*$$



parameter sensitivity wrt n, m, ψ_0^*

density growth - mass source

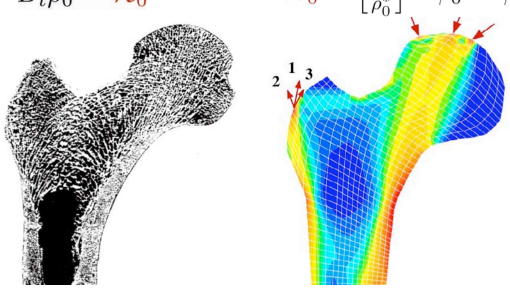
$$\mathrm{D}_t\rho_0 = \mathcal{R}_0 \qquad \mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m}\psi_0 - \psi_0^*$$



parameter insensitivity wrt $\rho_0^*, \Delta t$

density growth - mass source

$$\mathrm{D}_t\rho_0 = \mathcal{R}_0 \qquad \mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m}\psi_0 - \psi_0^*$$

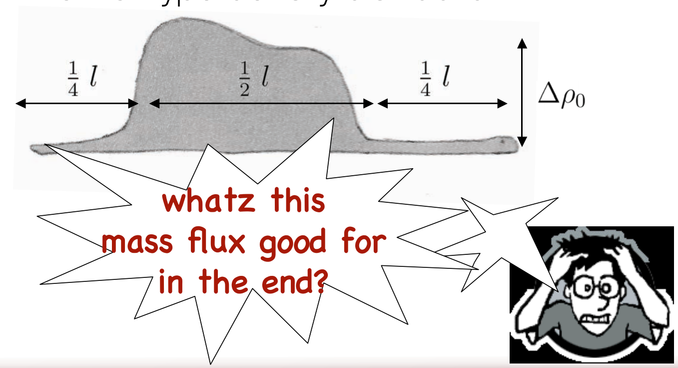


the density develops such that the tissue can just support the given mechanical load

density growth - mass flux

$$\mathrm{D}_t\rho_0 = \mathrm{Div}(\boldsymbol{R}) \qquad \boldsymbol{R} = R_0\,\nabla_X\rho_0$$

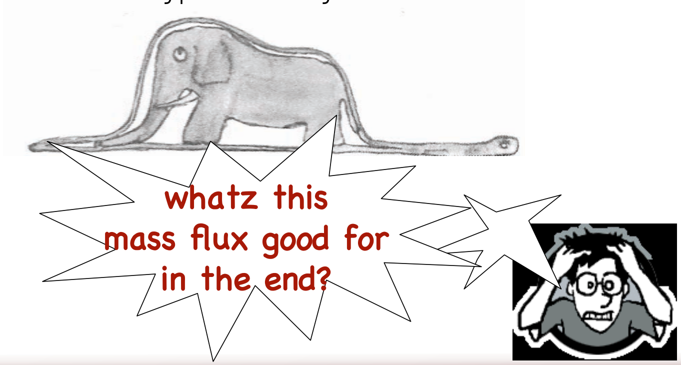
initial hat type density distriubtion



density growth - mass flux

$$D_t\rho_0 = \mathrm{Div}(\boldsymbol{R}) \qquad \boldsymbol{R} = R_0\, \nabla_X \rho_0$$

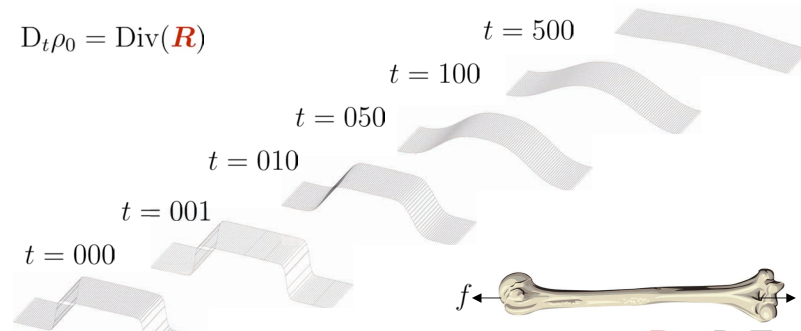
initial hat type density distriubtion



constitutive equations 37

density growth - mass flux

$$D_t\rho_0 = \mathrm{Div}(\boldsymbol{R})$$



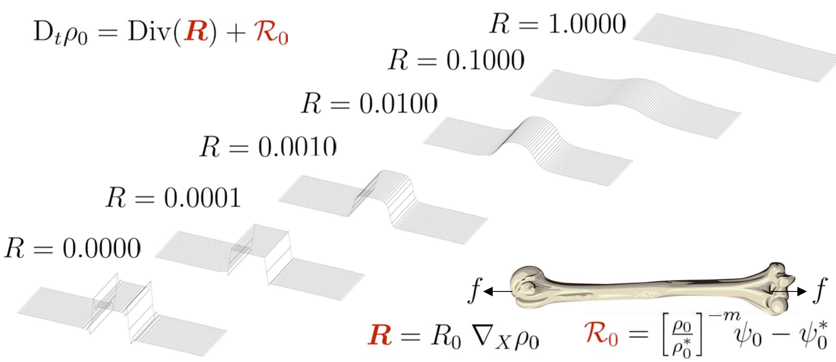
$$\boldsymbol{R} = R_0\, \nabla_X \rho_0$$

equilibration of concentrations

constitutive equations 38

density growth - mass flux & source

$$D_t\rho_0 = \mathrm{Div}(\boldsymbol{R}) + \mathcal{R}_0$$



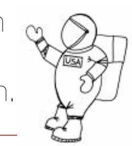
$$\boldsymbol{R} = R_0\, \nabla_X \rho_0 \qquad \mathcal{R}_0 = \left[\frac{\rho_0}{\rho_0^*}\right]^{-m} \psi_0 - \psi_0^*$$

smoothing influence of mass flux

constitutive equations 39

density growth - bone loss in space

human spaceflight to mars could become a reality within the next 25 years, but not until some physiological problems are resolved, including an alarming loss of bone mass, fitness and muscle strength. gravity at mars' surface is about 38 percent of that on earth. with lower gravitational forces, bones decrease in mass and density. the rate at which we lose bone in space is 10-15 times greater than that of a post-menopausal woman and there is no evidence that bone loss ever slows in space. further, it is not clear that space travelers will regain that bone on returning to gravity. druing a trip to mars, lasting between 13 and 30 months, unchecked bone loss could make an astronaut's skeleton the equivalent of a 100-year-old person.



http://www.acsm.org

example - bone loss in space 40

density growth - bone loss in space

$$D_t \rho_0 = \mathcal{R}_0 \qquad \mathcal{R}_0 = c\, \frac{\rho_0}{\psi_0^*} [\, \psi_0 - \psi_0^* \,]$$

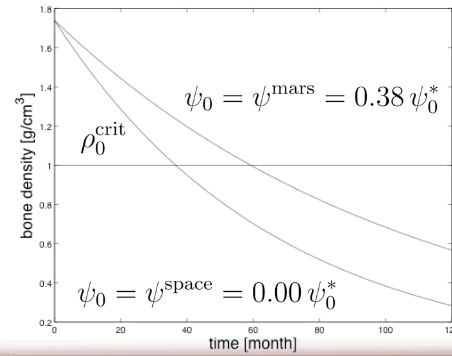
nasa has collected data that humans in space lose bone mass at a rate of $c = 1.5\%/\text{month}$ so far, no astronauts have been in space for more than 14 months but the predicted rate of bone loss seems constant in time. this could be a severe problem if we want to send astronauts on a 3 year trip to mars and back. how long could an astronaut survive in a zero-g environment if we assume the critical bone density to be $\rho_0^{\text{crit}} = 1.00 \frac{\text{g}}{\text{cm}^3}$? you can assume an initial density of $\rho_0^* = 1.79 \frac{\text{g}}{\text{cm}^3}$!

example - bone loss in space

density growth - bone loss in space

$$D_t \rho_0 = c\, \rho_0 [\, \frac{\psi_0}{\psi_0^*} - 1 \,] \qquad D_t \rho_0 = \frac{1}{\Delta t} [\rho_0^{n+1} - \rho_0^n]$$

$$\rho_0^{n+1} = \rho_0^n + c\, \rho_0^n [\, \frac{\psi_0}{\psi_0^*} - 1 \,] \Delta t \qquad \rho_0(t_0) = 1.79 \frac{\text{g}}{\text{cm}^3}$$



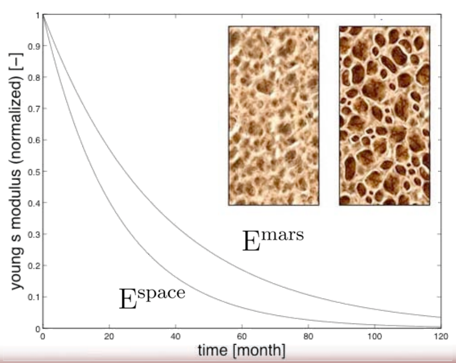
$$\rho_0(36) = 1.0098$$
$$\rho_0(37) = 0.9947$$

example - bone loss in space

density growth - bone loss in space

$$E = 3.790\, \rho_0^3\ \text{MPa} \quad \text{with } \rho_0 \text{ in g/cm}^3$$

carter & hayes [1977]



example - bone loss in space