Introduction

Different forms of growth

... who i am ...

... what i do ...

kinematic equations for finite growth
\[ F = F_0 \cdot F_y \]

balance equations for open systems
\[ D_t(p_0) = \text{Div}(R) + R_0 \]
\[ p_0 \cdot D_t\mathbf{v} = \text{Div}(P) + \mathbf{b}_0 \]

constitutive equations for living tissues
\[ P = P(p_0, F, F_y) \]

fe analyses for biological structures

continuum- & computational biomechanics
... why i do what i do ...  
kinematic equations for finite growth  
\[ F = F_e \cdot F_g \]
balance equations for open systems  
\[ D_t \rho_0 = \text{Div}(R) + R_0 \]
\[ \rho_0 D_t \nu = \text{Div}(P) + b_0 \]
constitutive equations for living tissues  
\[ P = P(\rho_0, F, F_g) \]
fe analyses for biological structures

... because biological structures are ...  

... what i do ...  5

... why i do what i do ...  
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... what i do ...  6

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... what i do ...  7

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constitutive equations for living tissues  
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fe analyses for biological structures

... because biological structures are ...  

... what i do ...  8
... why i do what i do...

kinematic equations for finite growth
\[ F = F_e \cdot F_g \]
balance equations for open systems
\[ D_t \rho + \nabla \cdot \mathbf{R} = 0 \]
\[ \rho_0 \nabla \cdot \mathbf{V} = \nabla \cdot (\mathbf{P} + \mathbf{b}_0) \]
constitutive equations for living tissues
\[ \mathbf{P} = P(\rho_0, \mathbf{F}) \]
fe analyses for biological structures

... because biological structures are...

highly deformable
living
nonlinear
inelastic
inhomogeneous

... what i do...

content

1... • introduction
2... • kinematic equations
3... • balance equations
4... • constitutive equations
5... • finite element method
6... • cool numerical examples

me337 - mechanics of growth
me - goals

In contrast to traditional engineering structures, living structures show the fascinating ability to grow and adapt their form, shape and microstructure to a given mechanical environment. This course addresses the phenomenon of growth on a theoretical and computational level and applies the resulting theories to classical biomechanical problems like bone remodeling, hip replacement, wound healing, atherosclerosis or in stent restenosis. This course will illustrate how classical engineering concepts like continuum mechanics, thermodynamics or finite element modeling have to be rephrased in the context of growth. Having attended this course, you will be able to develop your own problem-specific finite element based numerical solution techniques and interpret the results of biomechanical simulations with the ultimate goal of improving your understanding of the complex interplay between form and function.

me 337 - suggested reading


me 337 - syllabus

day date topic
(tue apr 03) introduction - different forms of growth
(thu apr 05) tensor calculus - tensor algebra
(tue apr 10) tensor calculus - tensor analysis
(thu apr 12) kinematic equations - large deformations and growth
(tue apr 17) balance equations - closed systems
(thu apr 19) balance equations - open systems
(tue apr 24) constitutive equations - density growth
(thu mai 01) finite element method - density growth - theory
(thu mai 03) examples - density growth
(tue mai 08) class project - growth of tennis player arms
(thu mai 10) finite element method - density growth - alternative formulation
(thu mai 15) constitutive equations - volume growth
(thu mai 17) finite element method - volume growth - theory
(tue mai 22) finite element method - volume growth - implementation
(thu mai 24) examples - volume growth
(tue mai 29) examples - remodeling
(thu mai 31) class project - growth of tennis player arms
(tue jun 05) class project - discussion, presentation, evaluation

what's growing?
classical engineering materials are not!
what's growing?

J. Cham "Piled higher and deeper", [1999]

grad student work output ;-) 

introduction

history - 19th century

Culmann & von Meyer „Graphic statics“ [1867]

history - 17th century

Galileo, "Discorsi e dimostrazioni matematiche", [1638]

..dal che e manifesto, che chi volesse mantener in un vastissimo gigante le proprizioni, che hanno le membra in un huomo ordinario, bisognerrebbe o trouver materia molto piu dura, e resistente per formarne lossa o vero ammettere, che la robustezza sua fusse a proporzione assai piu fisica, che negli huomini de statua mediecre, altimemente cresceandogli a smisurata altezza si vedrebbono del proprio peso oppressere, e cadere..."

introduction

Wolff „Gesetz der Transformation der Knochen“ [1892]

...es ist demnach unter dem gesetze der transformation der knochen dasjenige gesetz zu verstehen, nach welchem im gefolge primaerer abänderungen der form und inanspruchnahme bestimmte umwandlungen der inneren architectur und umwandlungen der auesseren form sich vollziehen..."
…whether it be the sweeping eagle in his flight or the open apple-blossom, the toiling work-horse, the blithe swan, the branching oak, the winding stream at its base, the drifting clouds, over all the coursing sun, form ever follows function, and this is the law…”

[Sullivan, "Form follows function" (1896)]

...the relationship between physical forces and the morphology of living things has piqued the curiosity of every artist, scientist, or philosopher who has contemplated a tree or drawn the human figure. Its importance was a concern of galileo and later thompson whose writings remind us that physical causation plays an inescapable role in the development of biological form…"

[Beaupré, Carter & Orr, "Theory of bone modeling & remodeling" (1990)]

...the system consisting of only the porous structure without its entrained perfusant is open with respect to momentum transfer as well as mass, energy, and entropy transfer. we shall write balance and constitutive equations for only the bone…"

[Cowin & Hegedus, "Theory of adaptive elasticity" (1976)]

"hypertrophy of the heart: comparison of cross sections of a normal heart (bottom), a heart chronically overloaded by an unusually large blood volume (left) and a heart chronically overloaded by an unusually large diastoic and systolic left ventricular pressure (right)"

[Fung, "Biomechanics - Motion, flow, stress, and growth" (1990)]
introduction

Fung “Biomechanics - Motion, flow, stress, and growth” [1990]

history - 20th century

hypertrophy of the heart: histology of a normal heart (left) and pressure overloaded heart (right) photographed at the same magnification - muscles in the hypertropic heart (right) are much bigger in diameter than those of the normal heart (left).”

Fung “Biomechanics - Motion, flow, stress, and growth” [1990]

introduction

hispthy - 21th century

Rodriguez, Holger & McCulloch [1994]

...the process of growth can be seen as an evolution of material point neighbourhoods in a fixed reference configuration. the growth process will cause the development of material inhomogeneities responsible for residual stresses in the body...”

Epstein & Maugin “Theory of volumetric growth” [2000]

introduction

growth, remodeling and morphogenesis

growth [gроо] which is defined as added mass, can occur through cell division (hyperplasia), cell enlargement (hypertrophy), secretion of extracellular matrix, or accretion @ external or internal surfaces. negative growth (atrophy) can occur through cell death, cell shrinkage, or resorption. in most cases, hyperplasia and hypertrophy are mutually exclusive processes. depending on the age of the organism and the type of tissue, one of these two growth processes dominates.

Taber “Biomechanics of growth, remodeling and morphogenesis” [1995]

remodeling [rɪˈmɔːd.lɪŋ] involves changes in material properties. These changes, which often are adaptive, may be brought about by alterations in modulus, internal structure, strength, or density. for example, bones, and heart muscle may change their internal structures through reorientation of trabeculae and muscle fibers, respectively.

Taber “Biomechanics of growth, remodeling and morphogenesis” [1995]
**introduction**

morphogenesis is the generation of animal form. usually, the term refers to embryonic development, but wound healing and organ regeneration are also morphogenetic events. morphogenesis contains a complex series of stages, each of which depends on the previous stage. during these stages, genetic and environmental factors guide the spatial-temporal motions and differentiation (specification) of cells. a flaw in any one stage may lead to structural defects.

Taber, "Biomechanics of growth, remodeling and morphogenesis" [1995]

**homework #1**

write a wikipedia article about the mechanics of growth about one page long. you may use taber's 1995 paper but any other sources are welcome (please cite). you can work in groups of two. at the end of this course, you will revise your article to see how your knowledge about growth has increased. finally, the class will decide which one to post on http://www.wikipedia.org/