mechanics of growth

me 337

ellen kuhl
mechanical engineering
stanford university

01 - introduction -
different forms of growth

… who i am …

since 01/07 - assistant professor me

… what i do …

kinematic equations for finite growth

$$\boldsymbol{F} = \boldsymbol{F}_{\mathrm{e}} \cdot \boldsymbol{F}_{\mathrm{g}}$$

balance equations for open systems

$$\mathrm{D}_t \rho_0 = \mathrm{Div}(\boldsymbol{R}) + \mathcal{R}_0$$

$$\rho_0 \, \mathrm{D}_t \boldsymbol{v} = \mathrm{Div}(\boldsymbol{P}) + \boldsymbol{b}_0$$

constitutive equations for living tissues

$$\boldsymbol{P} = \boldsymbol{P}(\rho_0, \boldsymbol{F}, \boldsymbol{F}_{\mathrm{g}})$$

fe analyses for biological structures

continuum- & computational biomechanics

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inhomo-geneous

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content

SPRING

me - goals

In contrast to traditional engineering structures living structures show the fascinating ability to grow and adapt their form, shape and microstructure to a given mechanical environment. This course addresses the phenomenon of growth on a theoretical and computational level and applies the resulting theories to classical biomechanical problems like bone remodeling, hip replacement, wound healing, atherosclerosis or in stent restenosis. This course will illustrate how classical engineering concepts like continuum mechanics, thermodynamics or finite element modeling have to be rephrased in the context of growth. Having attended this course, you will be able to develop your own problemspecific finite element based numerical solution techniques and interpret the results of biomechanical simulations with the ultimate goal of improving your understanding of the complex interplay between form and function.

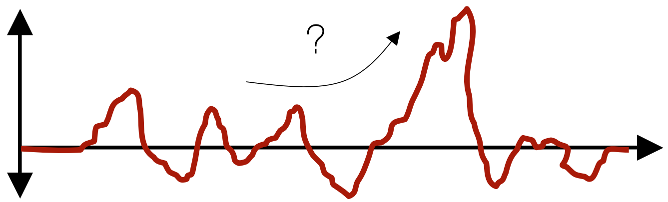
me 337 - syllabus

day	date		topic
tue	apr	03	introduction - different forms of growth
thu	apr	05	tensor calculus - tensor algebra
tue	apr	10	tensor calculus - tensor analysis
thu	apr	12	kinematic equations - large deformations and growth
tue	apr	17	balance equations - closed systems
thu	apr	19	balance equations - open systems
tue	apr	24	constitutive equations - density growth
thu	apr	26	finite element method - density growth - theory
tue	mai	01	finite element method - density growth - implementation
thu	mai	03	examples - density growth
tue	mai	08	class project - growth of tennis player arms
thu	mai	10	finite element method - density growth - alternative formulation
tue	mai	15	constitutive equations - volume growth
thu	mai	17	finite element method - volume growth - theory
tue	mai	22	finite element method - volume growth - implementation
thu	mai	24	examples - volume growth
tue	mai	29	examples - remodeling
thu	mai	31	class project - growth of tennis player arms
tue	jun	05	class project - discussion, presentation, evaluation

me 337 - suggested reading

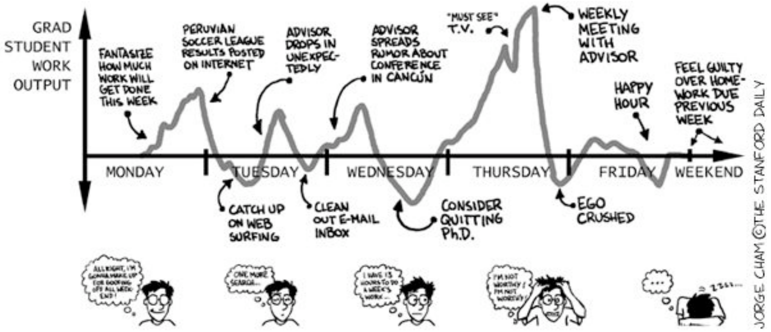
1. LA Taber: "Biomechanics of growth, remodeling and morphogenesis", *Appl. Mech. Rev.*, Vol 48, pp. 487-545, 1995
2. CR Jacobs, ME Levenston, GS Beaupré, JC Simo, DR Carter: "Numerical instabilities in bone remodeling simulations: The advantages of a node-based finite element approach", *Journal of Biomechanics*, Vol 28, pp. 449-459, 1995.
3. E Kuhl, A Menzel, P Steinmann: "Computational modeling of growth: A critical review, a classification of concepts and two new consistent approaches", *Computational Mechanics*, Vol 32, pp. 71-88, 2003.
4. EK Rodriguez, A Hoger, AD Mc~Culloch: "Stress--dependent finite growth in soft elastic tissues", *Journal of Biomechanics*, Vol 27, pp. 455-467,1994.
5. E Kuhl, R Maas, G Himpel, A Menzel: "Computational modeling of arterial wall growth: Attempts towards patient-specific simulations based on computer tomography", *Biomechanics and Modeling in Mechanobiology*, available online DOI 10.1007/s10237-006-0062-x, 2006.

what's growing?



classical engineering materials are not!

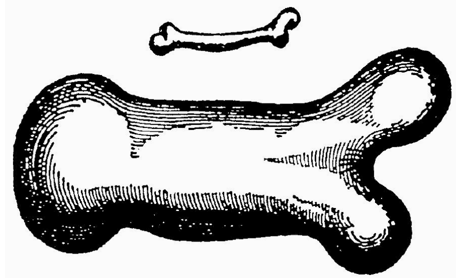
what's growing?



grad student work output ;-)

J. Cham "Piled higher and deeper", [1999]

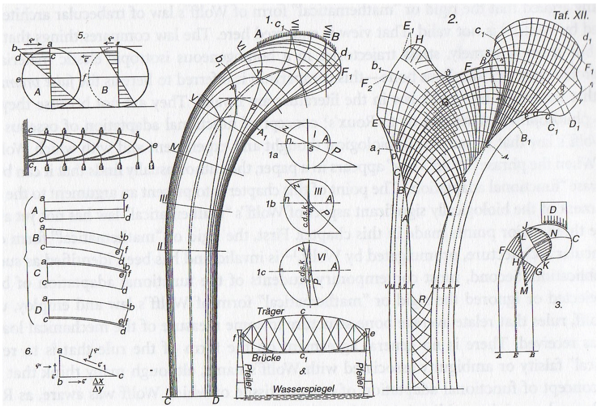
history - 17th century



„...dal che e manifesto, che chi volesse mantener in un vastissimo gigante le proporzioni, che hanno le membra in un huomo ordinario, bisognerebbe o trouar materia molto piu dura, e resistente per formarne l'ossa o vero ammettere, che la robustezza sua fusse a proporzione assai piu fiacca, che negli huomini de statura mediocre; altrimente crescendogli a smisurata altezza si vedrebbono dal proprio peso opprimere, e cadere..."

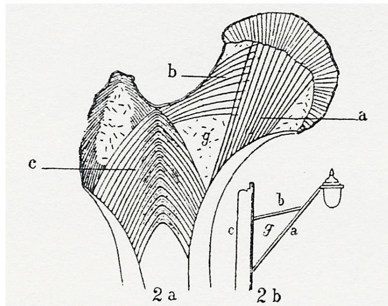
Galileo,"Discorsi e dimostrazioni matematiche", [1638]

history - 19th century



Culmann & von Meyer „Graphic statics" [1867]

history - 19th century



„...es ist demnach unter dem gesetze der transformation der knochen dasjenige gesetz zu verstehen, nach welchem im gefolge primaerer abaenderungen der form und inanspruchnahme bestimmte umwandlungen der inneren architectur und umwandlungen der aeusseren form sich vollziehen..."

Wolff „Gesetz der Transformation der Knochen" [1892]

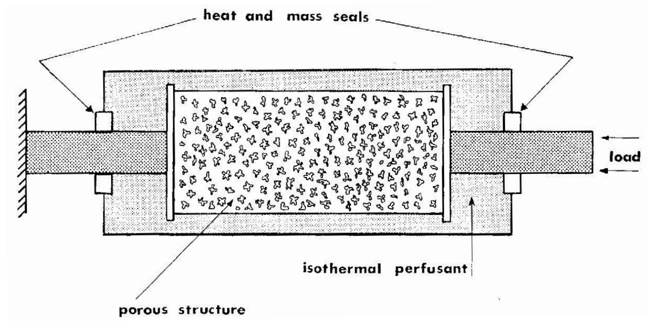
history - 19th century

carson pirie scott store
Sullivan[1904]

„…whether it be the sweeping eagle in his flight or the open apple-blossom, the toiling work-horse, the blithe swan, the branching oak, the winding stream at its base, the drifting clouds, over all the coursing sun, form ever follows function, and this is the law…"

Sullivan „Form follows function" [1896]

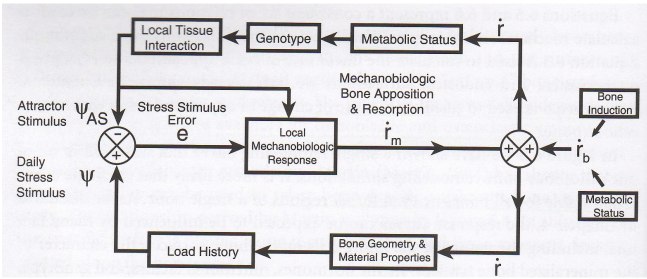
history - 20th century



„…the system consisting of only the porous structure without its entrained perfusant is open with respect to momentum transfer as well as mass, energy, and entropy transfer. we shall write balance and constitutive equations for only the bone…"

Cowin & Hegedus „Theory of adaptive elasticity" [1976]

history - 20th century



„…the relationship between physical forces and the morphology of living things has piqued the curiosity of every artist, scientist, or philosopher who has contemplated a tree or drawn the human figure. its importance was a concern of galileo and later thompson whose writings remind us that physical causation plays an inescapable role in the development of biological form…"

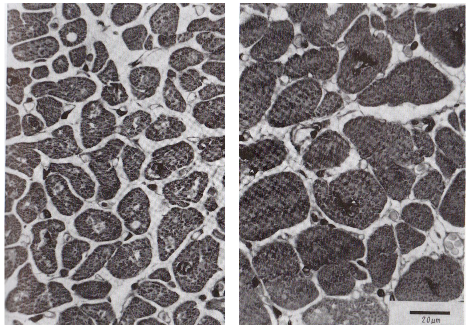
Beaupré, Carter & Orr „Theory of bone modeling & remodeling" [1990]

history - 20th century

„hypertrophy of the heart: comparison of cross sections of a normal heart (bottom), a heart chronically overloaded by an unusually large blood volume (left) and a heart chronically overloaded by an unusually large diastoic and systolic left ventricular pressure (right)"

Fung „Biomechanics - Motion, flow, stress, and growth" [1990]

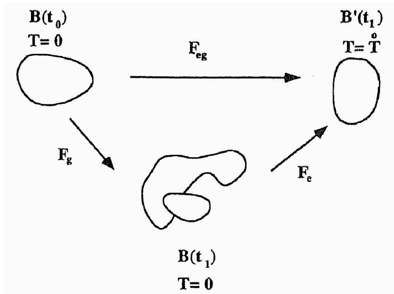
history - 20th century



„hypertrophy of the heart: histology of a normal heart (left) and pressure overloaded heart (right) photographed at the same magnification - muscles in the hypertropic heart (right) are much bigger in diameter than those of the normal heart (left)."

Fung „Biomechanics - Motion, flow, stress, and growth" [1990]

history - 21th century



Rodriguez, Holger & McCulloch [1994]

„…the process of growth can be seen as an evolution of material point neighbourhoods in a fixed reference configuration. the growth process will cause the development of material inhomogeneities responsible for residual stresses in the body…"

Epstein & Maugin „Theory of volumetric growth" [2000]

growth, remodeling and morphogenesis

growth [groʊθ] which is defined as added mass, can occur through cell division (hyperplasia), cell enlargement (hypertrophy), secretion of extracellular matrix, or accretion @ external or internal surfaces. negative growth (atrophy) can occur through cell death, cell shrinkage, or resorption. in most cases, hyperplasia and hypertrophy are mutually exclusive processes. depending on the age of the organism and the type of tissue, one of these two growth processes dominates.

Taber „Biomechanics of growth, remodeling and morphogenesis" [1995]

growth, remodeling and morphogenesis

remodeling [ri'mad. l.ɪŋ] involves changes in material properties. These changes, which often are adaptive, may be brought about by alterations in modulus, internal structure, strength, or density. for example, bones, and heart muscle may change their internal structures through reorientation of trabeculae and muscle fibers, respectively.

Taber „Biomechanics of growth, remodeling and morphogenesis" [1995]

growth, remodeling and morphogenesis

morphogenesis [mɔːr.foˈdʒen.ə.sɪs] is the generation of animal form. usually, the term refers to embryonic development, but wound healing and organ regeneration are also morphogenetic events. morphogenesis contains a complex series of stages, each of which depends on the previous stage. during these stages, genetic and environmental factors guide the spatial-temporal motions and differentiation (specification) of cells. a flaw in any one stage may lead to structural defects.

Taber „Biomechanics of growth, remodeling and morphogenesis" [1995]

mechanics of growth

write a wikipedia article about the **mechanics of growth** [məˈkæn.ɪks ɑːv groʊθ] about one page long. you may use taber's 1995 paper but any other sources are welcome (please cite). you can work in groups of two. at the end of this course, you will revise your article to see how your knowledge about growth has increased. finally, the class will decide which one to post on **http://www.wikipedia.org/**

Taber „Biomechanics of growth, remodeling and morphogenesis" [1995]