05 - balance equations - closed systems

balance equations

balance equations [ˈbæl.əns ɪˈkweɪ.ʒəns] of mass, momentum, angular momentum and energy, supplemented with an entropy inequality constitute the set of conservation laws. the law of conservation of mass/matter states that the mass of a closed system of substances will remain constant, regardless of the processes acting inside the system. the principle of conservation of momentum states that the total momentum of a closed system of objects is constant.

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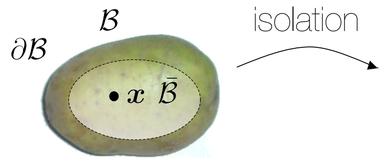
balance equations

balance equations [ˈbæl.əns ɪˈkweɪ.ʒəns] of mass, linear momentum, angular momentum and energy apply to all material bodies. each one gives rise to a field equation, holing on the configurations of a body in a sufficiently smooth motion and a jump condition on surfaces of discontinuity. like position, time and body, the concepts of mass, force, heating and internal energy which enter into the formulation of the balance equations are regarded as having primitive status in continuum mechanics.

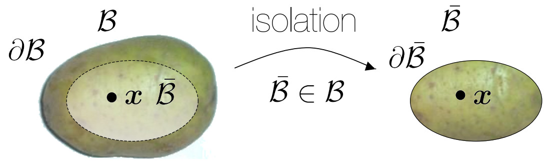
Chadwick „Continuum mechanics" [1976]

potato - balance equations



[1] isolation of subset $\bar{\mathcal{B}}$ from $\mathcal{B}$

potato - balance equations



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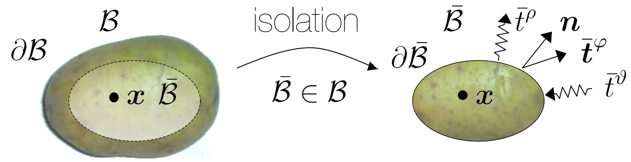
[2] characterization of influence of remaining body through phenomenological quantities - contact fluxes $\bar{t}^\rho$, $\bar{\boldsymbol{t}}^\varphi$ & $\bar{t}^\vartheta$

potato - balance equations



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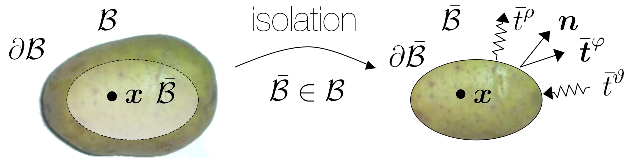
potato - balance equations



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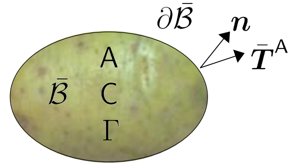
[4] postulation of balance of these quantities

potato - balance equations



general format

A . . . balance quantity

B . . . flux $\quad \mathbf{B} \cdot \boldsymbol{n} = \bar{\boldsymbol{T}}^{\mathsf{A}}$

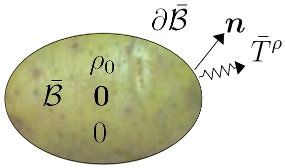
C . . . source

Γ . . . production

$$\mathrm{D}_t \mathsf{A} = \mathrm{Div}(\mathbf{B}) + \mathsf{C} + \Gamma$$

potato - balance equations

balance of mass



ρ_0 . . . density

$\mathbf{0}$ . . . no mass flux $\qquad \bar{T}^\rho = 0$

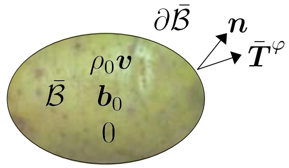
0 . . . no mass source

0 . . . no mass production

continuity equation $\quad \mathrm{D}_t \rho_0 = 0$

potato - balance equations

balance of momentum



$\rho_0\,\boldsymbol{v}$ . . . linear momentum density

$\boldsymbol{P}$ . . . momentum flux - stress $\qquad \boldsymbol{P}\cdot\boldsymbol{n} = \bar{\boldsymbol{T}}^\varphi$

$\boldsymbol{b}_0$ . . . momentum source - force

0 . . . no momentum production

equilibrium equation $\quad \mathrm{D}_t(\rho_0\boldsymbol{v}) = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0$

compare

NEWTON'S THREE LAWS OF GRADUATION

First published in 1679, Isaac Newton's "**Procrastinare Unnaturalis Principia Mathematica**" is often considered one of the most important single works in the history of science. Its Second Law is the most powerful of the three, allowing mathematical calculation of the duration of a doctoral degree.

SECOND LAW

"The age, a, of a doctoral process is directly proportional to the flexibility, f, given by the advisor and inversely proportional to the student's motivation, m"

Mathematically, this postulate translates to:

$$age_{PhD} = \frac{flexibility}{motivation}$$

$$a = F / m$$

$$\therefore F = m\,a$$

This Law is a quantitative description of the effect of the forces experienced by a grad student. A highly motivated student may still remain in grad school given enough flexibility. As motivation goes to zero, the duration of the PhD goes to infinity.

PHD.STANFORD.EDU

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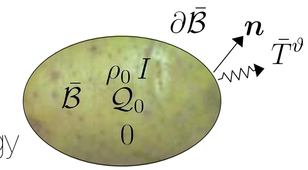
$\mathrm{D}_t(\rho_0\boldsymbol{v}) = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0 \quad$ mass point $\quad m\,\mathrm{D}_t\boldsymbol{v} = m\,\boldsymbol{a} = \boldsymbol{F}$

potato - balance equations

balance of internal energy



$\rho_0\,I$ . . . internal energy density

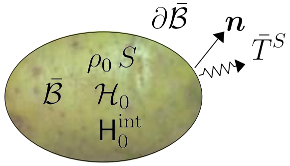
$\boldsymbol{Q}$ . . . heat flux $\qquad -\boldsymbol{Q}\cdot\boldsymbol{n} = \bar{T}^\vartheta$

$\mathcal{Q}_0$ . . . heat source

0 . . . no heat production

energy equation $\quad \mathrm{D}_t(\rho_0\,I) = \mathrm{Div}(-\boldsymbol{Q}) + \mathcal{Q}_0$

potato - balance equations

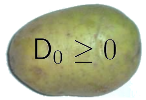


balance of entropy

$\rho_0\, S$ ... entropy density
$\boldsymbol{H}$ ... entropy flux $\quad -\boldsymbol{H}\cdot\boldsymbol{n} = \bar{T}^S$
$\mathcal{H}_0$ ... entropy source
$\mathsf{H}_0^{\mathrm{int}}$ ... entropy production $\quad \mathsf{H}_0^{\mathrm{int}} \geq 0$

entropy inequality $\quad \mathrm{D}_t(\rho_0 S) = \mathrm{Div}(-\boldsymbol{H}) + \mathcal{H}_0 + \mathsf{H}_0^{\mathrm{int}}$

potato - dissipation inequality



- dissipaltion inequality

$$\mathsf{D}_0 := \vartheta \mathsf{H}_0^{\mathrm{int}} = \vartheta\rho_0 \mathrm{D}_t S + \vartheta \mathrm{Div}(\boldsymbol{H}) - \vartheta\mathcal{H}_0 \geq 0$$

- identification $\quad \boldsymbol{H} = \frac{1}{\vartheta}\boldsymbol{Q} \qquad \mathcal{H}_0 = \frac{1}{\vartheta}\mathcal{Q}_0$
- with legendre-fenchel transform $\quad \psi = I - \vartheta\, S$

$$\mathsf{D}_0 = \boldsymbol{P} : \mathrm{D}_t\boldsymbol{F} - \rho_0 \mathrm{D}_t\psi - \rho_0\, S\, \mathrm{D}_t\vartheta + \boldsymbol{Q}\cdot\nabla_X \ln(\vartheta) \geq 0$$

- free energy $\quad \psi = \psi\,(\boldsymbol{F},\vartheta) \qquad \mathrm{D}_t\psi = \mathrm{D}_{\boldsymbol{F}}\psi : \mathrm{D}_t\boldsymbol{F} + \mathrm{D}_\vartheta\psi\; \mathrm{D}_t\vartheta$
- definition of stress and entropy

$$\boldsymbol{P} = \rho_0 \mathrm{D}_{\boldsymbol{F}}\psi \qquad S = -\rho_0 \mathrm{D}_\vartheta\psi$$

- thermodynamic restriction

$$\boldsymbol{Q}\cdot\nabla_X \ln(\vartheta) \geq 0$$

thermodynamic systems

isolated system [ˈaɪ.sə.leɪ.tɪd ˈsɪs.təm] thermodynamical system which is not allowed to have any interaction with its environment. enclosed by a rigid, adiabatic, impermeable membrane.

thermodynamic systems

adiabatic closed system[ˈə.dɪ.æ.bæ.tɪk kloʊzd ˈsɪs.təm] thermodynamic system which is allowed to exchange exclusively mechanical work, typically $\boldsymbol{P} = \boldsymbol{P}(\nabla\varphi,...)$, with its environment. enclosed by a deformable, adiabatic, impermeable membrane. characterized through its state of deformation φ .

closed system [kloʊzd ˈsɪs.təm] thermodynamic system which is allowed to exchange mechanical work and heat, typically $\boldsymbol{P} = \boldsymbol{P}(\nabla\varphi, ...)$ and $\boldsymbol{Q} = \boldsymbol{Q}(\nabla\theta, ...)$, with its environment. enclosed by a deformable, diathermal, impermeable membrane. characterized through its state of deformation φ and temperature θ.