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TECHNICAL NOTE

NUMERICAL INSTABILITIES IN BONE REMODELING SIMULATIONS: THE ADVANTAGES OF A NODE-BASED FINITE ELEMENT APPROACH

Christopher R. Jacobs,*†‡ Marc E. Levenston,*† Gary S. Beaupré,*†
Juan C. Simo† and Dennis R. Carter*†

*Rehabilitation Research and Development Center, Palo Alto V. A. Medical Center; †Department of Mechanical Engineering, Stanford University, California; and ‡Musculoskeletal Research Laboratory, Dept. of Orthopaedics, Pennsylvania State University, Hershey, Pennsylvania, U.S.A.

Abstract—Long bone structure occurs in two distinct forms. The bone mass near the joint is primarily found in a distributed, porous trabecular structure, while in the diaphyses a tubular cortical structure is formed. It seems likely that these two observed morphologies come about, at least in part, as a mechanical adaptation to the different mechanical demands in the two regions. Mathematical formulations of this dependency have been proposed, thus facilitating numerical simulations of bone adaptation. Recently two types of discontinuities have been observed in these simulations. The first type (near-field) appears in areas near distributed load application and is characterized by a 'checkerboard' pattern of density wherein adjacent remodeled elements alternate between low and high density. The second type of discontinuity (far-field) appears remote from the load application and is characterized by strut or column-like regions of elements which become fully compact bone while adjacent regions are fully resorbed. In fact, the far-field discontinuity is an accurate representation of bone physiology and morphology since it is consistent with the appearance of cortical bone in the diaphysis. On the other hand, the near-field discontinuity, appears in a region where continuous distributions of intermediate apparent densities (trabecular bone) are expected. This finding may cause some to question whether a single continuum formulation of bone remodeling can predict both discontinuous far-field behavior and continuous near-field behavior. We describe a node-based implementation of current continuum bone remodeling theories which eliminates the spurious near-field discontinuities and preserves the anatomically correct far-field discontinuities, thus indicating that a single biological process may be at work in forming and maintaining both far-field and near-field morphologies.

INTRODUCTION

The structure of long bones of the appendicular skeleton in the diaphysis is distinctly different from that seen adjacent to the bone ends. Bone morphology makes a dramatic transition from a distributed trabecular structure near the ends to a hollow cortical structure in the shaft. It has been proposed that this is a functional adaptation to the differing structural demands in the two regions (Curry, 1984). In particular, it is suggested that the purely cortical structures found in the diaphysis are well suited to the simple resultant loads in that region, while the more complex distributed loading applied near the joints are more efficiently supported by a continuous trabecular structure. It is well known that bone undergoes morphological changes in response to both increased and decreased mechanical demands by modifying external geometry as well as internal structure and many mathematical formulations have been proposed that relate these morphological changes to the local mechanical demands (Beaupré *et al.*, 1990b; Carter *et al.*, 1987; Cowin and Hegedus, 1976; Huiskes *et al.*, 1987; Kummer, 1972). This paper addresses stress-related internal bone remodeling only.

The advent of modern computer capabilities and numerical stress analysis techniques have allowed researchers to relate bone continuum mechanics to the observed bone structure (Beaupré *et al.*, 1990b; Carter, 1987; Carter *et al.*, 1987; Cowin and Hegedus, 1976; Hart and Davy, 1989; Huiskes *et al.*, 1987; Kummer, 1972). Generally, stress-related internal bone remodeling theories take the form of an expression for the local time rate-of-change of bone density as a nonlinear function of mechanical quantities such as stress, strain, or strain energy density. The numerical implementation of these relationships with the finite element method allows simulation of the continuum level morphology and morphological changes a bone will undergo due to loading.

Carter *et al.* (1989) observed that when their simulation of femoral bone remodeling was allowed to progress past the first few iterations, 'The method employed appears to be converging toward a condition in which most of the elements will either be saturated . . . or be completely resorbed.' In particular, we and others (Harrigan and Hamilton, 1992; Weinans *et al.*, 1992a) have observed that previous bone remodeling implementations tend towards discontinuous density patterns. In regions (e.g. the diaphysis of long bones) discontinuous cortical structures are predicted that match well with the observed morphology. However, close to the applied loads, alternating patterns of high and low density elements were predicted that resemble a checkerboard.

We propose that a single bone remodeling formulation can describe the development and maintenance of both types of morphology (continuous and discontinuous). In particular, such a formulation would predict discontinuous structures when applied to the simple load resultants experienced in the diaphysis (what we will term 'far-field' behavior), and

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Address correspondence to: Chris Jacobs, Dept. of Orthopaedics, P.O. Box 850; 500 University Drive, Hershey, PA 17033, U.S.A.

yet could be driven to a continuous structure when applied to the more complex loading found near the joints (what we will term 'near-field' behavior).

We propose that in the near-field case, the current continuum equations indeed lead to continuous densities, but that a numerical difficulty associated with the finite element discretization introduces the anomalous checkerboard density pattern. In other works, the current continuum formulation is able to model both the far-field discontinuous result as well as the near-field continuous result, but in the near-field case, the numerical procedure admits a spurious 'checkerboard' mode that is not inherently present in the continuum formulation. Therefore, a single biologic adaptation mechanism may be responsible for both cortical and trabecular bone structures as well as the transition between these structures as it is seen in long bones.

METHODS

Trabecular bone is a porous structure and, given current computing resources, stress analysis of whole bones can only be carried out on an 'average' or continuum level (Harrigan *et al.*, 1988). Much progress has been made in direct modeling of cancellous microstructure (Fyhrie and Hamid, 1993; Fyhrie *et al.*, 1992), but to date these techniques have not advanced sufficiently to model individual trabeculae in a whole bone simulation. Thus, both the mechanical variables and the biological response must be spatially averaged to yield appropriate apparent continuum variables (Aoubiza *et al.*, 1992; Hollister *et al.*, 1992) that can be used in a mathematical description of mechanically induced bone remodeling. In this paper we will confine ourselves to formulations where the homogenized continuum stress is related to the average trabecular stress through a power of the apparent density. It has been shown that the particular form of this scaling has a dramatic influence on the predicted remodeling behavior (Harrigan and Hamilton, 1992).

In this paper we explore the behavior of the remodeling formulation advanced by Beaupré *et al.* (1990b) as a particular example of strain-energy-based remodeling theories (see Appendix). According to these authors bone is assumed to respond to changes in a daily tissue level stress stimulus, Ψ_b, such that:

$$\frac{d\rho}{dt} = A(\Psi_b - \Psi_{bAS})S_v(\rho), \quad 0 \leqslant \rho \leqslant \rho_c,$$

$$\frac{d\rho}{dt} = 0, \quad \text{otherwise}, \tag{1}$$

where ρ is the volumetric mass density (apparent density), ρ_c is the maximum observed density of cortical bone, Ψ_{bAS} is the constant level of daily tissue level stress stimulus that will result in no net change in bone density (termed the *attractor state stimulus*), A is a rate constant or function and may or may not include a dead-zone effect, and S_v is the surface area per unit volume at the current density (based upon an empirically determined fifth-order polynomial in porosity (Martin, 1984)). In this model the daily tissue level stress stimulus is defined as

$$\Psi_b = \left(\sum_{day} n_i \bar{\sigma}_{b_i}^m\right)^{1/m}, \tag{2}$$

where n_i is the number of daily cycles of load type i, $\bar{\sigma}_{b_i}$ is a tissue level effective stress scalar, and the stress exponent, m, is an empirical constant. To be useful for bone remodeling simulation, Ψ_b in equation (1) must be scaled from continuum level quantities (Carter *et al.*, 1987). By assuming that this scaling takes the same form as ultimate stress (Carter and Hayes, 1977) we have

$$\frac{d\rho}{dt} = A\left(\Psi\left(\frac{\rho_c}{\rho}\right)^2 - \Psi_{bAS}\right)S_v(\rho), \quad 0 \leqslant \rho \leqslant \rho_c,$$

$$\frac{d\rho}{dt} = 0 \quad \text{otherwise}, \tag{3}$$

where

$$\Psi = \left(\sum_{day} n_i \bar{\sigma}_i^m\right)^{1/m} \tag{4}$$

is the continuum level effective stress stimulus, $\bar{\sigma}_i = \sqrt{2EU_i}$, $\bar{\sigma}_i$ the continuum level effective stress, E is the average Young's modulus, and U is the continuum level strain energy density.

Finally, density changes must be related to changes in material properties. Generally, an empirically derived relationship between density and material properties is employed. A common form is that of linear isotropic elasticity such that

$$E = D\rho^n, \tag{5}$$

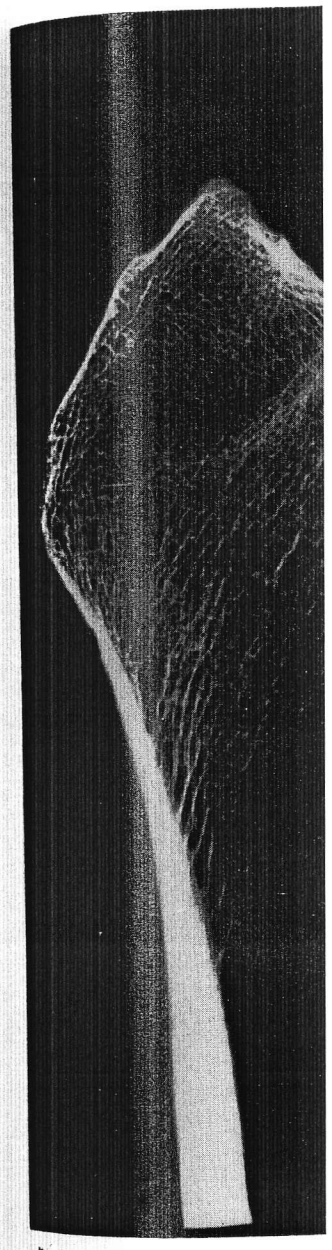
(Carter and Hayes, 1977), where E is the Young's modulus and D and n are experimentally derived parameters (which may vary with density) with n usually ranging from two to three. This relation has been refined to yield better agreement with current experimental evidence as it becomes available. However, the results described here are insensitive to the exact form of equation (5). As a result of equation (5) the matrix of elastic constants, $\mathbf{C}$, in the linear elastic constitutive equation

$$\sigma = \mathbf{C}(\rho) : \varepsilon \tag{6}$$

depends on density which implicitly depends on the stress and strain history. Equations (3), (4), (5) and (6), together with the equilibrium equation, make up a rate-dependent nonlinear continuum model for bone remodeling. With prescribed tractions or displacements on the boundaries and initial conditions for the density, the problem is in principle properly defined and may be solved numerically.

Traditionally a very straightforward method is applied to numerically solve the remodeling equations with the finite element method. The procedure is based on the approximation that each element has a uniform density. The element densities become state variables that determine the current configuration of the model (Fig. 2(a)). From one model configuration, the solution is advanced in time by solving the elasto-static problem using the known density distribution with the finite element method. The required mechanical variable (e.g. strain-energy density) is then computed at each element centroid by averaging the results from the integration points. The model configuration is advanced using the density rate-of-change from the remodeling theory and a time stepping algorithm (typically forward Euler). This strategy falls within the conventional finite element treatment of inelastic continuum models (Simo and Hughes, 1992).

This approach is 'element-based' in the sense that bone density is computed at element centroids and then assumed to be constant throughout the element. The traditional approach from computational inelasticity would be to use the densities at the integration points as state variables (Fig. 2(b)). Also, one might increase the order of interpolation of the displacements (Fig. 2(c)), with the corresponding increase in the number of integration points. Our approach was to use a 'node-based' implementation. Accordingly we applied equation (3) and kept track of the resulting density predictions at nodes rather than element centroids (Fig. 2(d)). Material properties, and therefore densities, are required at the integration points to perform conventional gauss-quadrature and assemble the finite element stiffness matrix. By interpolating nodal density values over the element interiors using the element shape functions, the integration point densities were computed. This is entirely analogous to the standard interpolation of nodal displacements or



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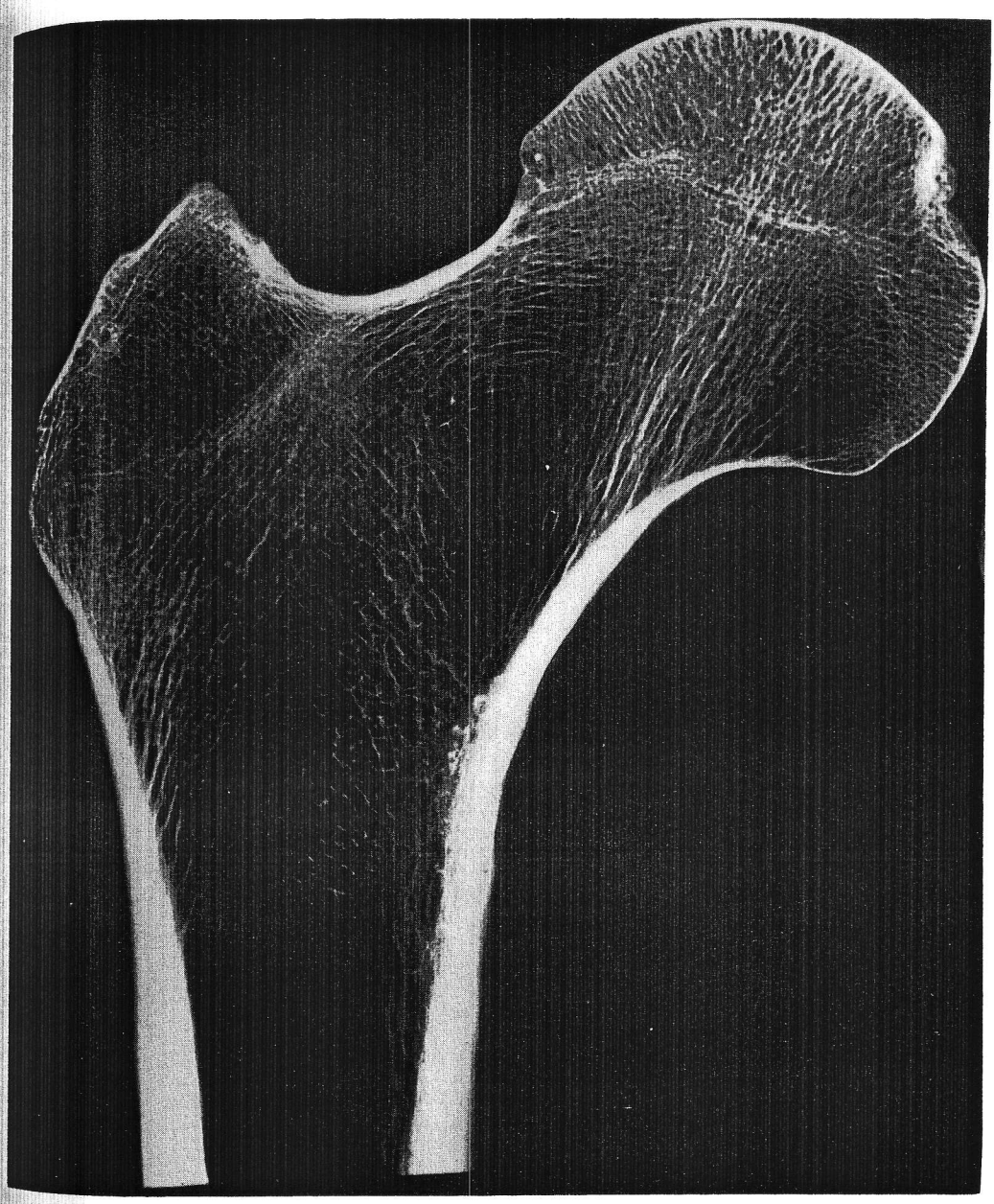

Fig. 1. This contact radiograph of a coronal slice through the proximal femur clearly shows the dramatic transition from primarily cortical bone distally to primarily trabecular bone proximally. The hypothesis investigated in this work is that a single mathematical formulation of adaptive bone remodeling can produce both types of bony structure. Some doubt has recently been raised due to the observation that current mathematical formulations seem to have a predisposition towards discontinuous structures, and do not seem to be capable of predicting the continuous apparent densities observed in the femoral head and neck regions.

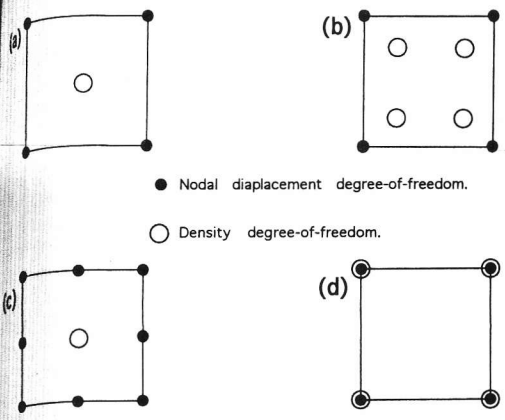

Fig. 2. The displacement and density interpolation schemes for the various methods are shown schematically for a quadrilateral parent element: (a) the common element-based approach; (b) traditional inelasticity approach; (c) higher-order element-based approach; (d) proposed node-based approach. The fundamental distinction between the methods is the location of displacement and density degrees of freedom, which is denoted by the closed and open circles respectively. Each implementation will be evaluated, in turn, for its ability to predict continuous near-field results, while maintaining cortical structures in the far-field case.

temperatures. Density continuity is ensured across element boundaries in the same way that it is for displacements. The only difficulty with implementing this approach is that the stress and strain quantities needed to apply equation (3) are not unique at the nodes. That is, most elasto-static elements employ piecewise continuous (C^0) interpolation functions. This leads to discontinuities in stress and strain at element boundaries. To overcome this, one of many smoothing techniques must be employed (Hughes, 1987, Section 4.4.1; Lee *et al.*, 1979; Lee and Gresho, 1978). In our case, the strain-energy-density was extrapolated from the integration points to the nodes by inverting the bilinear nodal interpolations within each element. The contributions at each node were then averaged (this is accomplished automatically by ABAQUS when nodal averaged results are requested). This technique can be shown to be equivalent to a least-squares (L_2) projection to the nodes similar to the one used by many finite element post-processors when plotting contours of elemental results.

Incremental calls to the commercial finite element code ABAQUS (v4.9, running on an HP/Apollo 9000 Model 730 workstation) were made in which the current material configuration was specified in the form of the current bone density at each node together with a table of values relating density to material properties (Young's modulus and Poisson's ratio). As the stiffness matrix was assembled, the density values were interpolated from the nodes to the integration points. The material table was then used to obtain

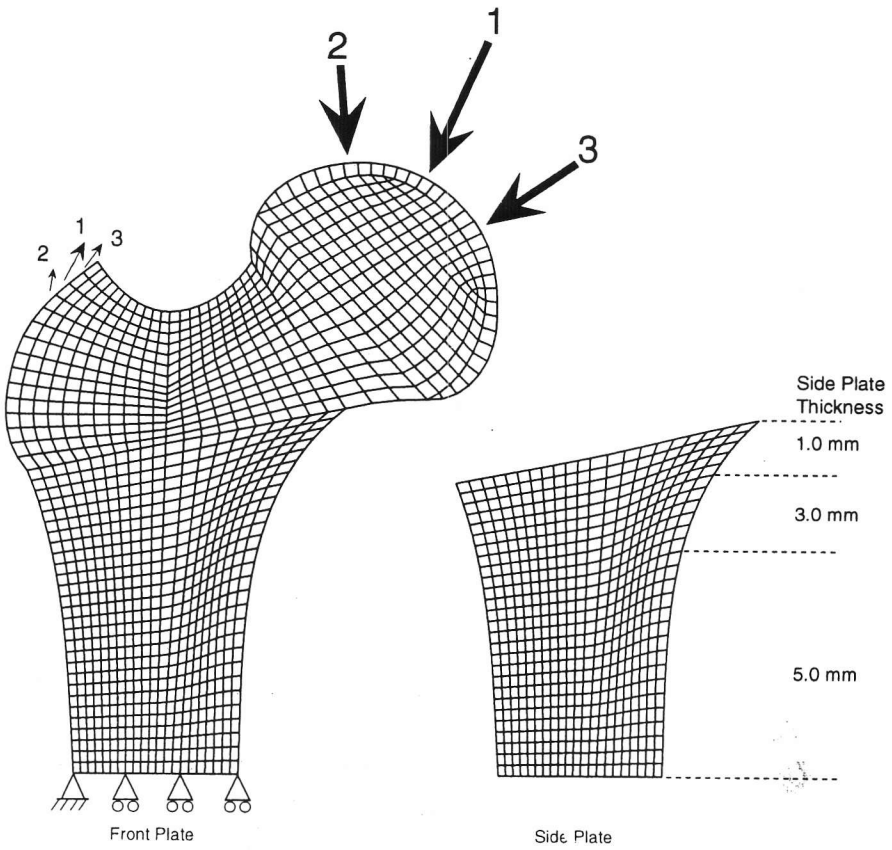

Fig. 3. The two-dimensional proximal femur model used to study the stability of bone remodeling algorithms employed the external geometry and loading of Beaupré *et al.* (1990a). A side-plate of cortical bone was coupled to the distal portion to account for the three-dimensional connectivity of the cortex. The side-plate and front-plate were connected via the three lateral and medial most nodes along each edge. The front-plate was assumed to be 40.0 mm thick. The side-plate was not allowed to remodel. Three cyclically applied load cases were selected to characterize a typical loading history. Six thousand cycles of load case one and two thousand cycles of load cases two and three were applied. The bottom edge was supported on rollers with the left-most node fully constrained to eliminate lateral rigid-body translations.

material properties. After a solution had been obtained, the results reported at nodes were used to advance the solution to the next time increment through numerical integration of equation (3) at the nodes. Forward Euler integration was used and the time step was selected to be well below the stability limit (Beaupré *et al.*, 1990a; Orr, 1990).

We considered four finite element implementations (Fig. 2) as follows: element-based with bilinear elements, integration-point-based with bilinear elements, element-based with bi-quadratic elements, and node-based with bilinear elements. Each of the four methods was applied to the example finite element problem in turn (Fig. 3). A side-plate of cortical bone of varying thickness was used to account for the three-dimensionality of the cortical structures in the proximal femur (Weinans *et al.*, 1992b). In each case, the simulation was started with a homogeneous distribution of an intermediate density (0.6 g cm^{-3}). A linear rate-relation was used in these simulations with the rate constant taken from Beaupré *et al.* (1990a). All of the simulations were carried out for a large enough period of time (500 ten-day increments) for density changes to approach zero (the largest change in density for the final time increment was less than 0.01 g cm^{-3} in all cases) in order for the spatial stability of each method to become fully evident. In each case, the resulting density predictions were plotted directly as predicted by the appropriate interpolation functions. That is, no smoothing from the post-processor has taken place. Finally, for reference purposes, the results from the traditional element-centroid method (Fig. 2(a)) were used to produce a traditional contour plot after the density data had been projected onto the nodes.

RESULTS

The phenomena which raised the initial biological question is clearly demonstrated by the results of the traditional centroidal method (Fig. 4), which also clearly shows the formation of cortices far from the applied load. In this region, the predicted morphology agrees well with observed morphology. However, near the applied load we see the formation of the alternating checkerboard pattern. Note that upon refinement of the mesh, the checkerboard remains. Where there was one element of a certain density in the coarser mesh, there are now four elements of alternating density in the refined mesh. This mesh dependency suggests that the discontinuities in this region result from the numerical procedure employed.

Furthermore, the second two numerical techniques investigated (Fig. 2(b) and (c)) failed to yield the biologically expected continuous result. In the case of densities computed at integration points (Fig. 5), the checkerboard pattern once again occurred. However, there were very large density gradients superimposed on top of the original pattern which results in predicted densities that violate the density limits imposed by the remodeling theory when they are extrapolated throughout the element interior, although each individual density is within the density bounds of zero and cortical bone. When eight-noded elements were used, the density distribution seems more appropriate (Fig. 6). Notice that the checkerboard pattern has been greatly suppressed, although some discontinuities remain in the femoral head and neck regions where they are not observed anatomically.

The results for the nodal method (Fig. 7) support the hypothesis that a single mathematical model may indeed describe both continuous and discontinuous structures simultaneously, and that current strain-energy-based continuum formulations like the one employed here are capable of just that. Specifically note that in regions where the interaction between the remodeling equations and the finite element discretization had produced the checkerboard patterns (Fig. 4) we find continuous distributions of intermediate densities. Furthermore, the formation of cortices far from the region of load application is maintained indicating that both

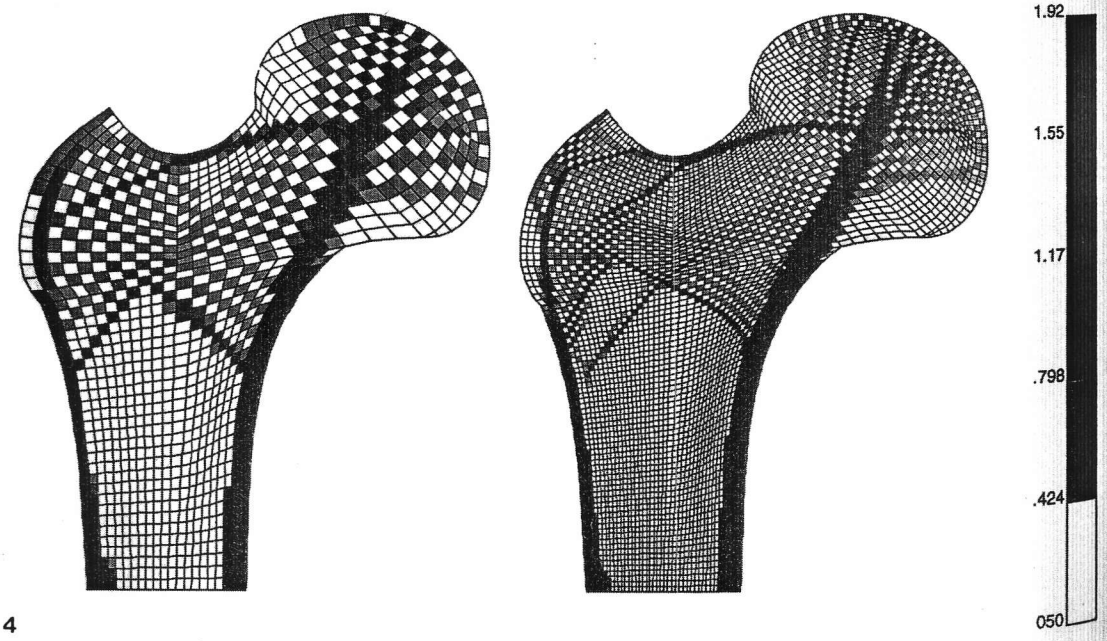

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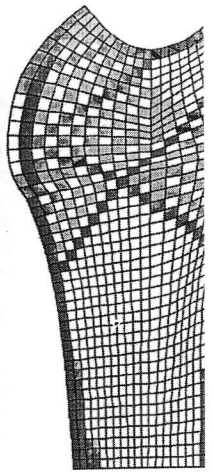
Fig. 4. The density distribution resulting from a bone remodeling simulation carried out using the traditional element-based algorithm (Fig. 2(a)). This type of behavior is clearly nonbiological in nature and motivates the question: are the current strain-energy-based continuum formulations incapable of predicting the expected continuous results near bone ends (and thus, biologically inappropriate), or is this difficulty technical in nature to be overcome with appropriate numerical implementation? Note that far from the region of applied load, strut-like structures develop the appropriately emulate the cortical structures seen in the diaphyses. On the other hand, in the proximal region of the model an anomalous checkerboard pattern of density develops. Upon refinement, note that the cortical structures remain intact, while the checkerboard pattern is clearly mesh dependent, at least at this level of refinement.



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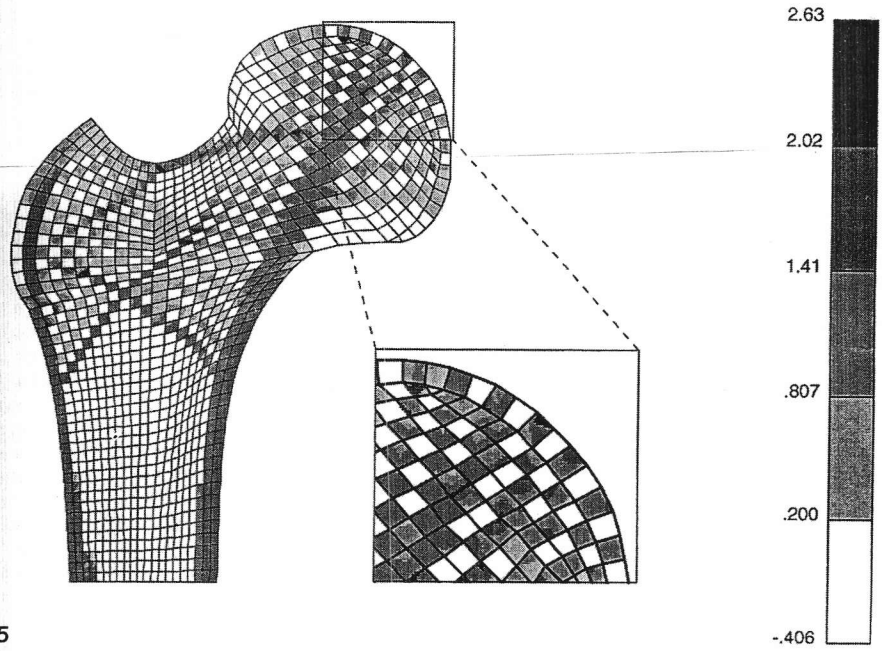


Fig. 5. If the remodeling equations are applied at integration points rather than element centroids (Fig. 2(b)) very large density gradients are predicted across element interiors. When these results are compared with Fig. 4 it is clear that the global density distribution has not been significantly altered. Instead, elements that had remodeled to a constant density value using the element-based procedure have large density gradients with roughly the same average density. Extrapolating the bone densities from the integration points throughout the element interiors, yield densities which violate the physical limitations on porosity.

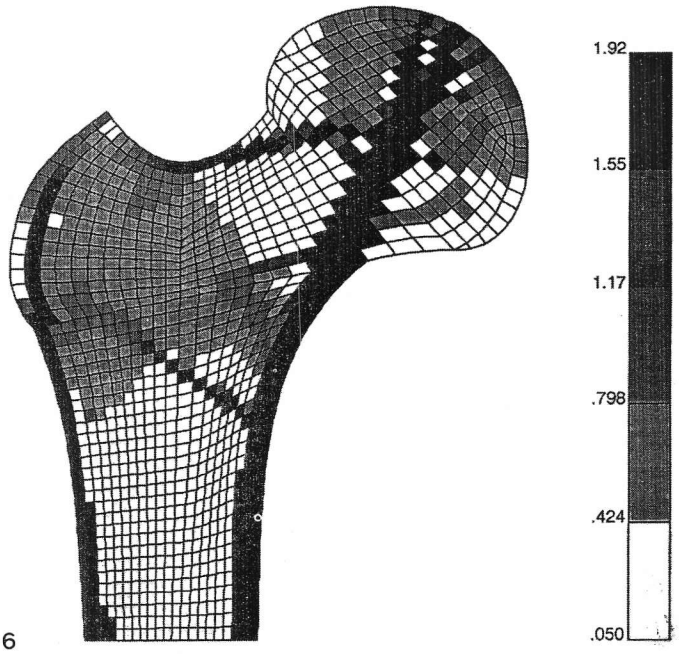


Fig. 6. Using a higher-order element-based formulation (see Fig. 2(c)) eliminates much of the checkerboarding. However, significant discontinuities in the femoral head and neck region remain. This approach requires a much greater investment of computational resources with no resulting refinement in the predicted density distribution.

stable and unstable behavior (i.e. continuous and discontinuous predicted density distributions) can occur regionally due to biological adaptation, depending on the character of the local loading conditions.

Finally, to facilitate a comparison of our results with those in the literature, a nodal projection and contour plot was prepared from the data predicted with the element-based method applied to the four-node mesh (Fig. 8).

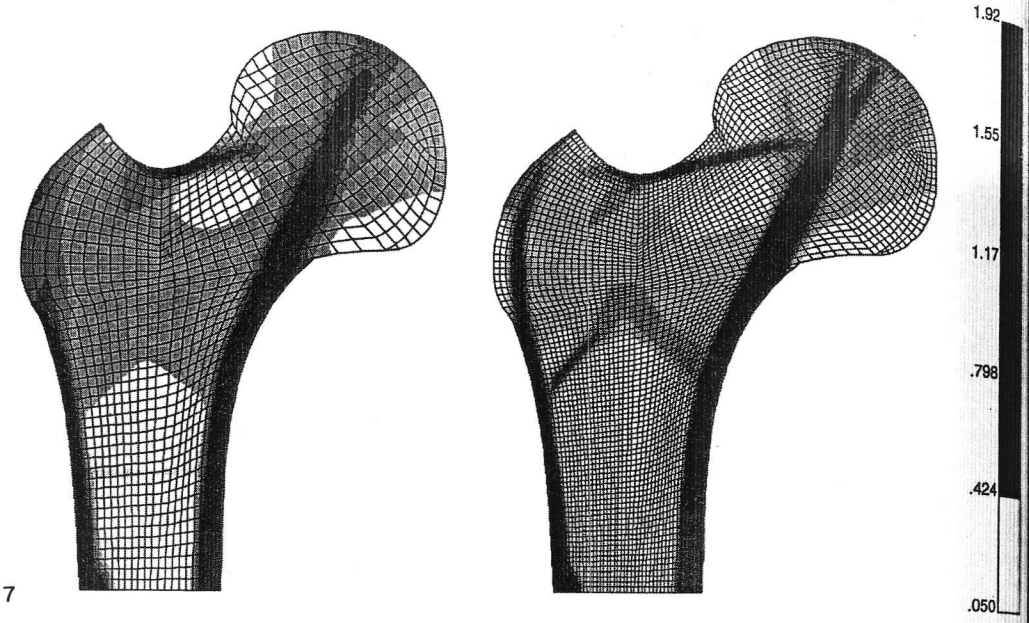


Fig. 7. The density distribution resulting from the node-based method. Although this is a contour plot, no smoothing fro the post-processor has occurred. Notice that the checkerboarding has been replaced by a continuous density distributio Strut formation far from the load application is still found and the cortical structures drop to a low density in t intramedullary space across one element. These results support the interpretation that the current continuum formulatio yield appropriately continuous solutions in near-field loading situations, and that previously observed discontinuities occ due to purely numerical considerations. This, in turn, supports the biologic hypothesis that the same adaptive bon remodeling mechanism that leads to discontinuous cortical structures can also produce continuous cancellous structur Refining the mesh reveals more details of the internal femoral structure (note the intriguing bifurcation near the joint surfac

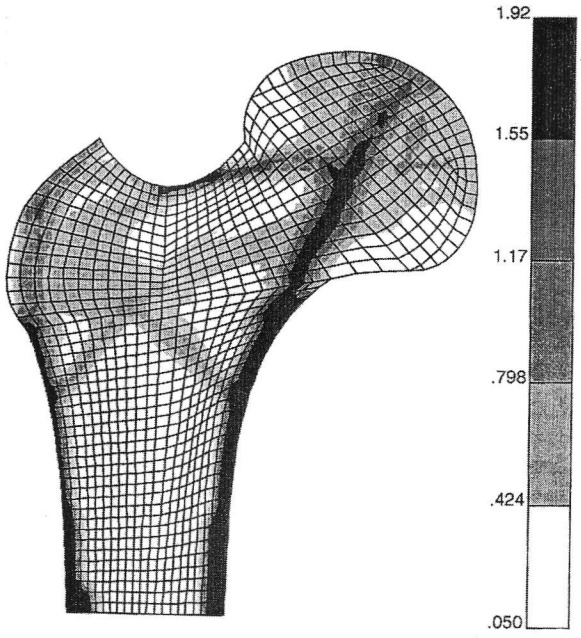


Fig. 8. For comparison, the element centroid densities from the final iteration of the element-based approach, using the coarse mesh (Fig. 4), were projected to the nodes and a contour plot was prepared. This was accomplished via the 'fringe plot' option in Patran. Note that the result is quite similar to Fig. 7, indicating that the bulk of the simulations currently in the literature do not need to be re-analyzed.

DISCUSSION

We have shown that a single mathematical description o continuum level adaptive bone remodeling can lead to di continuous far-field behavior and continuous near-field be havior, which supports the proposal that the same mechan biological adaptive mechanisms are involved in the forma tion and maintenance of both type of structures. To see how this is possible, notice that the stress stimulus term in equation (3) will in general depend on the global densit distribution because the *local* stress and strain (which dete mine the stress stimulus) are influenced by *remote* changes in density. Thus, the continuum equations may have continu ous solutions in one case and discontinuous solutions in another case depending on how the local stress and strain depend on the global density distribution. In fact, in all bu the simplest of cases, it is extremely difficult or impossible to determine the continuity of the continuum solution, since analytical solutions of the complete nonlinear rate-dependent problem are not available.

Weinans *et al.* (1992) studied the simplified far-field example of an axial resultant load. Harrigan and Hamilton (1992) investigated continuum solutions as well as numerical solutions (the similarities between the formulation used in these studies and the Beaupré *et al.* (1990b) formulation are explored in the Appendix). Both of these studies found the system to be spatially unstable, which agrees well with the far-field results found here. This configuration is an accurate model of the mechanical environment of the mid-shaft region of most long bones far from the applied load and is what we have termed the 'far-field' behavior. In this region we do not find any intermediate values of density, thus the continuum equations agree well with observations.

However, in regions where the local loading conditions are sufficiently complex, these conclusions may not apply

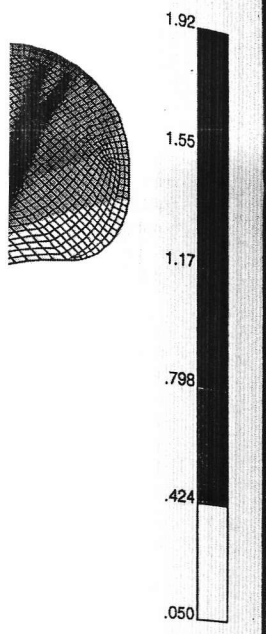
since the relation between local stress global density distribution may be quite the continuum solution to be a continuo distribution adjacent to applied distribu impossible to ignore end effects and is w near-field' behavior. In the region near t an alternating pattern of bone densities i ical results of traditional methods (Fig. havior has been termed a 'patchwork' (W or a 'checkerboard' pattern (Jacobs a Unlike the far-field discontinuities, this mesh-dependent and strikingly reminisc stabilities arising in marginally stable fin (Simo and Armero, 1992). In the present tinuities seem to arise due to an inter remodeling equations and the discrete n ical method used to simulate them.

Another approach is to apply the r and update density changes at integratio at element centroids. This implementati standard finite element implementatio nelasticity (Simo and Hughes, 1992). Ho to the bone remodeling equations, th density gradients across element interi cantly changing the global solution (Fig

Higher-order elements may be empl producing better numerical performanc ments, if the remodeling equations are ap tion points, large density gradients acro are found. If the remodeling equations element centroids of higher-order elem eliminate the near-field discontinuities a able density patterns. However, this a sub-optimal element (Hughes, 1987) number of displacement degrees-of-fre ciated with an increased level of den achieve the result presented here the mo displacement degrees of freedom compa mesh with four-node elements. This lead performance and wasted computer resou this may not seem to be a crucial iss solutions may be obtained with this me in mind that as finite element models bec refined and particularly when they ar dimensions, computational efficiency ca consideration.

Certain interesting parallels exist bet ing simulation and the finite element in with other nonliving nonadapting m these parallels helps to relate the approa the broad number of approaches assoc ment implementations in general. Severa method formulations for incompressib element formulations that exhibit ran produce spurious pressure modes (Hug 4.II; Strang and Fix, 1973). This difficu checkerboard pressure modes introduc implementation that are not present in A common solution is to use an averag 'filtering' (Hughes, 1987, Section 4.4.1; Lo Although somewhat *ad hoc* in nature, i pressure-smoothed elements are a goo the continuum behavior. Essentially w same approach to smooth the disco strain quantities in computing the nod ority needed in the node-based bone rem tion.

Bone remodeling simulation is also the simulation of materials that have behavior in the plastic region. These un present numerical difficulties that have reformulate the continuum equations. local formulation (where the material



tour plot, no smoothing from
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SSION

e mathematical description of
e remodeling can lead to dis-
and continuous near-field be-
posal that the same mechano-
ns are involved in the forma-
type of structures. To see how
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lepend on the global density
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conclusions may not apply

since the relation between local stress and strain and the global density distribution may be quite different. We expect the continuum solution to be a continuous apparent density distribution adjacent to applied distributed loads where it is impossible to ignore end effects and is what we have termed 'near-field' behavior. In the region near the load application an alternating pattern of bone densities is seen in the numerical results of traditional methods (Fig. 4). This type of behavior has been termed a 'patchwork' (Weinans *et al.*, 1992a) or a 'checkerboard' pattern (Jacobs and Beaupré, 1992). Unlike the far-field discontinuities, this behavior is clearly mesh-dependent and strikingly reminiscent of numerical instabilities arising in marginally stable finite element methods (Simo and Armero, 1992). In the present context the discontinuities seem to arise due to an interaction between the remodeling equations and the discrete nature of the numerical method used to simulate them.

Another approach is to apply the remodeling relations and update density changes at integration points rather than at element centroids. This implementation is analogous to a standard finite element implementation of computational inelasticity (Simo and Hughes, 1992). However, when applied to the bone remodeling equations, this introduces large density gradients across element interiors without significantly changing the global solution (Fig. 5).

Higher-order elements may be employed in the hope of producing better numerical performance. As with linear elements, if the remodeling equations are applied at the integration points, large density gradients across element interiors are found. If the remodeling equations are applied at the element centroids of higher-order elements it is possible to eliminate the near-field discontinuities and produce acceptable density patterns. However, this technique leads to a sub-optimal element (Hughes, 1987) since the increased number of displacement degrees-of-freedom are not associated with an increased level of density refinement. To achieve the result presented here the model contained 10,758 displacement degrees of freedom compared with 3670 for the mesh with four-node elements. This leads to poor numerical performance and wasted computer resources. On the surface, this may not seem to be a crucial issue since acceptable solutions may be obtained with this method. However, keep in mind that as finite element models become more and more refined and particularly when they are extended to three dimensions, computational efficiency can become a critical consideration.

Certain interesting parallels exist between bone remodeling simulation and the finite element implementations used with other nonliving nonadapting materials. Illustrating these parallels helps to relate the approach presented here to the broad number of approaches associated with finite element implementations in general. Several mixed and penalty method formulations for incompressible materials lead to element formulations that exhibit rank-deficiencies which produce spurious pressure modes (Hughes, 1987, Appendix 4.II; Strang and Fix, 1973). This difficulty exhibits itself as checkerboard pressure modes introduced by the numerical implementation that are not present in the continuum case. A common solution is to use an averaging procedure called 'filtering' (Hughes, 1987, Section 4.4.1; Lee and Gresho, 1978). Although somewhat *ad hoc* in nature, it can be shown that pressure-smoothed elements are a good approximation to the continuum behavior. Essentially we have adopted the same approach to smooth the discontinuous stress and strain quantities in computing the nodal strain energy density needed in the node-based bone remodeling implementation.

Bone remodeling simulation is also strikingly similar to the simulation of materials that have a strain softening behavior in the plastic region. These unstable materials also present numerical difficulties that have led investigators to reformulate the continuum equations. In particular, a nonlocal formulation (where the material behavior at a point depends on the stress and/or strain in some surrounding region) has been recently employed (Brekelmans, 1993). In the field of bone remodeling simulation, two reformulations have been suggested recently; an adjustment of the parameters used in the model (leading to a stable material formulation) (Harrigan and Hamilton, 1992), and a nonlocal formulation (Mullender *et al.*, 1993). Both approaches can lead to acceptable density distributions and are certainly worth consideration. However, reformulation of the continuum equations should primarily be undertaken in order to model biological processes more faithfully, rather than being motivated by convenient numerical behavior once they have been discretized.

The node-based implementation described here seems to overcome the numerical difficulties of the near-field behavior associated with the original finite element implementation of the continuum formulation, while maintaining the desired far-field discontinuous solutions in the sense that nodal densities predicted at adjacent nodes still change abruptly from the saturated cortical value to zero. Although the density interpolations used in this method are continuous for this transition, this does not imply that the method is not capable of representing discontinuous cortical structures. As with the displacement predictions near a material interface in a conventional elasto-static simulation, the predictions are only accurate at particular points, and the only way to get more information between these points is to refine the mesh. The element-based method may be more appealing in this regard since it will yield a density discontinuity at the element boundary, however both methods predict some sort transition between two points (a node or element centroid); the only way to get more information about the density transition is to refine the mesh.

It must be noted that several factors may mitigate the near-field numerical difficulties associated with an element-based implementation. Many investigators now incorporate a 'dead-zone' or 'lazy-zone' (Beaupré *et al.*, 1990a; Huiskes *et al.*, 1987) to better simulate the biologic response. This has the additional effect of slowing down or preventing the full development of the density discontinuous. Although it is important to include a lazy-zone in a more physiologic simulation, for our purposes it would have the effect of obscuring the discontinuities we are interested in observing.

In the results presented here, the simulations were continued for a very long time. Note that the formation of a dense cortical strut in the femoral head is a result of oversimplified loads applied for an extreme duration. In a more physiologic situation it would be unrealistic to assume that a simple set of three load cases can represent the load history over the very long term. Thus, in a clinical application (e.g. predicted density changes due to the altered loading induced by total hip replacement) the simulation duration may be too short for inappropriate near-field discontinuities to manifest themselves.

Also, in most cases a graphical post-processor is used to generate contour plots for visualization of results. When centroidal results are plotted by many post-processors they are first extrapolated to and averaged at the nodes. Also known as an L_2-projection, this smooths the raw data, tending to obscure the near-field type of discontinuity. The method we have presented also depends on projecting the stress and strain data to the nodes at each time step (and therefore smoothing them). It is difficult to distinguish end configuration contour plots of the element-based approach (where smoothing of density takes place once at the end of the analysis) from the node-based approach (where smoothing of stress and strain take place at each time step). Thus, the bulk of the data in the literature is not likely to require reinterpretation.

In the node-based implementation, no features specific to ABAQUS have been employed. In fact, any code that can solve thermoelastic problems can be used by simply specifying a material with a temperature dependence, using the

density in place of temperature. At each increment one specifies the density distribution throughout the model where the code expects to find the temperature.

In this treatment, we have made a distinction between near-field and far-field bone remodeling behavior. Although the transition from cortical to trabecular morphology can be quite dramatic, this distinction is admittedly artificial. As a conceptual construction, however, it is helpful to express certain concepts of bone remodeling, namely that a single formulation can be used in both situations, as well as regions of transition from one to the other.

Long bone structure transitions from a primarily nonporous cortical form in the diaphysis to a highly porous trabecular form in the epiphyses. Many investigators have explicitly or implicitly suggested that this is an adaptation to changing mechanical demands in the two different regions, which suggests that a single biological mechanism of stress-adaptation may explain the formation of both types of bony morphology. We have presented evidence that currently employed continuum formulations of bone remodeling on the apparent level can produce both continuous and discontinuous solutions, depending on the character of the local mechanical loading, when appropriate numerical methods are employed. In this way, current formulations are attractive since they hypothesize a single strain-energy-based mechano-biologic mechanism which leads to continuous or discontinuous morphologies depending on the local loading environment. This property seems to agree well with the observation of two distinct types of bone (cortical and cancellous) which occur regionally in response to the local mechanical requirements. The node-based method for simulating bone remodeling has attractive properties in this area as well as computational efficiency and thus, we believe it to be the method of choice.

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APPENDIX

By assuming particular values for para
modeling formulation used in this paper
arrive at a formulation that is very similar
Weinans *et al.* (1992) and Harrigan and H
we assume that each load case is applied fo
of cycles, N, and the stress exponent, m
becomes

$$\Psi = \sqrt{2NE \sum_{\text{day}} U}.$$

Next, if we assume that $n=3$ in equatio
becomes

$$\frac{d\rho}{dt} = A\left(\left(\frac{\rho_c}{\rho}\right)^2 \sqrt{2ND\rho^3 \sum_{\text{day}} U} - \right.$$
$$\times S_v(\rho), \quad 0 \leqslant \rho \leqslant \rho_c,$$

$$\frac{d\rho}{dt} = 0, \quad \text{otherwise.}$$

Weinans, H., Huiskes, R. and Grootenboer, H. J. (1992b) Effects of material properties on femoral hip components on bone remodeling. *J. orthop. Res.* **10,** 845–853.

APPENDIX

By assuming particular values for parameters in the re modeling formulation used in this paper it is possible to arrive at a formulation that is very similar to those used by Weinans *et al.* (1992) and Harrigan and Hamilton (1992). If we assume that each load case is applied for an equal number of cycles, N, and the stress exponent, $m=2$, equation (2) becomes

$$\Psi=\sqrt{2NE\sum_{\text{day}}U}. \tag{A.1}$$

Next, if we assume that $n=3$ in equation (5), equation (3) becomes

$$\frac{d\rho}{dt}=A\left(\left(\frac{\rho_c}{\rho}\right)^2\sqrt{2ND\rho^3\sum_{\text{day}}U}-\Psi_{\text{bAS}}\right)\times S_v(\rho),\quad 0\leqslant\rho\leqslant\rho_c,$$

$$\frac{d\rho}{dt}=0,\quad \text{otherwise.} \tag{A.2}$$

Ignoring the specific surface area effect and consolidating the constants yields

$$\frac{d\rho}{dt}=\hat{A}\left(\sqrt{\frac{\sum_{\text{day}}U}{\rho}}-\hat{k}\right)S_v(\rho),\quad 0\leqslant\rho\leqslant\rho_c,$$

$$\frac{d\rho}{dt}=0,\quad \text{otherwise.} \tag{A.3}$$

When this is compared with the formulation taken from Weinans *et al.* (1992),

$$\frac{d\rho}{dt}=B\left(\frac{\frac{1}{n}\sum_{i=1}^{n}U_i}{\rho}-k\right),\quad 0\leqslant\rho\leqslant\rho_c,$$

$$\frac{d\rho}{dt}=0,\quad \text{otherwise,} \tag{A.4}$$

it is clear that the two sets of equations (A.3 and A.4) are very similar. Thus, it is not surprising that the stability behavior of the method is also similar. The equations analyzed by Harrigan and Hamilton (1992a) are a restricted case of equation (A.4).