4.4 the cytoskeleton - tensegrity model

assuming we know the mechanical properties of the individual filaments, what does that actually tell us about the assembly of filaments that we find in the cell?
• could we then predict the stiffness of the overall assembly?
• how does the filament microstructure affect cytoskeletal properties?
• how can we calculate the macroscopic network properties from the individual microscopic filament properties?

Figure 4.1: The cytoskeleton provides structural stability and is responsible for forces during cell locomotion. Microtubules are thick hollow cylinders reaching out from the nucleus to the membrane, intermediate filaments can be found anywhere in the cytosol, and actin filaments are usually concentrated close to the cell membrane.

me239 mechanics of the cell

4.1 mechanics of the cytoskeleton
red blood cells

Erythrocytes, red blood cells are essential to deliver oxygen to the body via the blood flow through the circulatory system. They take up oxygen in the lungs and release it while squeezing through the body's capillaries. Adult humans have about \(2-3 \times 10^{13}\) or 20-30 trillion, red blood cells comprising about a quarter of the total amount of cells in the human body.

4.3 network model for red blood cells

Homogenization - Hill-Mandel condition

**Aim.** To determine the overall material properties \(\kappa\) and \(\mu\) of the network of spectrin chains in terms of the spectrin chain stiffness \(k\).

**Energy Approach**

\[
W^{\text{mac}} = W^{\text{mic}}
\]

It has been shown how the central problem is reducible to the calculation of average stress or strain in one or other phase. A more versatile approach stems directly from classical theorems in elasticity and focuses attention on strain energies.


Figure 4.6: Microstructural architecture of the cell membrane of a red blood cell. A six-fold connected network of spectrin tetramers which are crosslinked through short actin filaments, anchored to the phospholipid bilayer, provides structural support to the inner cell membrane.

Figure 4.7: Microstructural architecture of a six-fold and four-fold connected network. The theory of homogenization helps to explain why nature prefers a six-fold connected network geometry.
single spring energy

free energy $W^{spr}$ of a single spring

$$W^{spr} = \frac{1}{2} k \delta^2 = \frac{1}{2} k [l - l_0]^2$$

where $\delta = l - l_0$

$$\text{unstretched spring}$$

$$\text{stretched spring}$$

$k$ ... spring stiffness

Figure 1.7: Spectrin can be modeled as a Gaussian chain which we can conceptually replace by an equivalent linear entropic spring with a spring stiffness $k = 3 kT N / L$. The strain energy of this spring can then be expressed as $W^{spr} = \frac{1}{2} k \delta^2$.

4.3 network model for red blood cells

equivalent macroscopic energy

$$W^{mac} = \frac{1}{2} k [\epsilon_{xx} + \epsilon_{yy}]^2$$

$$+ \frac{1}{2} \mu [\epsilon_{xx} - \epsilon_{yy}]^2 + 2 \mu \epsilon_{xy}$$

micro-to-macro kinematics

$\epsilon_{xx} = \epsilon_{yy} = \delta / l_0$; $\epsilon_{xy} = 0$

$$W^{mac} \equiv W^{mic}$$

$$\frac{1}{2} k [\delta / l_0]^2 = \sqrt{3} k \left[ \frac{\delta}{l_0} \right]^2$$

$$\kappa = \frac{1}{2} \sqrt{3} k$$

discrete microscopic network energy

$$W^{mic} = \sum_{i=1}^{3} W^{spr}$$

$$\sum_{i=1}^{3} \frac{1}{2} k \left[ l - l_0 \right]^2$$

$$h_0 = \frac{1}{2} \sqrt{3} l_0$$

$$\sum_{i=1}^{3} A^{spr} = 3 A^{spr} = \frac{1}{2} \sqrt{3} l_0$$

$$W^{mic} = \frac{3}{2} k \delta^2$$

$$\frac{1}{2} \sqrt{3} l_0$$

4.3 network model for red blood cells
4.3 network model for red blood cells

$W^{\text{mac}} = \frac{1}{2} k \left[ \varepsilon_{xx} + \varepsilon_{yy} \right]^2 + \frac{1}{2} \mu \left[ \varepsilon_{xx} - \varepsilon_{yy} \right]^2 + 2 \mu \varepsilon_{xy}^2$

Micro-to-macro kinematics

\[ \varepsilon_{xx} = 0 \quad \varepsilon_{yy} = 0 \]

\[ \varepsilon_{xy} = \frac{1}{2} \left( \frac{\delta}{\sqrt{3} l_0} + 0 \right) = \frac{1}{\sqrt{3} l_0} \delta \]

\[ W^{\text{mac}} = W^{\text{mic}} \]

\[ 2 \mu \left( \frac{1}{\sqrt{3} l_0} \right)^2 = \frac{\sqrt{3}}{6} k \left[ \frac{\delta}{l_0} \right]^2 \]

\[ \mu = \frac{1}{4} \sqrt{3} k \]

4.4 tensegrity model for cells

The term tensegrity was first coined by Buckminster Fuller to describe a structure in which continuous tension in its members forms the basis for structural integrity. Fuller most famously demonstrated the concept of tensegrity in architecture through the design of geodesic domes while his student, the artist Kenneth Snelson, applied the concept of tensegrity to creating sculptures that appear to defy gravity. Snelson’s tensegrity sculptures are minimal in components and achieve their stability through dynamic distribution of tension and compression forces amongst their members to create internal balance. It was upon viewing Snelson’s art that Donald Ingber became inspired by the sculpture’s structural efficiency and dynamic force balance to adopt tensegrity as a paradigm upon which to analyze cell structure and mechanics. It has been 30 years since the premier appearance of the cellular tensegrity model. Although the model is still largely under discussion, empirical evidence suggests that the model may explain a wide variety of phenomena ranging from tumor growth to cell motility.

Tensegrity = tension + integrity

Tensegrity bike held together by wires

The Architecture of Life

A universal set of building rules seems to guide the design of organic structures—from simple carbon compounds to complex cells and tissues

by Donald E. Ingber

Life is the ultimate example of complexity at work. An organism, whether it is a bacterium or a baboon, develops through an incredibly complex series of interactions involving a vast number of different components. These components, or subsystems, are themselves made up of smaller molecular components, which independently exhibit their own dynamic behavior, such as the ability to catalyze chemical reactions. Yet when they are combined into some larger functioning unit—such as a cell or tissue—utterly new and unpredictable properties emerge, including the ability to move, to change shape and to grow.

Tensegrity = tension + integrity

Tensegrity bike held together by wires

looy alter [2003]
4.4 tensegrity model for cells

Tensegrity = tension + integrity

Balanced interplay between tension and compression

Example - geodesic domes

Geodesic domes carry load with minimum material

Ingber [1998]

Pollen grains are geodesic domes

Ingber [1998]

Cell design contest 2007

**bryan’s cell…**

- number of microtubules: 8
- number of filaments: 36
- fiber length: 27.4135 \( \text{d} \)
- volume fraction: 0.00755 \( V \)
- tensile stiffness: 0.488149 N/\( \text{d} \)
- compressive stiffness: 4.86e-11 N/\( \text{d} \)
- shear stiffness: 4.91e-05 N/\( \text{d} \)

... is super stiff and yet compliant

**cell design contest 2007**

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**lizzie’s cell**

- number of microtubules: 6
- number of filaments: 36
- fiber length: 23.7215 \( \text{d} \)
- volume fraction: 0.00609 \( V \)
- tensile stiffness: 0.145106 N/\( \text{d} \)
- compressive stiffness: 0.000000 N/\( \text{d} \)
- shear stiffness: 0.024453 N/\( \text{d} \)

... is a typical tensegrity structure

**cell design contest 2007**

---

**takane’s cell**

- number of microtubules: 8
- number of filaments: 24
- fiber length: 28.9282 \( \text{d} \)
- volume fraction: 0.00630 \( V \)
- tensile stiffness: 0.388568 N/\( \text{d} \)
- compressive stiffness: 0.714010 N/\( \text{d} \)
- shear stiffness: 0.016585 N/\( \text{d} \)

... is modeled after a sculpture and really cool

**cell design contest 2007**

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**lizzie’s cell**

- number of microtubules: 90
- number of filaments: 60
- fiber length: 48.1597 \( \text{d} \)
- volume fraction: 0.03099 \( V \)
- tensile stiffness: 0.000000 N/\( \text{d} \)
- compressive stiffness: 0.000000 N/\( \text{d} \)
- shear stiffness: 0.000000 N/\( \text{d} \)

... looks really cool but has no stiffness at all

**cell design contest 2007**
lizzie’s cell

- number of microtubules: 90
- number of filaments: 60
- fiber length: 48.1597 d_{cell}^{max}
- volume fraction: 0.03099 V_{cell}^{max}

... needs triangular substructures for support

amir’s cell

- number of microtubules: 12 (should have been 6!)
- number of filaments: 30
- fiber length: 21.7719 d_{cell}^{max}
- volume fraction: 0.00624 V_{cell}^{max}

... typical structure that needs prestress

cell design contest 2007

zubin’s and joey’s cells

- number of microtubules: 607
- number of filaments: 1815
- fiber length: 145.3161 d_{cell}^{max}
- volume fraction: 0.093529 V_{cell}^{max}

... membrane models

chunhua’s cell

- number of microtubules: 3
- number of filaments: 62
- fiber length: 11.4853 d_{cell}^{max}
- volume fraction: 0.00321 V_{cell}^{max}

... and the winner is: the smallest tensegrity structure

cell design contest 2007
4.4 tensegrity model for cells

Figure 1: Tensegrity structures, Snelson’s sculpture ‘Mozart’ on Stanford campus, wood-rubber model, and cytoskeleton of the cell.

4 trusses
6 trusses
12 trusses
8 trusses
6 trusses and 1 nucleus
3 trusses

4.4 tensegrity model for cells

tensegrity models for cells

Figure 4.12: Kinematics of simple tensegrity cell model consisting of six compressive trusses (grey) and 24 tensile ropes (black). In the original state, all trusses are of the same length $l_0$, the rope lengths are $l_0 = \sqrt{3}/2 l_0$, and the distances between two parallel trusses are $s_0 = 1/2 l_0$. 
4.4 tensegrity model for cells

- **kinematics**
  \[ l_0 = \left[ \frac{L_0 - s_0}{2} \right]^2 + \left[ \frac{s_0}{2} \right]^2 + \left[ \frac{L_0}{2} \right]^2 \]

- **equilibrium**
  \[ \sum F = 0 : \ T + 4 F_{AB} \left[ \frac{L_0 - s_0}{2l_0} \right] - 4 F_{AC} \left[ \frac{s_0}{2l_0} \right] = 0 \]

- **constitutive equation**
  \[ F = k [ l - l_0 ] \]

\[ T = 0 \] special case
\[ s_0 = L_0 / 2 \] from the equilibrium equation
\[ l_0 = \sqrt{3}/8L_0 \] from the kinematic equation

\[ \text{microscopic free energy density} \]
\[ W^{\text{mic}} = \frac{1}{V_0} \int_{s_0}^{s_x} T \, dx \]

\[ \text{tensile force} T \text{ acting along the changing length } \int_{s_0}^{s_x} \text{d}x \text{ scaled by volume of the tensegrity cell } V_0 \]

4.4 tensegrity model for cells

\[ W^{\text{mac}} = W^{\text{mic}} \quad \text{or} \quad \frac{\partial W^{\text{mac}}}{\partial s_x} = \frac{\partial W^{\text{mic}}}{\partial s_x} \]

\[ W^{\text{mac}} = \frac{1}{V_0} \int_{s_0}^{s_x} T \, dx \]

\[ E = \frac{2\sqrt{3}}{3} \frac{T}{s_x - s_0} \]

\[ E_0 = \frac{5.85}{2} \frac{F_0}{l_0} \frac{1 + 4\epsilon_0}{1 + 12\epsilon_0} \]

4.4 tensegrity model for cells

\[ \text{equivalent macroscopic energy} \]

\[ W^{\text{mac}} = \frac{1}{2} E \epsilon \]

\[ \frac{\partial W^{\text{mac}}}{\partial s_x} = \frac{\partial W^{\text{mac}}}{\partial \epsilon} = E \epsilon \frac{\partial \epsilon}{\partial s_x} \]

\[ \text{micro-to-macro kinematics / strain} \]

\[ \epsilon = \frac{s_x - s_0}{s_0} \quad \text{such that} \quad \frac{\partial \epsilon}{\partial s_x} = \frac{1}{s_0} \]

\[ W^{\text{mac}} = W^{\text{mic}} \]

\[ W^{\text{mac}} = \frac{1}{V_0} \int_{s_0}^{s_x} T \, dx \]

\[ E = \frac{2\sqrt{3}}{3} \frac{T}{s_x - s_0} \]

\[ E_0 = \frac{5.85}{2} \frac{F_0}{l_0} \frac{1 + 4\epsilon_0}{1 + 12\epsilon_0} \]
Tensegrity models for prestress

Prestress. Tensegrity models are an extremely elegant way to model prestress through the application of initial tension in the rope members. In fact, prestress is inherent to tensegrity structures in that they stabilize themselves through their own weight balanced by prestress. Prestress, very common to biological structures, is a design concept that we have adopted from nature. For example, in the form of prestressed reinforced concrete, prestressed concrete was patented by a San Francisco engineer in 1886.

4.4 Tensegrity model for cells

Prestress - analytically predicted

- Assume prestress is approximately equal in all three directions:
  \[ P \approx \frac{1}{3} V_{\text{actin}} \rho_{\text{actin}} \]
- Volume fraction of actin filaments:
  \[ \rho_{\text{actin}} = \frac{24 A_{\text{actin}} I_0}{[5 \sqrt{2}] / [3 \sqrt{3}] I_0^2} = \frac{24 A_{\text{actin}}}{1.3608 I_0^2} \]
- Stress in a typical actin filament:
  \[ \gamma_{\text{actin}} = \frac{F_0}{A_{\text{actin}}} \]
- Approximation of prestress:
  \[ P \approx \frac{1}{3} V_{\text{actin}} \rho_{\text{actin}} = \frac{1}{3} \frac{24 A_{\text{actin}}}{1.3608 I_0^2} \frac{F_0}{A_{\text{actin}}} \]
  \[ P \approx 5.85 \frac{F_0}{I_0^2} = E \]

Prestress is of the same order as Young’s modulus.

4.4 Tensegrity model for cells

Prestress - experimentally measured

\[ E = 3G \quad P \approx 5.85 \frac{F_0}{I_0^2} = E \]

Prestress is of the same order as Young’s modulus.

Wang, Naruse, Stamenovic, Fredberg, Mijailovich, Tolic-Norrelykke, Potte, Mannix, Ingber (2001)

Tensegrity structures on campus

Lightweight and strong

Snelson (1986)