balance equations

**balance equations** of mass, momentum, angular momentum and energy, supplemented with an entropy inequality constitute the set of conservation laws. The law of conservation of mass/matter states that the mass of a closed system of substances will remain constant, regardless of the processes acting inside the system. The principle of conservation of momentum states that the total momentum of a closed system of objects is constant.

balance equations

**balance equations** [balance equations of mass, linear momentum, angular momentum and energy apply to all material bodies. Each one gives rise to a field equation, holding on the configurations of a body in a sufficiently smooth motion and a jump condition on surfaces of discontinuity. Like position, time and body, the concepts of mass, force, heating and internal energy which enter into the formulation of the balance equations are regarded as having primitive status in continuum mechanics.]

Chadwick, *Continuum mechanics* [1976]

potato - balance equations

[1] isolation of subset $B$ from $B$
balance equations

[1] isolation of subset \( \mathcal{B} \) from \( \mathcal{B} \)
[2] characterization of influence of remaining body through phenomenological quantities - contact fluxes \( \mathbf{i}^p, \mathbf{i}^x, \mathbf{i}^\rho \)

[3] definition of basic physical quantities - mass, linear and angular momentum, energy
[4] postulation of balance of these quantities

general format

\[ \partial_t A = \text{Div}(\mathbf{B}) + C + \Gamma \]
balance of mass
\[ \rho_0 \ldots \text{density} \]
\[ \mathbf{0} \ldots \text{no mass flux} \]
\[ T^p = 0 \]
\[ 0 \ldots \text{no mass source} \]
\[ 0 \ldots \text{no mass production} \]

continuity equation
\[ D_t \rho_0 = 0 \]

balance of internal energy
\[ \rho_0 \ldots \text{internal energy density} \]
\[ \mathbf{Q} \ldots \text{heat flux} \]
\[ -\mathbf{Q} \cdot n = T^\theta \]
\[ Q_0 \ldots \text{heat source} \]
\[ 0 \ldots \text{no heat production} \]

energy equation
\[ D_t (\rho_0 I) = \text{Div}(-\mathbf{Q}) + Q_0 \]

balance of momentum
\[ \rho_0 \mathbf{v} \ldots \text{linear momentum density} \]
\[ \mathbf{P} \ldots \text{momentum flux - stress} \]
\[ \mathbf{P} \cdot n = T^\theta \]
\[ \mathbf{b}_0 \ldots \text{momentum source - force} \]
\[ 0 \ldots \text{no momentum production} \]

equilibrium equation
\[ D_t (\rho_0 \mathbf{v}) = \text{Div} (\mathbf{P}) + \mathbf{b}_0 \]
balance equations

Isolated system: A thermodynamic system which is not allowed to have any interaction with its environment. Enclosed by a rigid, adiabatic, impermeable membrane.

Potato - balance equations

Potato - dissipation inequality

Potato - entropy inequality

Balance of entropy

\[ \frac{dH}{dt} = \frac{d}{dt} \left( \int_V \phi \, dV + \int_{\partial V} \psi \, ds \right) \]

Entropy density

\[ \rho S \frac{dS}{dt} + \frac{\partial (\rho S \psi)}{\partial x} = \frac{\partial \phi}{\partial t} + \nabla \cdot \psi \]

Entropy source

\[ \rho S \frac{dS}{dt} + \frac{\partial (\rho S \psi)}{\partial x} = \frac{\partial \phi}{\partial t} + \nabla \cdot \psi \]

Entropy production

\[ \rho S \frac{dS}{dt} + \frac{\partial (\rho S \psi)}{\partial x} = \frac{\partial \phi}{\partial t} + \nabla \cdot \psi \]

Free energy

\[ \frac{d}{dt} \left( \rho \psi \right) = \frac{\partial (\rho \psi S)}{\partial x} + \rho \psi \frac{dS}{dt} - \frac{\partial \phi}{\partial t} - \frac{\partial \psi}{\partial x} \nabla \cdot \psi \]

Thermodynamic restriction

\[ \frac{d}{dt} \left( \rho \psi \right) = \frac{\partial (\rho \psi S)}{\partial x} + \rho \psi \frac{dS}{dt} - \frac{\partial \phi}{\partial t} - \frac{\partial \psi}{\partial x} \nabla \cdot \psi \]

Thermodynamic inequality

\[ \frac{d}{dt} \left( \rho \psi \right) = \frac{\partial (\rho \psi S)}{\partial x} + \rho \psi \frac{dS}{dt} - \frac{\partial \phi}{\partial t} - \frac{\partial \psi}{\partial x} \nabla \cdot \psi \]

Balance of free energy

\[ \frac{d}{dt} \left( \rho \psi \right) = \frac{\partial (\rho \psi S)}{\partial x} + \rho \psi \frac{dS}{dt} - \frac{\partial \phi}{\partial t} - \frac{\partial \psi}{\partial x} \nabla \cdot \psi \]

Potato - work equality

\[ P = D \nabla \psi \]

Work equality

\[ P = D \nabla \psi \]

Potato - heat equality

\[ Q = \nabla \cdot \psi \]

Heat equality

\[ Q = \nabla \cdot \psi \]

Isothermal closed system: A thermodynamic system which is allowed to exchange exclusively mechanical work, typically enclosed by a deformable, adiabatic, impermeable membrane. Characterized through its state of deformation \( \psi \).

Adiabatic closed system: A thermodynamic system which is allowed to exchange exclusively mechanical work, typically enclosed by a deformable, adiabatic, impermeable membrane. Characterized through its state of deformation \( \psi \).

Enclosed by a deformable, adiabatic, impermeable membrane. Characterized through its state of deformation \( \psi \).
closed system [klozd sýstém] thermodynamic system which is allowed to exchange mechanical work and heat, typically $P = P(\nabla \varphi, \ldots)$ and $Q = Q(\nabla \theta, \ldots)$, with its environment. enclosed by a deformable, diathermal, impermeable membrane. characterized through its state of deformation $\varphi$ and temperature $\theta$. 

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