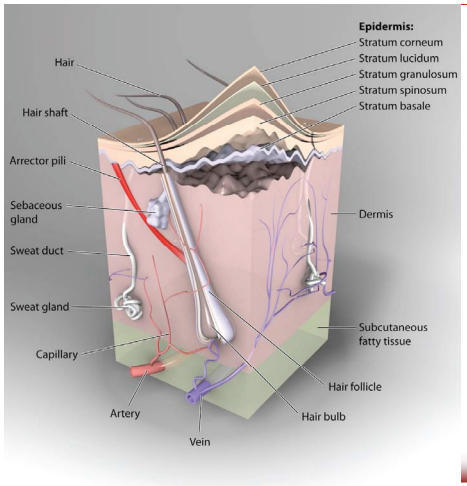
11 - volume growth
skin expansion & growth

skin

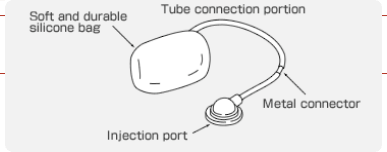


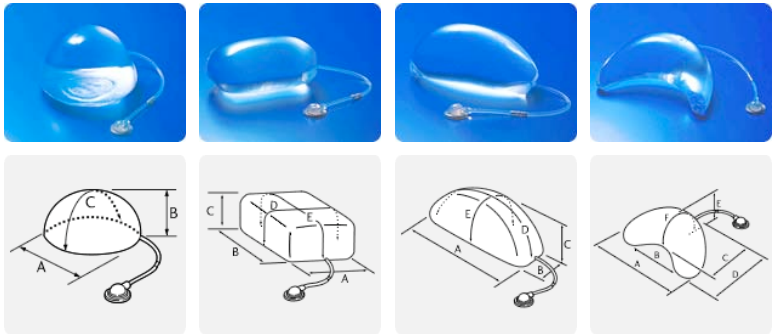
skin is the soft outer covering of an animal, in particular a vertebrate. in mammals, the skin is the largest organ of the system made up of multiple layers of ectodermal tissue, and guards the underlying muscles, bones, ligaments, and internal organs. because it interfaces with the environment, skin plays a key role in protecting the body. the adjective cutaneous literally means of the skin.

skin expansion

skin expansion is a technique used by plastic and restorative surgeons to cause the body grow additional skin. keeping living tissues under tension causes new cells to form and the amount of tissue to increase. in some cases, this may be accomplished by the implantation of inflatable balloons under the skin. by far the most common method, the surgeon inserts the inflatable expander beneath the skin and periodically, over weeks or months, injects a saline solution to slowly stretch the overlying skin. the growth of tissue is permanent, but will retract to some degree when the expander is removed. within the past 30 years, skin expansion has revolutionized reconstructive surgery. typical applications are breast reconstruction, burn injuries, and pediatric plastic surgery.

skin expanders





skin expansion and growth

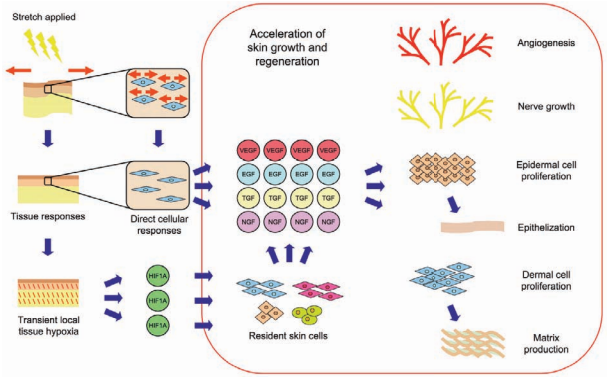


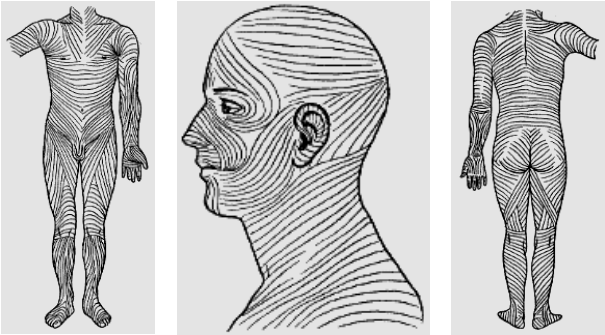
Fig 5. Proposed parallel mechanisms for skin growth. It is likely that there are mechanisms promoting tissue growth on both the cellular as well as the macroscopic level. Previous studies have implicated the role of induced cellular growth through direct stimulation of the cell and related mechanotransduction pathways.

chin, ogawa, lancerotto, pietramaggiori, schomacker, methews, scherer,duyn,prsa,…,orgill [2010]

langer's lines - anisotropy of human skin

langer's lines, sometimes called cleavage lines, are topological lines drawn on a map of the human body. they are defined by the direction in which the human skin would split when struck with a spike. langer's lines correspond to the natural orientation of collagen fibers in the dermis and epidermis. knowing the direction of langer's lines within a specific area of the skin is important for surgical procedures. particularly cosmetic surgery involving the skin. if a surgeon has a choice about where and in what direction to place an incision, he may choose to cut in the direction of langer's lines. incisions made parallel to langer's lines may heal better and produce less scarring than those cut across.

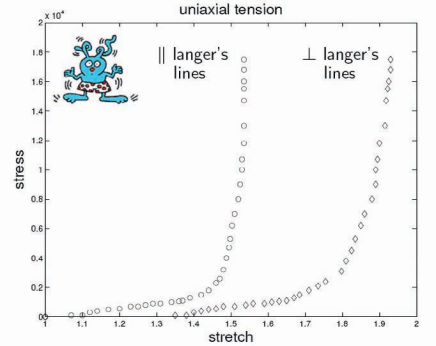
langer's lines - anisotropy of human skin



lines of tension - orientation of collagen fiber bundles

carl ritter von langer [1819-1887]

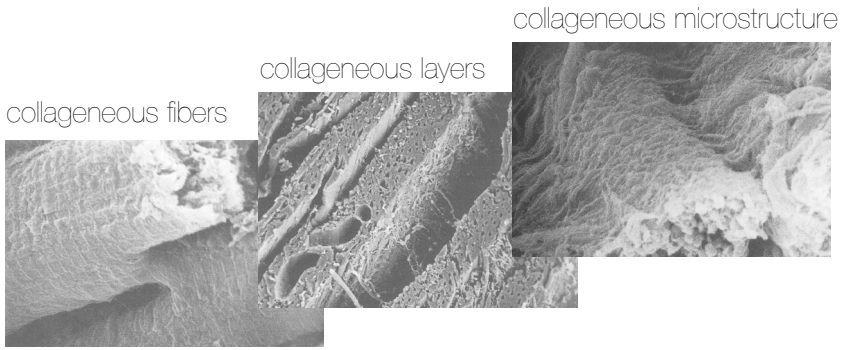
langer's lines - anisotropy of rabbit skin



stiffer || to langer's lines - stress locking @crit stretch

lanir & fung [1974]

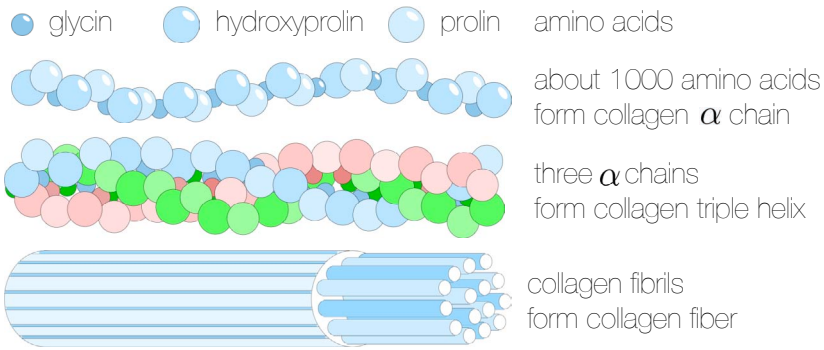
what is it that makes skin anisotropic? collagen fibers



directional strengthening due to collagen fibers

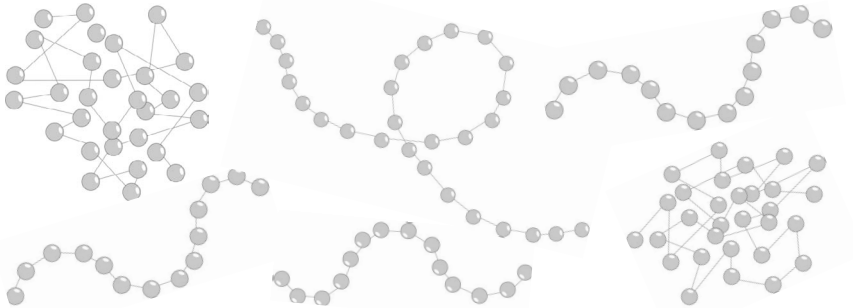
humphrey [2002]

collagen fibers - hierarchical microstructure



directional strengthening due to collagen fibers

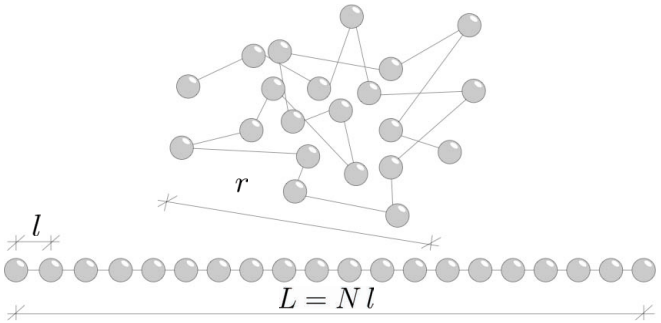
statistical mechanics of long chain molecules



entropic elasticity - entropy increases upon stretching

kuhn [1936], [1938], porod [1949], kratky & porod [1949], treolar [1958], flory [1969], bustamante, smith, marko & siggia [1994], marko & siggia [1995], rief [1997], holzapfel [2000], bischoff, arruda & grosh [2000], [2002], ogden, saccomandi & sgura [2006]

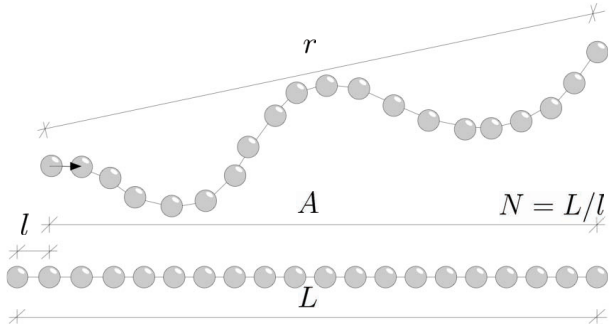
uncorrelated chain model - freely jointed chain



$$\psi^{\text{fjc}} = k\,\theta\,N\left[\frac{r}{L}\mathcal{L}^{-1} + \ln\left(\frac{\mathcal{L}^{-1}}{\sinh(\mathcal{L}^{-1})}\right)\right]$$

micromechanically motivated parameter - contour length L

correlated chain model - wormlike chain

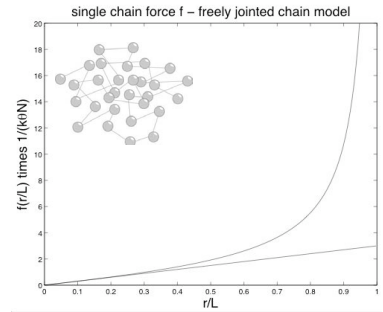


$$\psi^{\mathrm{wlc}} = \frac{k\,\theta\,\boldsymbol{L}}{4\,\boldsymbol{A}}\left[2\,\frac{r^2}{\boldsymbol{L}^2} + \frac{1}{[1-r/\boldsymbol{L}]} - \frac{r}{\boldsymbol{L}}\right]$$

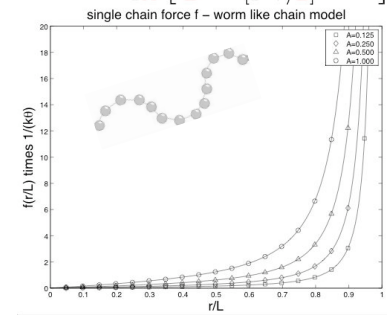
micromechanically motivated parameters - contour length $\boldsymbol{L}$ and persistence length $\boldsymbol{A}$

uncorrelated vs correlated chain model

$$f^{\mathrm{fjc}} = k\,\theta\,N\mathcal{L}^{-1}\left(\frac{r}{L}\right)$$



$$f^{\mathrm{wlc}} = \frac{k\,\theta}{4\,A}\left[\frac{4\,r}{L} + \frac{1}{[1-r/L]^2} - 1\right]$$

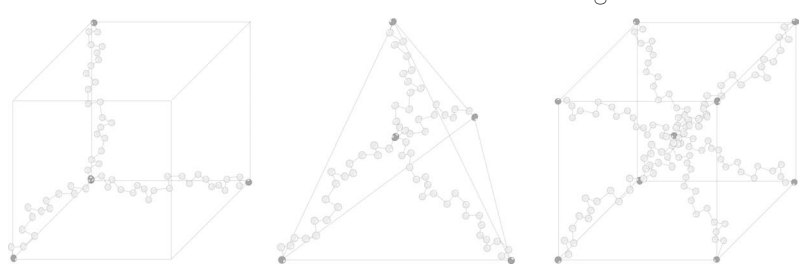


characteristic locking behavior - initial stiffness of wlc

micromechanically motivated parameters - contour length $\boldsymbol{L}$ and persistence length $\boldsymbol{A}$

chain network models

three chain model | four chain model | eight chain model

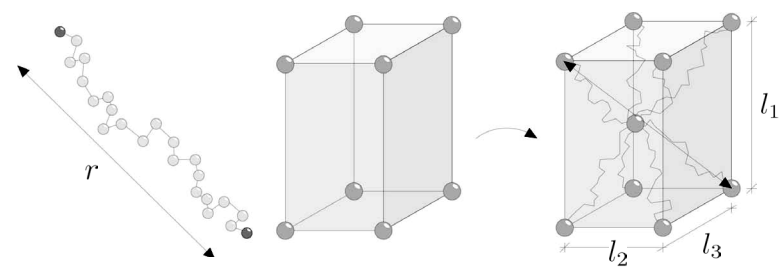


representative isotropic network of cross-linked chains

flory & rehner [1943], james & guth [1943], wang & guth [1952], treloar [1958], arruda & boyce [1993], wu & van der giessen [1993], boyce [1996], boyce & arruda [2000], bischoff, arruda & grosh [2002], miehe, göktepe & lulei [2004]

generalized eight chain model

eight single chains | isotropic cell matrix | eight chain model



$$\Psi^{\mathrm{chn}} = \tfrac{1}{8}\,\gamma^{\mathrm{chn}}\sum_{i=1}^{8}\psi^{\mathrm{wlc}}(r) \qquad \text{with} \qquad r = r(\boldsymbol{F})$$

$$\Psi^{\mathrm{iso}} = \tfrac{1}{2}\,\lambda\ln^2(\det(\boldsymbol{F})) + \tfrac{1}{2}\,\mu[\,\boldsymbol{F}^{\mathrm{t}}:\boldsymbol{F} - n^{\mathrm{dim}} - 2\,\ln(\det(\boldsymbol{F}))\,]$$

micromechanically motivated parameters - chain density γ^{chn} and cell dimensions l_1, l_2, l_3

generalized eight chain model

- general case orthotropic network model

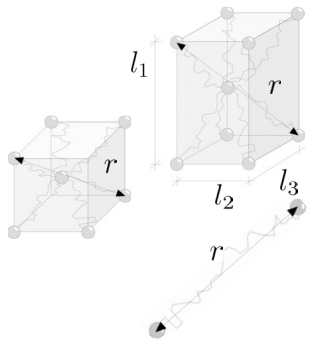
  $l_1 \neq l_2 \neq l_3 \qquad r = \sqrt{l_I^2 \, \bar{I}_I^C}$

- special case isotropic network model

  $l_1 = l_2 = l_3 = l \qquad r = l\sqrt{I_1^C}$

- special case transversely isotropic model

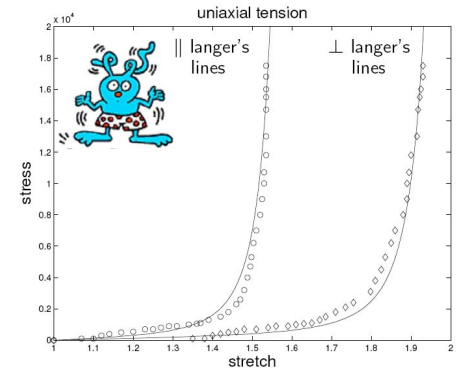
  $l_2 = l_3 = 0 \qquad r = l_1\sqrt{\bar{I}_1^C}$



traditional arruda boyce model as special case

invariants $I_1^C = \boldsymbol{C} : \boldsymbol{I}$ and $\bar{I}_I^C = \boldsymbol{n}_I \cdot \boldsymbol{C} \cdot \boldsymbol{n}_I$

experiment vs simulation - rabbit skin



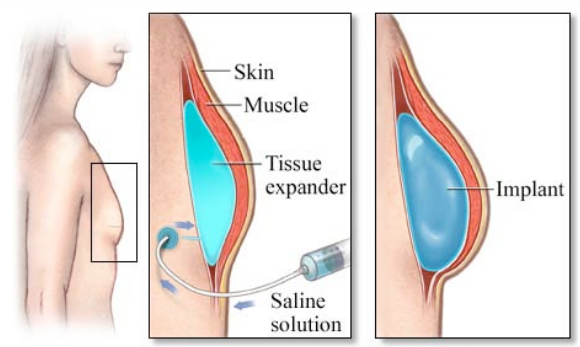
stiffer || to langer's lines - stress locking @crit stretch

lanir & fung [1974], kuhl, garikipati, arruda & grosh [2005]

skin expansion and growth - partial ear reconstruction

partial reconstruction of right ear. fig 1. preoperative status of partial traumatic amputation of right ear. fig 2. a rubber balloon is inserted in the subcutaneous tissue. fig 3. the rubber balloon is inflated gradually over a period of six weeks. fig 4. upon removal of the balloon, a c-shaped autogenous graft was introduced and covered by a double pedicled tubed flap fashioned from the skin expanded by balloon inflation. neumann [1957]

skin expansion and growth - breast reconstruction



within the past 30 years, tissue expansion has revolutionized reconstrutive surgery. typical application are breast reconstruction, burn injuries, and pediatric plastic surgery. natural tissue expansion can be observed in pregnancy, where the local tissue expands and growth in area in response to tension.

skin expansion and growth - facial reconstruction

eight-year-old boy who had a nevus removed as an infant. tissue expansion is completed in approximately ten weeks. the use of tissue expansion in cosmetic precedures is often limited by the significant deformity the patient must temporarily accept during the four to six week long procedure.

mangubat [2010]

skin expansion and growth - facial reconstruction

in this study of reconstruction of the forehead in children, the average number of surgical procedures required to complete reconstruction was six, involving an average of three tissue expansion proecures.

gosain & cortes [2007]

skin expansion and growth - ear gauging

there are many different methods you can choose from to stretch your ears. always wait at least one month between stretching. failure to stick to this could result in your earlobe puckering, being very thin, or even tearing completely apart. **tapering** is the most common way to stretch ears today. it involves the use of a taper, a rod that is larger at one end, specifically made for this purpose. the taper is lubricated and pushed though the hole until the larger end is flush with the earlobe. rings are then pushed though, parallel to the end of the taper. no equipment is used **dead stretching**. larger jewelry is just forced though the existing piercing. large **weights** can be used to stretch the piercing.

skin expansion and growth - lip plates

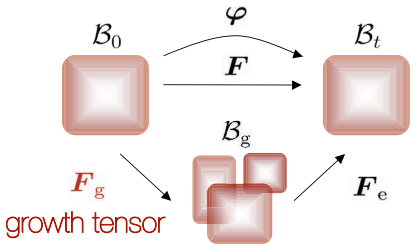
among the surma and mursi in ethiopia, about 6 to 12 months before marriage the woman's lip is pierced, usually at around the age of 15 to 18. the initial piercing is done as an incision of the lower lip of 1 to 2 cm length, and a simple wooden peg is inserted. after the wound has healed, which usually takes 2 or 3 weeks, the peg is replaced with a slightly bigger one. at a diameter of about 4 cm the first lip plate made of clay is inserted. every woman crafts her plate by herself and takes pride in including some ornamentation. the final diameter ranges from about 8 cm to over 20 cm. the plate's size is a sign of social or economical importance in some tribes.

http://www.mursi.org

motivation - skin growth

kinematics of finite growth

$$\boldsymbol{F} = \boldsymbol{F}_{\mathrm{e}} \cdot \boldsymbol{F}_{\mathrm{g}}$$



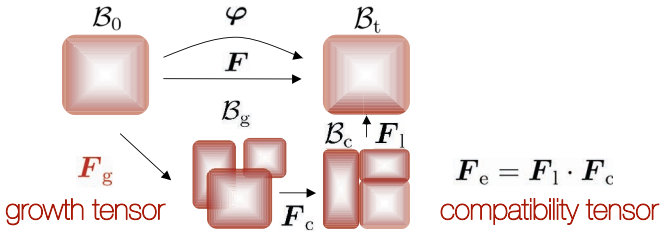
multiplicative decomposition

lee [1969], simo [1992], rodriguez, hoger & mc culloch [1994], epstein & maugin [2000], humphrey [2002], ambrosi & mollica [2002], himpel, kuhl, menzel & steinmann [2005]

kinematic equations

kinematics of finite growth

$$\boldsymbol{F} = \boldsymbol{F}_{\mathrm{l}} \cdot \boldsymbol{F}_{\mathrm{c}} \cdot \boldsymbol{F}_{\mathrm{g}}$$



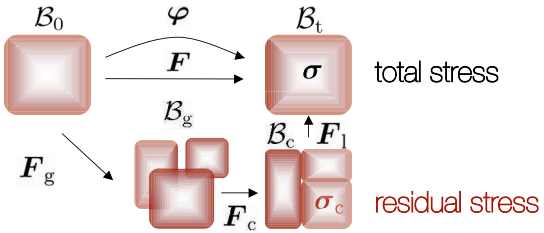
multiplicative decomposition

lee [1969], simo [1992], rodriguez, hoger & mc culloch [1994], epstein & maugin [2000], humphrey [2002], ambrosi & mollica [2002], himpel, kuhl, menzel & steinmann [2005]

kinematic equations

kinematics of finite growth

$$\boldsymbol{F} = \boldsymbol{F}_{\mathrm{l}} \cdot \boldsymbol{F}_{\mathrm{c}} \cdot \boldsymbol{F}_{\mathrm{g}}$$



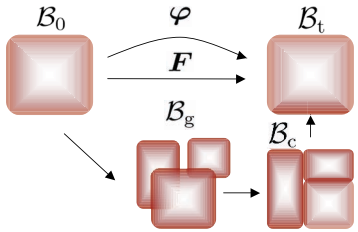
residual stress

the additional deformation of squeezing the grown parts back to a compatible configuration gives rise to residual stresses (see thermal stresses)

kinematic equations

skin expansion and growth

$$\boldsymbol{F} = \boldsymbol{F}_{\mathrm{l}} \cdot \boldsymbol{F}_{\mathrm{c}} \cdot \boldsymbol{F}_{\mathrm{g}}$$



growth tensor

$$\boldsymbol{F}_{\mathrm{g}} = \begin{bmatrix} \lambda_{\mathrm{m}}^{\mathrm{g}} & 0 & 0 \\ 0 & \lambda_{\mathrm{p}}^{\mathrm{g}} & 0 \\ 0 & 0 & \lambda_{h}^{\mathrm{g}} \end{bmatrix}$$

elastic tensor

$$\boldsymbol{F}_{\mathrm{l}} = \begin{bmatrix} \lambda_{\mathrm{m}}^{\ell} & 0 & 0 \\ 0 & \lambda_{\mathrm{p}}^{\ell} & 0 \\ 0 & 0 & \lambda_{h}^{\ell} \end{bmatrix}$$

compatibility restoring tensor

$$\boldsymbol{F}_{\mathrm{c}} = \begin{bmatrix} \lambda_{\mathrm{m}}^{\mathrm{eg}} & 0 & 0 \\ 0 & \lambda_{\mathrm{p}}^{\mathrm{eg}} & 0 \\ 0 & 0 & \lambda_{h}^{\mathrm{eg}} \end{bmatrix}$$

socci, pennati, gervaso, vena [2007]

skin expansion and growth

- growth law $\dot{\mathbf{F}}^{\mathrm{g}} = \dot{\mathbf{U}}^{\mathrm{g}} = k_{\lambda} \begin{bmatrix} \lambda_{\mathrm{m}}^{\ell} - 1 & 0 & 0 \\ 0 & \lambda_{\mathrm{p}}^{\ell} - 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- increase in mass $\dot{m} = \delta \int_V \mathrm{div}(\mathbf{v}^{\mathrm{g}}) \mathrm{d}V = \delta \int_V (D_{11}^{\mathrm{g}} + D_{22}^{\mathrm{g}} + D_{33}^{\mathrm{g}}) \mathrm{d}V$
- rate of growth deformation tensor $\mathbf{D}^{\mathrm{g}} = \frac{1}{2}\left[\dot{\mathbf{U}}^{\mathrm{g}} (\mathbf{U}^{\mathrm{g}})^{-1} + (\mathbf{U}^{\mathrm{g}})^{-1} \dot{\mathbf{U}}^{\mathrm{g}}\right]$
- rate of mass increase $\dot{m} = \delta k_{\lambda} \int_V \left[\frac{(\lambda_{\mathrm{m}}^{\mathrm{l}} - 1)}{\lambda_{\mathrm{m}}^{\mathrm{g}}} + \frac{(\lambda_{\mathrm{p}}^{\mathrm{l}} - 1)}{\lambda_{\mathrm{p}}^{\mathrm{g}}} \right] \mathrm{d}V$

socci, pennati, gervaso, vena [2007]

skin expansion and growth

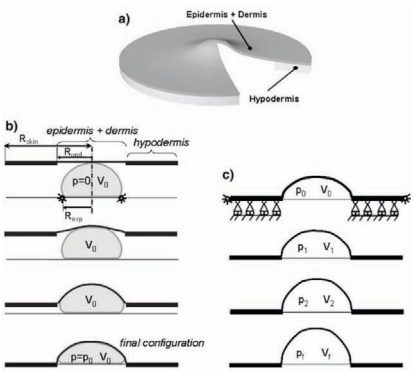


Fig 2. 2a. Sketch of model of expanded skin showing the two considered layers. 2b. First step of simulation of skin expansion: thee successive phases of skin-expander interaction. 2c. Second step of simulation of skin expansion: three successive phases of skin-expander model.

socci, pennati, gervaso, vena [2007]

skin expansion and growth

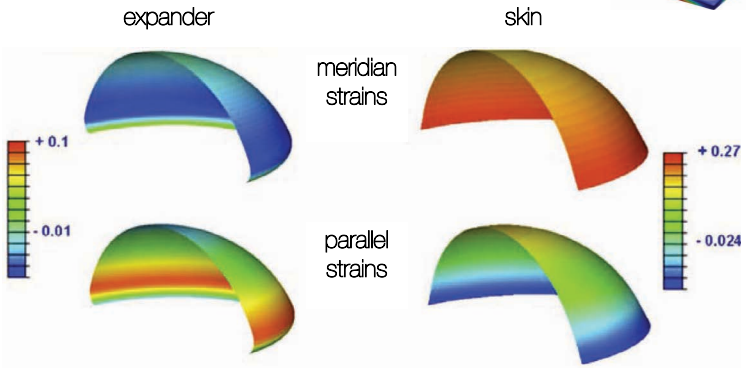


Fig 7. Contour plot of logarithmic principal strains for expander (left) and skin (right) at volume of 550 ml.

socci, pennati, gervaso, vena [2007]

skin expansion and growth

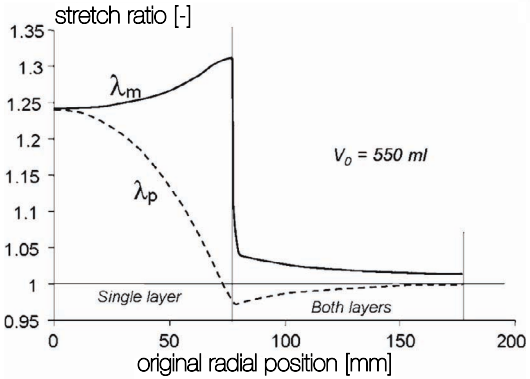


Fig 8. Meridian and parallel stretch ratios vs. distance from the axis of symmetry of the two skin regions (single layer and two layers) after expander injection at reference volume V_0.

socci, pennati, gervaso, vena [2007]

skin expansion and growth

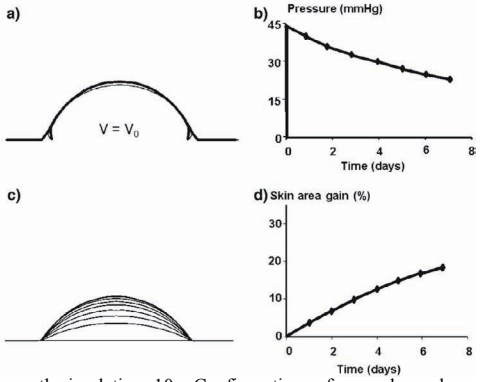


Fig 10. Results of skin growth simulation. 10a. Configurations of expander and expander skin immediately after inflation (thin line) and one week post inflation (thick line). 10b. Pressure decrease during one week after inflation. 10c. Different stress-free configurations at times d0 to d7 at increments of one day. 10d. Percentage of skin area gain.

socci, pennati, gervaso, vena [2007]

volume growth at constant density

- deformation gradient

$$\boldsymbol{F} = \boldsymbol{F}^{\mathrm{e}} \cdot \boldsymbol{F}^{\mathrm{g}} \qquad \text{with} \qquad \boldsymbol{F} = \nabla_{\boldsymbol{X}} \boldsymbol{\varphi}$$

- jacobians … remember: volume change

$$J = J^{\mathrm{e}}\, J^{\mathrm{g}} \qquad \text{with} \qquad J = \det(\boldsymbol{F})$$
$$J^{\mathrm{e}} = \det(\boldsymbol{F}^{\mathrm{e}}) \text{ and } J^{\mathrm{g}} = \det(\boldsymbol{F}^{\mathrm{g}})$$

- cofactor … remember: area change

$$\vartheta = \vartheta^{\mathrm{e}}\, \vartheta^{\mathrm{g}} \qquad \text{with} \qquad \vartheta = ||\operatorname{cof}(\boldsymbol{F}) \cdot \boldsymbol{n}_0||$$
$$\vartheta^{\mathrm{e}} = ||\operatorname{cof}(\boldsymbol{F}^{\mathrm{e}}) \cdot \boldsymbol{n}_0|| \text{ and } \vartheta^{\mathrm{g}} = ||\operatorname{cof}(\boldsymbol{F}^{\mathrm{g}}) \cdot \boldsymbol{n}_0||$$

- growth tensor … growth = area change

$$\boldsymbol{F}^{\mathrm{g}} = \sqrt{\vartheta^{\mathrm{g}}}\, \boldsymbol{I} + [\, 1 - \sqrt{\vartheta^{\mathrm{g}}}\,]\, \boldsymbol{n}_0 \otimes \boldsymbol{n}_0$$

the adrian model [2010]

time integration - euler backward

- finite difference approximation

$$\dot{\vartheta}^{\mathrm{g}} = \frac{1}{\Delta t}\,[\,\vartheta^{\mathrm{g}} - \vartheta^{\mathrm{g}}_{\mathrm{n}}\,] = k^{\mathrm{g}}(\vartheta^{\mathrm{g}})\, \phi^{\mathrm{g}}(\vartheta^{\mathrm{e}})$$

- residual of discrete evolution equation

$$\mathsf{R}^{\mathrm{g}} = \vartheta^{\mathrm{g}} - \vartheta^{\mathrm{g}}_{\mathrm{n}} - k^{\mathrm{g}}\, \phi^{\mathrm{g}}\, \Delta t \;\doteq 0$$

- linearized residual for local newton iteration

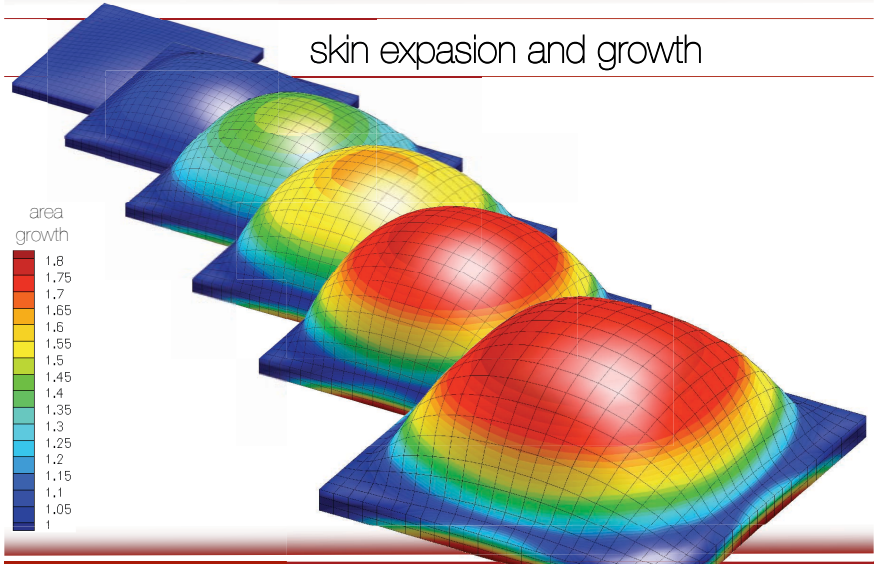
$$\mathsf{K}^{\mathrm{g}} = \frac{\partial \mathsf{R}^{\mathrm{g}}}{\partial \vartheta^{\mathrm{g}}} = 1 - \left[\frac{\partial k^{\mathrm{g}}}{\partial \vartheta^{\mathrm{g}}}\, \phi^{\mathrm{g}} + k^{\mathrm{g}}\, \frac{\partial \phi^{\mathrm{g}}}{\partial \vartheta^{\mathrm{g}}}\right] \Delta t$$

- iterative update of growth multiplier

$$\vartheta^{\mathrm{g}} \leftarrow \vartheta^{\mathrm{g}} - \mathsf{R} \,/\, \mathsf{K}$$

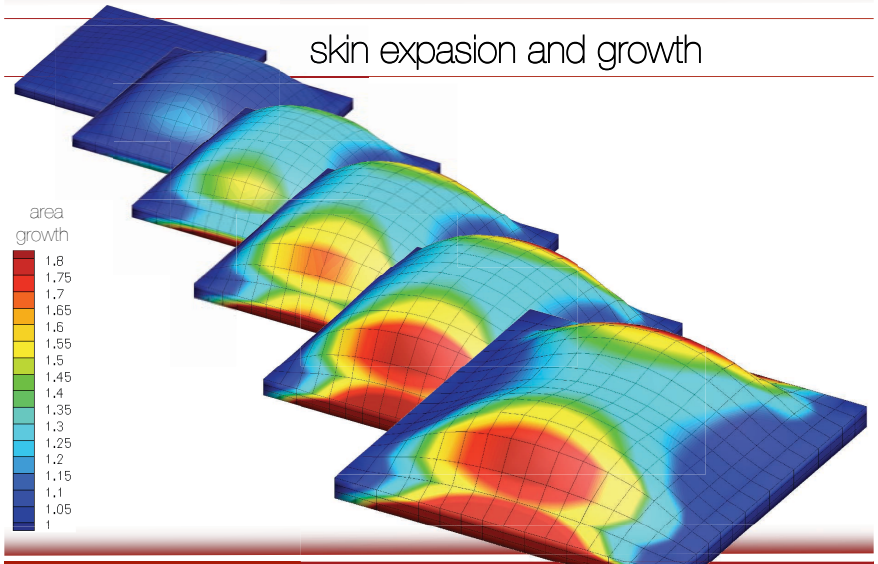
the adrian model [2010]

skin expasion and growth



example - skin expansion

skin expasion and growth



example - skin expansion