mechanics of growth

ellen kuhl
mechanical engineering
stanford university

01 - motivation -
everything grows!

... what we do ...

kinematic equations for finite growth
\( \mathbf{F} = \mathbf{F}_c \cdot \mathbf{F}_g \)

balance equations for open systems
\( D_t \rho_0 = \text{Div}(\mathbf{R}) + \mathcal{R}_0 \)
\( \rho_0 \ D_t \mathbf{v} = \text{Div}(\mathbf{P}) + \mathbf{b}_0 \)

constitutive equations for living tissues
\( \mathbf{P} = \mathbf{P}(\rho_0, \mathbf{F}, \mathbf{F}_g) \)

fe analyses for biological structures
continuum- & computational biomechanics

01 - introduction

... why we do what we do ...

kinematic equations for finite growth
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constitutive equations for living tissues
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fe analyses for biological structures

... because biological structures are ...

... what we do ...

... what we do ...

... what we do ...
... why we do what we do...

Kinematic equations for finite growth:
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Balance equations for open systems:
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FE analyses for biological structures

... because biological structures are ...

... what we do ...

Highly deformable

... why we do what we do ...

Kinematic equations for finite growth:
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FE analyses for biological structures

... because biological structures are ...

... what we do ...

Living

... why we do what we do ...

Kinematic equations for finite growth:
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Constitutive equations for living tissues:
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FE analyses for biological structures

... because biological structures are ...

... what we do ...

Nonlinear

... why we do what we do ...

Kinematic equations for finite growth:
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Balance equations for open systems:
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FE analyses for biological structures

... because biological structures are ...

... what we do ...

Inelastic

... why we do what we do ...

Kinematic equations for finite growth:
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Balance equations for open systems:
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Constitutive equations for living tissues:
\[ P = P(\rho_0, \mathbf{F}, F_g) \]
FE analyses for biological structures

... because biological structures are ...

... what we do ...

Highly nonlinear
me337 - goals

in contrast to traditional engineering structures living structures show the fascinating ability to grow and adapt their form, shape and microstructure to a given mechanical environment. this course addresses the phenomenon of growth on a theoretical and computational level and applies the resulting theories to classical biomechanical problems like bone remodeling, hip replacement, wound healing, atherosclerosis or in stent restenosis. this course will illustrate how classical engineering concepts like continuum mechanics, thermodynamics or finite element modeling have to be rephrased in the context of growth. having attended this course, you will be able to develop your own problemspecific finite element based numerical solution techniques and interpret the results of biomechanical simulations with the ultimate goal of improving your understanding of the complex interplay between form and function.

me 337 - syllabus

<table>
<thead>
<tr>
<th>day</th>
<th>date</th>
<th>topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>tue</td>
<td>sep 21</td>
<td>motivation - everything grows</td>
</tr>
<tr>
<td>thu</td>
<td>sep 23</td>
<td>basics and maths - notation and tensors</td>
</tr>
<tr>
<td>tue</td>
<td>sep 28</td>
<td>class project '07 - growth of tennis player arms</td>
</tr>
<tr>
<td>thu</td>
<td>sep 30</td>
<td>guided reading - no class</td>
</tr>
<tr>
<td>tue</td>
<td>oct 5</td>
<td>basics and mechanics - kinematics and balance equations of growth</td>
</tr>
<tr>
<td>thu</td>
<td>oct 7</td>
<td>guided reading - no class</td>
</tr>
<tr>
<td>thu</td>
<td>oct 12</td>
<td>density growth - growing bones</td>
</tr>
<tr>
<td>thu</td>
<td>oct 14</td>
<td>density growth - Finite elements for growth / theory</td>
</tr>
<tr>
<td>thu</td>
<td>oct 19</td>
<td>density growth - Finite elements for growth / matlab</td>
</tr>
<tr>
<td>thu</td>
<td>oct 21</td>
<td>density growth - growing bones</td>
</tr>
<tr>
<td>thu</td>
<td>oct 26</td>
<td>density growth - Finite elements for growth</td>
</tr>
<tr>
<td>thu</td>
<td>oct 26</td>
<td>midterm</td>
</tr>
<tr>
<td>thu</td>
<td>nov 1</td>
<td>volume growth - growing tumors</td>
</tr>
<tr>
<td>thu</td>
<td>nov 4</td>
<td>volume growth - Finite elements for growth / theory</td>
</tr>
<tr>
<td>thu</td>
<td>nov 9</td>
<td>volume growth - Finite elements for growth / matlab</td>
</tr>
<tr>
<td>thu</td>
<td>nov 11</td>
<td>volume growth - growing arteries</td>
</tr>
<tr>
<td>thu</td>
<td>nov 16</td>
<td>volume growth - growing hearts</td>
</tr>
<tr>
<td>thu</td>
<td>nov 18</td>
<td>remodeling - remodeling arteries and tendons</td>
</tr>
<tr>
<td>thu</td>
<td>nov 30</td>
<td>class project - discussion, presentation, evaluation</td>
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<tr>
<td>thu</td>
<td>dec 2</td>
<td>class project - discussion, presentation, evaluation</td>
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<tr>
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<td>dec 5</td>
<td>written part of final project due</td>
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The phenomenon of twisted growth: humeral torsion in dominant arms of high performance tennis players


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This manuscript is driven by the need to understand the fundamental mechanisms that cause twisted bone growth and shoulder pain in high performance tennis players. Our ultimate goal is to predict bone mass density in the humerus through computational analysis. The underlying study spans a unique four level complete analysis consisting of a high-speed video analysis, a musculoskeletal analysis, a finite element based density growth analysis and an X-ray based bone mass density analysis. For high performance tennis players, critical loads are postulated to occur during the serve. From high-speed video analyses, the serve phases of maximum external shoulder rotation and ball impact are identified as most critical loading situations for the humerus. The corresponding posts from the video analysis are reproduced with a musculoskeletal analysis tool to determine muscle attachment points, muscle force vectors and overall forces of relevant muscle groups. Collective representative muscle forces of the deltoid, latissimus dorsi, pectoralis major and triceps are then applied as external loads in a fully 3D finite element analysis. A problem specific nonlinear finite element based density analysis tool is developed to predict functional adaptation over time. The density profiles in response to the identified critical muscle forces during serve are qualitatively compared to X-ray based bone mass density analyses.

Keywords: bone mass density changes; functional adaptation; musculoskeletal analysis; finite element analysis; sports medicine

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Computational modeling of arterial wall growth

Attempts towards patient-specific simulations based on computer tomography

E. Kuhl  -  R. Maas  -  G. Hinney  -  A. Menzel

Original Paper

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Abstract  The present manuscript documents our first experiences with a computational model for stress-induced arterial wall growth and in-stent restenosis related to atherosclerosis. The underlying theoretical framework is provided by the kinematics of finite growth combined with open system thermodynamics. The computational simulation is embedded in a finite element approach where the growth is essentially captured by a single scalar-valued growth factor introduced as internal variable on the integration point level. The conceptual simplicity of the model enables straightforward implementation into standard commercial finite element codes. Qualitative studies of stress-induced changes of the arterial wall thickness in response to balloon angioplasty or stenting are presented to illustrate the features of the suggested growth model. First attempts towards a patient specific simulation based on realistic artery morphologies generated from computer tomography data are discussed.
Computational modeling of hip replacement surgery:
Total hip replacement vs. hip resurfacing

E. Kuhl & F. Balle

The motivation of the present work is the computational simulation of hip replacement surgery by means of a finite element approach based on open system thermodynamics. Its key feature is a non-constant material density, which is allowed to adapt with respect to changes in the mechanical loading environment. From a computational point of view, the density is treated as an internal variable. Its evolution is governed by a first order rate equation, the balance of mass, which is enhanced by an additional mass production term to account for growth. An implicit Euler backward scheme is suggested for its time discretization. The algorithmic determination of the material density based on a local Newton iteration is presented. To ensure quadratic convergence of the global Newton Raphson solution scheme, a consistent linearization of the discrete algorithmic equations is carried out. Finally, two alternative medical techniques in hip arthritis are compared, the conventional total hip replacement strategy and the more recent hip resurfacing technology. The result of the suggested remodeling algorithm is shown to agree remarkably well with clinically observed phenomena.
Computational modeling of growth
Systemic and pulmonary hypertension in the heart

Abstract We introduce a novel constitutive model for growing soft biological tissues and study its performance in two characteristic cases of mechanically induced wall thickening of the heart. We adopt the concept of an incompatible growth configuration introducing the multiplicative decomposition of the deformation gradient into an elastic and a growth part. The key feature of the model is the definition of the evolution equation for the growth tensor which we motivate by pressure-overload induced sarcomere stretch. In response to the deposition of sarcomere units on the molecular level, the individual heart muscle cells increase in diameter, and the wall of the heart becomes progressively thicker. We present the underlying constitutive equations and their algorithmic implementation within an implicit nonlinear finite element framework. To demonstrate the features of the proposed approach, we study two classical growth phenomena in the heart: left and right ventricle wall thickening in response to systemic and pulmonary hypertension.

Keywords Biomechanics; growth; remodeling; finite element; hypertension; hypertrophy

why does the heart wall get thicker?

J. Cham “Piled higher and deeper”, [1999]
history - 17th century

"...dal che e manifesto, che chi volesse mantener in un vastissimo gigante le proporzioni, che hanno le membra in un huomo ordinario, bisognerebbe o trovare materia molto più dura, e resistente per formare fossa o vero ammietere, che la robustezza sua fusse a proporzione assai più fia cca, che negli huomini de statu media: cremente crescendo, a smisurata altezza si vedrebbono dal proprio peso opprime re, e cadere..."

Galileo, "Discorsi e dimostrazioni matematiche", [1638]

history - 19th century

Culmann & von Meyer, "Graphic statics" [1867]

introduction

history - 19th century

"...es ist demnach unter dem gesetze der transformation der knochen dasjenige gesetz zu verstehen, nach welchem im gefolge primarer abanderungen der form und transpruchnahme bestimmte umwandlungen der inneren architektur und umwandlungen der aeuseren form sich vollziehen..."

Wolff, "Gesetz der Transformation der Knochen" [1892]

introduction

history - 19th century

"...whether it be the sweeping eagle in his flight or the open apple-blossom, the toiling work-horse, the billie swan, the branching osk, the winding stream at its base, the drifting clouds, over all the coursing sun, form ever follows function, and this is the law...

Sullivan, "Form follows function" [1896]

introduction

history - 19th century

carson pirie scott store
Sullivan [1904]
...the system consisting of only the porous structure without its entrained perfusant is open with respect to momentum transfer as well as mass, energy, and entropy transfer, we shall write balance and constitutive equations for only the bone..."

Cowin & Hagedus „Theory of adaptive elasticity“ [1976]

"...the relationship between physical forces and the morphology of living things has puzzled the curiosity of every artist, scientist, or philosopher who has contemplated a tree or drawn the human figure, its importance was a concern of galileo and later thompson whose writings remind us that physical causation plays an inescapable role in the development of biological form..."

Beaupré, Carter & Orr „Theory of bone modeling & remodeling“ [1990]

"hypertrophy of the heart; comparison of cross sections of a normal heart (bottom), a heart chronically overloaded by an unusually large blood volume (left) and a heart chronically overloaded by an unusually large diastolic and systolic left ventricular pressure (right)"

Fung „Biomechanics - Motion, flow, stress, and growth“ [1990]
remodeling involves changes in material properties. These changes, which often are adaptive, may be brought about by alterations in modulus, internal structure, strength, or density. For example, bones, and heart muscle may change their internal structures through reorientation of trabeculae and muscle fibers, respectively.

Taber „Biomechanics of growth, remodeling and morphogenesis“ [1995]
**Athlete’s Heart - Patient-Specific Simulation**

- Growth beyond normal size due to **significant exercise**
- Training-induced changes are typically **reversible**
- Adaptation driven by **elevated pressure** and increased filling
- Cardiac output increases from 6 l/min at rest up to 40 l/min
- Cardiac mass increases up to 50%

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**How Do Skeletal Muscles Grow When We Train?**

- **Figure**: Response of rat soleus muscle to overstretch (left). Sarcomere length (top) and sarcomere number (bottom). During the initial phase of loading, days 4 and 8, sarcomere length progressively increases. Beyond this point, however, sarcomere length remains constant and there is a progressive increase of the number of sarcomeres in series. Response of rodent skeletal muscle to overload (right). Myofiber cross section area increases with increased loading.

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**Group Project - Athlete’s Heart**

**Group Project - Getting Stronger**

**Group Project - Mouse Heart**

**Group Project - Sarcomerogenesis**

**Heart Attack! How Does the Heart Grow?**

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**How Do Cells Grow?**

- How do cardiomyocytes change their length?
- Left ventricle **enlarged 4-28%** and **dilated 14% at week one** after volume overload
- LV dilation by **cardiomyocyte elongation**
- Myocyte elongation by **serial sarcomere deposition**
- Sarcomere number increases **linearly** from 62 to 85
- Sarcomere deposition rate is linear in weeks 1 to 4 & decays smoothly from week 4 to saturation at 26

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**A Literature Study / Review Paper**