# Homework III

due Thu 02/24/11, 4:30pm, 200-034

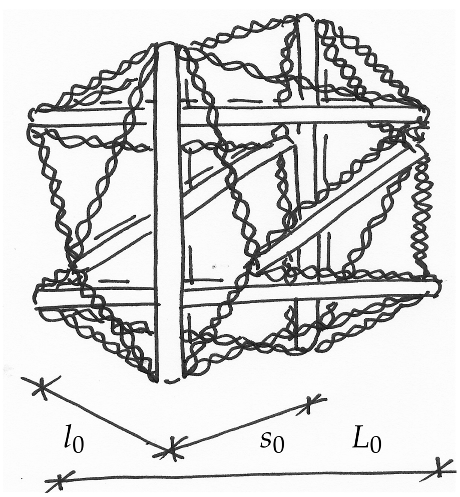
Late homework can be dropped off in a box in front of Durand 217. Please mark clearly with date and time @drop off. We will take off 10 points for each 24 hours late.

## Problem 1 - The tensegrity model

Derive upper and lower bounds for the incremental cell modulus $E_0$ predicted by the simple six-truss tensegrity model illustrated in Figure 1. Use the following relation

$$E_0 = 15.6 \, \frac{F_0}{L_0^2} \, \frac{1 + 4\,\varepsilon_0}{1 + 12\,\varepsilon_0}$$

between the incremental modulus $E_0$, the resting force in the actin filaments $F_0$, the microtubules length $L_0$, and the actin filament strain $\varepsilon_0$. Recall that for this simplified model, the resting length of the actin filaments is $l_0 = \sqrt{3/8}\, L_0$.




$E_0$ ... incremental modulus
$F_0$ ... resting force in actin filaments
$L_0$ ... length of microtubules
$l_0$ ... resting length of actin filaments
$\varepsilon_0$ ... strain in actin filaments

**Figure 1.** Kinematics of simple tensegrity cell model consisting of six compressive trusses and 24 tensile ropes. In the original state, all trusses are of the same length $L_0$, the rope lengths are $l_0 = \sqrt{3/8}\, L_0$, and the distances between two parallel trusses are $s_0 = 1/2\, L_0$.

(1.1) Assume an upper bound for $E_0$ can be obtained when the actin filaments are almost about to break in the resting position. Use their effective radius $r^{\text{actin}} = 3.5\,\text{nm}$, their Young's modulus of $E^{\text{actin}} = 1.9 \cdot 10^9\,\text{N/m}^2$, and their average force at filament breaking $F^{\text{actin}} = 400\,\text{pN}$ to estimate the critical strain $\varepsilon^{\text{actin}}$ at which the actin filaments will break.

(1.2) Plot $E_0$ versus $L_0$ for $L_0 = 1\text{–}6\,\mu$m using the estimate for the critical resting strain calculated in (1.1). You may want to use a log scale for the $E_0$ axis. This graph represents the upper bound for the tensegrity cell modulus $E_0$.

(1.3) Assume a lower bound for $E_0$ can be obtained when the microtubules are almost about to buckle in the resting position. Assume the resting force in the microtubules is related to the resting force in the actin filaments as $F_0^{\text{mtube}} = \sqrt{6}\,F_0$. Express $E_0$ in terms of the microtubules flexural rigidity, the initial actin filament strain, and the microtubules length $L_0$, assuming the critical state of microtubules buckling.

(1.4) When microtubules are about to buckle, the actin filament strains $\varepsilon_0$ are relatively small and contribute negligibly to the estimate of $E_0$. Using the same graph of (1.2), plot $E_0$ versus $L_0$ for $L_0 = 1\text{–}6\,\mu$m for the case of small strain and microtubules buckling. Assume a flexural rigidity of microtubules of $364 \cdot 10^{-25}\,\text{Nm}^2$. This graph represents the lower bound for the tensegrity cell modulus $E_0$.

(1.5) Cell stiffness for various cell types measured with various different techniques is illustrated in Table 1. Herein, $L_0$ denotes the probe diameter which you can assume to be of the order of the microtubules length $L_0$ in our model. Plot the experimental data of Table 1 into your graph form (1.2) and (1.4) with the upper and lower bounds for $E_0$ predicted by the tensegrity model.

(1.6) Comment on the model prediction and on the experimental results.

| cell type | measurement technique | $L_0$ [$\mu$m] | $E_0$ [dyne/cm$^2$] |
|---|---|---:|---:|
| spread fibroblast | atomic force microscopy | 2.0 | 16 000 |
| sheared endothelial cell | micropipette aspiration | 3.0 | 1 575 |
| round endothelial cell | micropipette aspiration | 3.0 | 750 |
| spread epithelial cell | magnetic bead rheometry | 4.0 | 75 |
| spread smooth muscle cell | magnetic bead rheometry | 5.5 | 110 |
| spread endothelial cell | magnetic bead rheometry | 5.5 | 45 |
| round endothelial cell | magnetic bead rheometry | 5.5 | 22 |

**Table 1.** Young's moduli for different cell types and measurement techniques, adopted from Stamenovic & Coughlin and Ethier & Simmons.

## Problem 2 - Mechanotransduction

The recent review *"Mechanotransduction gone awry"* by Jaalouk and Lammerding discusses defects in mechanotransduction and their effects on various different disease types.

(2.1) Read the manuscript carefully and summarize it in approximately 150 words.

(2.2) Select your favorite mechanotransduction pathway and describe how it is altered under diseased conditions.

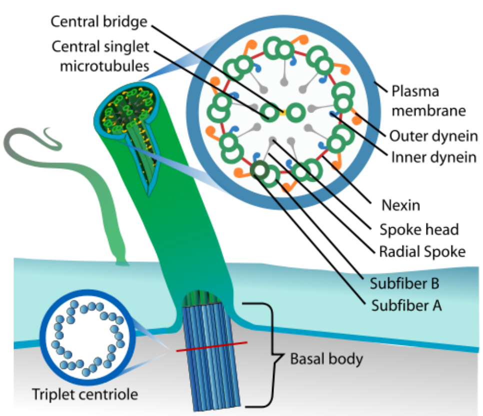
(2.3) What are the implications of this article? Write a paragraph of about 150 words.

## Problem 3 - Midterm preparation

For the mid term exam, you are allowed to use one letter format cheat sheet, either handwritten or printed. Prepare your cheat sheet as part of this homework.

## Problem 4 - Cilia ... optional ;-)

A cilium of length $L = 2\mu$m and a diameter of $0.4\mu$m sweeps through extracellular fluid with $\mu = 1$cP at a constant angular velocity $\omega$. You can model this motion as a rotation of the cilium around its base, assuming that the cilium has the form of a rigid cylinder.




$M_0$ ... torque at the cilium base
$L$ ... cilium length
$\omega$ ... angular velocity
$F^{\text{drag}}$ ... drag force per unit length
$F^{\text{max}}$ ... maximum dimer force

**Figure 2.** Cilium bending. In a cross-section a cilium is seen to consist of a regular array of microtubules. Two tubules run down the centre of the shaft and nine pairs of microtubules surround the central two these called "nine-plus-two".

(1.1) Show that the torque $M_0$ exerted on the base of the cilium by the cell body is $M_0 = k\,\mu\,\omega\,L^3/3$. You can assume that the drag force exerted per unit length of the cilium is $F^{\text{drag}} = k\,\mu\,v$, where $k$ is a constant and $v$ is the local fluid velocity passing over the cylinder.

(1.2) Assume that the nine tubulin dimers are evenly spaced around the periphery of the cilium as shown in Figure 2. Further assume that the left half of the perimeter of the cross section is the leading edge and the right half is the trailing edge. All dimers to in the left are under maximum compression, $-F^{\text{max}}$, and all dimers to the right are under maximum tension, $+F^{\text{max}}$. Compute the value of $F^{\text{max}}$ at the base of the cilium, assuming that the dimers transfer the entire torque $M_0$ from the cell body to the cilium. You can assume that $k$ =2 and $\omega = 0.2$ rad/2. Make other assumptions as needed.

(1.3) Give examples where we can find cilia.

## Final project

A layout style file for your final project has been uploaded on coursework (`final project - layout`). You will also find a sample file from one of last year's students (`final project - example`) and an evaluation spreadsheet that we will use to grade your final presentation (`final project - judging guidelines`). Oral presentations will be 5-10 minutes long, to be given in class on March 8th and 10th. Assignments to either day are listed in the judging guideline sheet. Written reports are due on March 11th, 11:59pm.