4. the cytoskeleton - fiber bundle models

Forward and Reverse Engineering of Molecular Motors

BioE Seminar/BioE 393
Tuesday, April 20, 2010, 4:15 pm, Clark S360
Prof. Zev Bryant, Department of Bioengineering

Molecular motors lie at the heart of cellular processes ranging from DNA replication to organelle transport. We are using single-molecule tracking and manipulation assays to characterize the structural dynamics of nanoscale machines such as DNA gyrase, an enzyme that harnesses the free energy of ATP hydrolysis to introduce superhelical strain into DNA. To further challenge our understanding of the relationships between molecular structures and mechanical functions, we are designing and building novel variants of biological molecular motors. Our design targets include myosin motors that can reversibly switch their direction of motion in response to an external signal.

3.1 biopolymers - motivation

Figure 3.1. Biopolymers. Characteristic length scales on the cellular and subcellular level.
3.2 biopolymers - polymerization

**polymerization of actin and tubulin**

**polymerization of actin and tubulin**

**NUCLEOTIDE HYDROLYSIS**

Each actin molecule carries a tightly bound ATP molecule that is hydrolyzed to a tightly bound ADP molecule soon after its assembly into polymer. Similarly, each tubulin molecule carries a tightly bound GTP that is converted to a tightly bound GDP molecule soon after the molecule assembles into the polymer.

**ATP CAPS AND GTP CAPS**

The rate of addition of subunits to a growing actin filament or microtubule can be faster than the rate at which their bound nucleotides is hydrolyzed. Under such conditions, the end has a “cap” of subunits bearing the nucleotide triphosphate—an ATP cap on an actin filament or a GTP cap on a microtubule.

**DYNAMIC INSTABILITY and TREADMILLING** are two behaviors observed in cytoskeletal polymers. Both are associated with nucleotide triphosphate hydrolysis. Dynamic instability is believed to predominate in microtubules, whereas treadmilling may predominate in actin filaments.

**Table 3.1: Measured rate constants of actin filaments and microtubules**

<table>
<thead>
<tr>
<th></th>
<th>$k^+_{on}$ [1/(μM) s]</th>
<th>$k^-_{off}$ [1/s]</th>
<th>$k^+_{on}$ [1/(μM) s]</th>
<th>$k^-_{off}$ [1/s]</th>
<th>$C^+_{crit}$ μM</th>
<th>$C^-_{crit}$ μM</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATP-actin</td>
<td>11.60</td>
<td>1.40</td>
<td>1.30</td>
<td>0.80</td>
<td>0.12</td>
<td>0.62</td>
</tr>
<tr>
<td>ADP-actin</td>
<td>3.80</td>
<td>7.20</td>
<td>0.16</td>
<td>0.27</td>
<td>1.90</td>
<td>1.70</td>
</tr>
<tr>
<td>GTP-tubulin</td>
<td>8.90</td>
<td>44.00</td>
<td>4.30</td>
<td>23.00</td>
<td>4.90</td>
<td>5.30</td>
</tr>
<tr>
<td>GDP-tubulin</td>
<td>0</td>
<td>733</td>
<td>0</td>
<td>915</td>
<td>n/a</td>
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</tr>
</tbody>
</table>

**Figure 3.4:** Eight rate constants for polymerization capture and release in non-symmetric actin and microtubule filaments.
3.3 biopolymers - energy

The concept of persistence length

- Stiffer filaments are straighter $\propto$ bending stiffness $EI$
- Cooler filaments are straighter $\propto$ inverse temperature $kT$

$$A = \frac{EI}{kT} \quad \text{... persistence length}$$

<table>
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<tr>
<th></th>
<th>$r$</th>
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<tr>
<td>microtubule</td>
<td>12.5 nm</td>
<td>1.9 x 10^8 N/m^2</td>
<td>364 x 10^{-25} Nm^2</td>
<td>8.800 mm</td>
</tr>
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<td>intermediate filament</td>
<td>5.0 nm</td>
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<td>0.240 mm</td>
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<td>0.048 mm</td>
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Table 3.6: Persistence lengths of major constituents of cytoskeleton at room temperature: microtubules, intermediate filaments and actin filaments

3.3 biopolymers - entropy

The concept of persistence length

- Stiffer filaments are straighter $\propto$ bending stiffness $EI$
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Table 3.6: Persistence lengths of major constituents of cytoskeleton at room temperature: microtubules, intermediate filaments and actin filaments

4.1 mechanics of the cytoskeleton

Assuming we know the mechanical properties of the individual filaments, what does that actually tell us about the assembly of filaments that we find in the cell?

- Could we then predict the stiffness of the overall assembly?
- How does the filament microstructure affect cytoskeletal properties?
- How can we calculate the macroscopic network properties from the individual microscopic filament properties?

- Elements of the cytoskeleton: microtubules, intermediate filaments, actin filaments

Figure 4.1: The cytoskeleton provides structural stability and is responsible for forces during cell locomotion. Microtubules are thick hollow cylinders reaching out from the nucleus to the membrane, intermediate filaments can be found anywhere in the cytosol, and actin filaments are usually concentrated close to the cell membrane.
from molecular level to cellular level

three examples

- **fiber bundle model** for filopodia
- **network model** for red blood cell membranes
- **tensegrity model** for generic cell structures

4.1 mechanics of the cytoskeleton

filopodia are **thin dynamic cytoplasmic projections** composed of **tight bundles of long actin filaments** extending from the leading edge of migrating cells. Sometimes, the name filopodia is used to describe all different kinds of cytoskeletal protrusions including thick filopodia, cell feet, and amoeba pseudopods. **Filopodia contain actin filaments cross-linked into bundles by actin-binding proteins such as fimbrin.**

Many types of motile cell such as fibroblasts or keratinocytes use filopodia for **cell locomotion**. Filopodia at the leading edge of a migrating cell seem to explore the extracellular matrix and surfaces of other cells. Once they have identified appropriate targets, they **form focal adhesions** linking the cell surface to the substratum further down the migratory pathway. The contraction of stress fibers then retracts the rear of the cell and the cell crawls forwards.

4.2 fiber bundle model for filopodia

**Filopodia and lamellipodia**

**Figure 4.2.1.** A crawling cell, drawn to scale, is shown with three areas enlarged to show the arrangement of actin filaments. The actin filaments are shown in red, with arrowheads pointing toward the plus end. Stress fibers are contractile and exert tension. The cell cortex underlies the plasma membrane. Filopodia are spike-like projections of the plasma membrane that allow a cell to explore its environments.

*Alberts, Johnson, Lewis, et al. [2002]*

**Figure 4.2.2.** Filopodia and lamellipodia are actin-rich protrusions important for cell motility. This electron micrograph shows exaggerated filopodia with club-like shape. Filopodia are filled with bundled actin filaments which were born in and converged from the lamellipodial network.

*Czech, Sakina, Yang [2007]*
4.2 fiber bundle model for filopodia

Filopodia and other fiber bundles of F-actin

Figure 4.2: Fiber bundles of F-actin. Ciliary bundle from the sensory epithelium of a bullfrog saccule consisting of stereocilia, filopodium protruding from the lamellipodium of a mouse melanoma cell, epithelial microvilli, and drosophila neurosensory micro- and macrochaete bristles.

Bathe, Heussinger, Claessens, Bausch, Frey [2008]

Pushing the envelope

This single-celled amoeba crawls around by using actin polymerization to push out pseudopods, or false feet, to explore new territory. At the same time, organelles move in complex patterns within the cell. Alberts, Johnson, Lewis, Raff, Roberts, Walter [2002]

Assembly of crosslinked actin filaments

Figure 4.3. Bundles of actin filaments tightly crosslinked through fascin are known as filopodia. The mechanical properties of filopodia play an essential role in various different physiological processes including hearing, cell migration, and growth. Filopodia are very thin structures approximately 0.2 um in diameter, they can easily extend up to 1.5um. They typically polymerize and depolymerize at rates of approximately 10 um/min. The mechanical properties of filopodia play an essential role in various different physiological processes, including hearing, cell migration, and growth. Despite their importance to cell function, the structural architecture responsible for their overall mechanical behavior remains largely unknown.

Pushing the envelope

Simplified model for cell locomotion

- Protrusion ... polymerization at the leading edge of the cell
- Attachment ... formation of focal adhesions to link the cell to the surface
- Retraction ... contraction of stress fibers to retract the rear of the cell

Figure 4.4: Single-celled amoeba crawling around by using actin polymerization to push out pseudopods to explore new territory. Organelles move in complex patterns within the cell. Alberts, Johnson, Lewis, Raff, Roberts, Walter [2002]
Figure 4.2.3. Cell migration is dependent on different actin filament structures. Motility is initiated by an actin-dependent protrusion of the leading edge, which is composed of lamellipodia and filopodia. a) These protrusive structures contain actin filaments, with elongating barbed ends oriented towards the plasma membrane. b) During cellular extension, new adhesions with the substratum are formed under the leading edge. c) Next, the nucleus and the cell body are translocated forward through actomyosin-based contraction forces that might be mediated by focal adhesion-linked stress fibers, which also mediate the attachment to the substratum. d) Then, retraction fibers pull the rear of the cell forward, adhesions at the rear of the cell disassemble and the trailing edge retracts. Mattila & Lappalainen (2008).

4.2 fiber bundle model for filopodia

pushing the envelope - membrane force

- membrane forces acting on a cylinder of radius $r_{fil}$
  $$F_{mem} \approx 5 r_{fil} \frac{pN}{nm}$$
- radius $r_{fil}$ of the filopodium
  $$A_{fil} = \pi r_{fil}^2$$
  $$A_{act} = n \pi r_{act}^2$$
- force exerted by the cell membrane on the filopodium cylinder
  $$F_{mem} \approx 5 \sqrt{n} r_{act} pN/nm$$

pushing the envelope - filament force

- critical force
  $$F_{crit} = \frac{\pi^2 EI}{L_{crit}^2} \quad \text{thus} \quad F_{fil} = \frac{\pi^2 EI}{[2L]^2} = \frac{\pi^2 EI}{4L}$$

Figure 4.5: The four Euler buckling modes. As the buckling length $L_{crit}$ decreases from $L_{crit} = 2L$ to $L_{crit} = 1/2L$, from left to right, the critical buckling force $F_{crit} = \pi^2 EI / L_{crit}^2$ or rather the resistance to buckling, increases.

4.2 fiber bundle model for filopodia

pushing the envelope - critical length

Newton’s third law: actio = reactio

- $F_{fil} \doteq F_{mem}$
  $$F_{fil} = \frac{\pi^2 EI}{[2L]^2} = \frac{\pi^2 EI}{4L}$$
  $$F_{mem} \approx 5 \sqrt{n} r_{act} pN/nm$$

- $L_{crit} = \frac{\pi}{2} \sqrt{\frac{EI}{5 \sqrt{n} r_{act} pN/nm}}$

- $E = 1.9 \cdot 10^9$ N/m$^2 = 1.9$ GPa
  - loose assembly
  - tightly crosslinked

- $r_{act} = 2.5$ moment of inertia $I$
case I - loosely assembled actin filaments

\[ L_{\text{crit}} = \frac{\pi}{2} \sqrt{\frac{EI}{5\sqrt{n} r_{\text{act}} pN/nm}} \]

moment of inertia \( I = n I_{\text{act}} \) with \( I_{\text{act}} = \frac{\pi r_{\text{act}}^4}{4} \)

\[ E = 1.9 \cdot 10^9 \text{ N/m}^2 = 1.9 \text{ GPa} \quad r_{\text{act}} = 2.5 \]

\[ n = 30 \text{ filaments} \quad L_{\text{crit}} = 0.416 \mu m \]

much too low - disagrees with observations of 2um

4.2 fiber bundle model for filopodia

pushing the envelope

4.2 fiber bundle model for filopodia

case II - tightly crosslinked actin filaments

\[ L_{\text{crit}} = \frac{\pi}{2} \sqrt{\frac{EI}{5\sqrt{n} r_{\text{act}} pN/nm}} \]

moment of inertia \( I = \frac{n^2 \pi r_{\text{fil}}^4}{4} \) with \( r_{\text{fil}} = \sqrt{n} r_{\text{act}} \)

\[ E = 1.9 \cdot 10^9 \text{ N/m}^2 = 1.9 \text{ GPa} \quad r_{\text{act}} = 2.5 \]

\[ n = 30 \text{ filaments} \quad L_{\text{crit}} = 2.278 \mu m \]

better model - agrees with observations of 2um

4.2 fiber bundle model for filopodia

pushing the envelope

Figure 4.2.4. A working model for filopodia formation. The model describes functions of key proteins at different stages during filopodia formation. a) A subset of uncapped actin filaments are targeted for continued elongation. The barbed ends of these elongating actin filaments are converged together through the motor activity of myosin-X, leading to the initiation of a filopodium. b) When the primary filopodium begins to push the plasma membrane, ISPp53 might further facilitate plasma membrane protrusion by directly deforming or tubulating the membrane. c) The incorporation of the actin crosslinking protein fascin in the shaft of the filopodium generates a stiff actin filament bundle. At this stage, myosin-X might localize adhesion molecules to the filopodium tip by barbed-end directed movement and attach the elongating actin filament barbed ends to the plasma membrane.

Figure 4.2.5. Model for mechanisms of regulation of lamellipodial versus filopodial protrusion by filament capping.
4.2 fiber bundle model for filopodia

Figure 4.2.6. Fibroblasts cultured on a very thin sheet of silicon rubber. Attachment of the cells, followed by contraction of their cytoskeleton, has caused the rubber substratum to wrinkle.

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