homework III - revise your final project

due 11/11/10, 11:11am, 420-040

Late homework can be dropped off in a box in front of Durand 217. Please mark clearly with date and time @drop off. We will take off 1/10 of points for each 24 hours late, every 12pm after due date. This homework will count 10% towards your final grade.

problem 1 - growth tensors

We have introduced different growth tensors $\boldsymbol{F}^{\text{g}}$ in class. Discuss the following growth tensors.

1.1 $\boldsymbol{F}^{\text{g}} = \vartheta\, \boldsymbol{I}$

1.2 $\boldsymbol{F}^{\text{g}} = \boldsymbol{I} + [\vartheta - 1]\, \boldsymbol{f}_0 \otimes \boldsymbol{f}_0$

1.3 $\boldsymbol{F}^{\text{g}} = \boldsymbol{I} + [\vartheta - 1]\, \boldsymbol{s}_0 \otimes \boldsymbol{s}_0$

1.4 $\boldsymbol{F}^{\text{g}} = \sqrt{\vartheta}\, \boldsymbol{I} + [1 - \sqrt{\vartheta}]\, \boldsymbol{n}_0 \otimes \boldsymbol{n}_0$

Herein, $\boldsymbol{f}_0$ denotes a distinct fiber direction, $\boldsymbol{s}_0$ is a sheet direction, $\boldsymbol{n}_0$ is the normal to a characteristic microstructural plane, and ϑ is a scalar-valued growth multiplier.

For each growth tensor, focus on: (i) its mechanical interpretation, e.g., isotropic, transversely isotropic, orthotropic, generally anisotropic; (ii) its microstructural interpretation, e.g., spherical growth, fiber lengthening, fiber thickening, area growth; (iii) its biological application, e.g., which type of tissue growth does it characterize, why does this ansatz make sense.

problem 2 - growth tensors

Assume the following microstructural vectors, $\boldsymbol{f}_0 = [1,0,0]^{\text{t}}$, $\boldsymbol{s}_0 = [0,1,0]^{\text{t}}$, and $\boldsymbol{n}_0 = [0,0,1]^{\text{t}}$ aligned with the cartesian coordinates, and a growth multiplier of $\vartheta = 2$.

2.1 Calculate the four growth tensors $\boldsymbol{F}^{\text{g}}$ from 1.1 to 1.4.

2.2 Calculate the volume change of a cube of unit length for all four growth tensors $\boldsymbol{F}^{\text{g}}$ from 1.1 to 1.4 using the Jacobian $J^{\text{g}} = \det(\boldsymbol{F}^{\text{g}})$.

2.3 Draw a cubic block of tissue of unit length in a three-dimensional coordinate system. Add the unit vectors $\mathrm{d}\boldsymbol{X}_1 = [1,0,0]^{\text{t}}$, $\mathrm{d}\boldsymbol{X}_2 = [0,1,0]^{\text{t}}$, and $\mathrm{d}\boldsymbol{X}_3 = [0,0,1]^{\text{t}}$. For each of the growth tensors $\boldsymbol{F}^{\text{g}}$ in 1.1 to 1.4, calculate and illustrate the deformed vectors $\mathrm{d}\boldsymbol{x}_1$, $\mathrm{d}\boldsymbol{x}_2$ and $\mathrm{d}\boldsymbol{x}_3$ using $\mathrm{d}\boldsymbol{x} = \boldsymbol{F}^{\text{g}} \cdot \mathrm{d}\boldsymbol{X}$. Illustrate the grown block.

problem 3 - growing bones in matlab

3.1 Download the matlab file package from the coursework website or from the website `http://biomechanics.stanford.edu`.

3.2 To run the femur example, open the main file `nlin_fem.m`, and make sure that all input file readings are commented out by a % sign in the first column of lines 6 through 19. The only active input line should be line 12 reading `ex_femur`.

3.3 In the command window, call the main file by typing `nlin_fem` and wait for the mesh to be generated.

3.4 Run the density evolution algorithm for 5 time steps by typing `step,,5`. Describe what you see in the command window and in the graphics window. How many iterations does a typical load step take to find the equilibrium of the nonlinear problem? Focus on load step 5. Report the residuals, i.e., the errors in solution, to demonstrate quadratic convergence of the Newton Raphson scheme. Then, run the algorithm for an additional 25 time steps by typing `step,,25` in the command window. Quit the algorithmic environment by typing `quit`.

3.5 There are two essential fields that describe the geometry of a finite element input file, in this case your femur. Type `q0` to show one of them and then `size(q0)`. What does the `q0` field contain? Type `edof` to show one of them and then `size (edof)`. What does the `edof` field contain?

3.6 Learn to fake your results! Most people show finite element results in terms of colorful plots. Here, the color figures are produced in the subroutine `plot_int.m`. You can easily manipulate a plot by changing the color axis. Type `caxis([-1.0 0.2])` to change the color axis. You can then turn off the shadows by `lighting none` and plot your final figure by `print('-depsc','-r300','figure01.eps')`.

[1]	midstance phase of gait	2317 N	24°	703 N	28°
[2]	extreme range of abduction	1158 N	-15°	351 N	-8°
[3]	extreme range of adduction	1548 N	56°	468 N	35°

3.7 You have now optimized the density of the proxima femur based on the three most significant load cases as shown in the figure above. Now, change the loading. Open the input file `ex_femur.m`. In lines 58 through 62, you can see how the load is generated. Manipulate the direction of the compressive load of load case 1 on node 723 which is currently `F_ext(723*2-1)=-0.9424`, i.e., -2317N sin(24^0) in the horizontal direction and `F_ext(723*2)=-2.1167`, i.e., -2317N cos(24^0) in the vertical direction. Rotate the compressive force of load case 1 by 21^0 in the clockwise direction, i.e., change its line of action to -2.317 sin(45^0) in the horizontal and -2.317 cos(45^0) in the vertical direction by modifying line 62 in the code accordingly. Save your input file `ex_femur.m`, and rerun the simulation `nlin_fem` for `step`,,`30` time steps and quit your algorithmic environment by typing `quit`. Then, type `caxis([-1.0 0.2])` to change the color axis, turn off the shadows by `lighting none`, and plot your final figure with `print('-depsc', '-r300','figure02.eps')`.

3.8 Compare both figures, `figure01.eps` and `figure02.eps`, in three to five sentences and attach them to your homework.

For this part of the homework, it is okay to work in groups, especially if you are not very familiar with matlab. Even if you create the results in a group, however, the results, interpretations, and discussions must be written individually by each group member. It goes without saying that each group member has to understand the matlab algorithm.

problem 4 - revise your final project

4.1 Download the style file `me337_project_sample.doc` from the coursework website and paste in your title, outline, opening sentence, introduction, schematic drawing, and references from homework II.

4.2 Expand the reference section to at least three key references and seven additional references. Make sure your citations all have the same style.

4.3 Revise your introduction and make sure that all your references are cited. The introduction should: (i) contain your catchy opening sentence with citation, (ii) motivate your work, and (iii) give an overview of the current state of the field. It should be one to two columns long.

4.4 Draft an outline of all the figures you would like to include in your manuscript. This is the most important step of drafting your paper, since most scientific papers are written around figures. For each figure, create a place holder or the figure itself. Create meaningful figure captions. The figure captions in the sample file are actually not a good example. In the biological literature, captions are usually more detailed and can be several lines long. When you adopt figures from the literature, cite your source. Remember that in a real journal paper, you cannot use other authors' figures without copyright agreement.