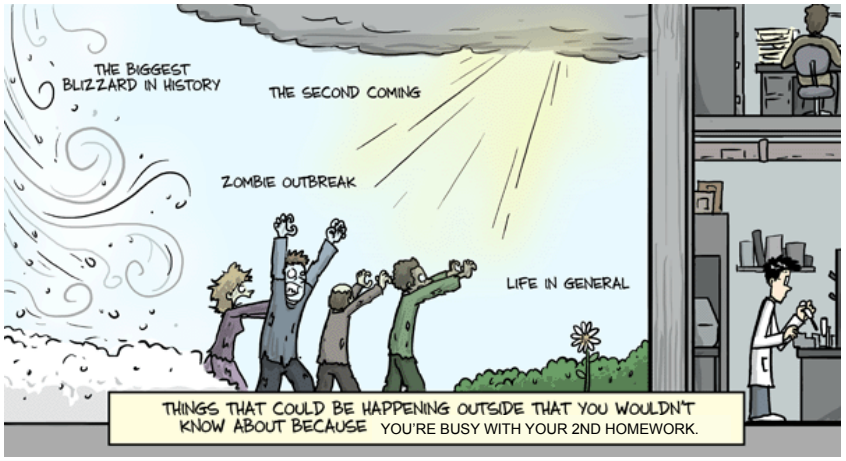
engr14 – intro to solid mechanics



tue/thu, 11:30-1:20pm, 550-200

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Day	Date	Topic	Reading	HW due
	W01	**Force Week**	**Ch 1-2**	
Tue	01/05	What's statics?	1.1-1.5	
Thu	01/07	What's a force?	2.1-2.9	
	W02	**Particle Week**	**Ch 3**	
Tue	01/12	What's a free body diagram at a particle?	3.1-3.2	
Thu	01/14	What's force equilibrium at a particle?	3.3-3.4	HW1
	W03	**Moment Week**	**Ch 4**	
Tue	01/19	What's a moment?	4.1-4.4	
Thu	01/21	What's a couple? What's distributed loading?	4.5-4.7	HW2
	W04	**Practice Week**	**Ch 1-4**	
Tue	01/26	Problems, problems, problems…	1.1-4.7	
Thu	01/28	**Midterm 1**, in class, closed book, 1 cheat sheet	1.1-4.7	
	W05	**2d Equilibrium Week**	**Ch 5**	
Tue	02/02	What's a free body diagram of a 2d system?	5.1-5.2	
Thu	02/04	What's force and moment equilibrium in 2d?	5.3-5.4	HW3

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	W06	**3d Equilibrium Week**	**Ch 5**	
Tue	02/09	What's a free body diagram of a 3d system?	5.5	
Thu	02/11	What's force and moment equilibrium in 3d?	5.6-5.7	HW4
	W07	**Structures Week**	**Ch 6**	
Tue	02/16	What's a truss?	6.1-6.3	
Thu	02/18	What's a frame?	6.6	HW5
	W08	**Internal Force Week**	**Ch 7**	
Tue	02/23	What's a shear and moment diagram?	7.1-7.3	
Thu	02/25	**Midterm 2**, all day take home, Thu-Fri 2pm	5.1-6.6	
	W09	**Friction & Center Week**	**Ch 8-9**	
Tue	03/01	What's friction?	8.1-8.2	
Thu	03/03	What's the center?	9.1-9.2	
	W10	**That's it Week**	**Ch 1-9**	
Tue	03/08	The Last Lecture	1.1-9.2	
Thu	03/10	Problems, problems, problems…	1.1-9.2	
Wed	03/16	**Final**, in class, closed book, 1 cheat sheet	1.1-9.2	

syllabus

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Homework II - Chapters 3 and 4

due Thursday, 01/21/16, 11:30am, 550-200

For late homework, you are responsible to arrange drop off with the teaching team. Once you have used up your three late days, you will no longer receive points for your homework. Here are our current office hours and contact information.

When	When	Where	Jean-claude	jcangles@stanford.edu
Mon	05:00 - 07:00pm	520-155	Jeanny	jeanny61@stanford.edu
Tue	05:00 - 07:00pm	520-155	Josh	joshuasiegel@stanford.edu
Wed	11:00 - 01:00pm	520-155	Kayla	kepowers@stanford.edu
Wed	05:00 - 07:00pm	520-155	Walter	wgoodwin@stanford.edu

For this homework, you need to be familiar with chapters 3 and 4 of your book!
All solutions for problems 1 - 4 must include a free body diagram!

homework #02

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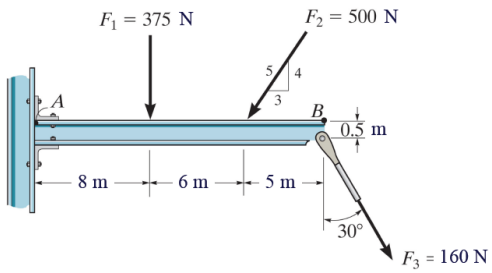
Problem 5 (Hibbeler 4-4 / 4-5)

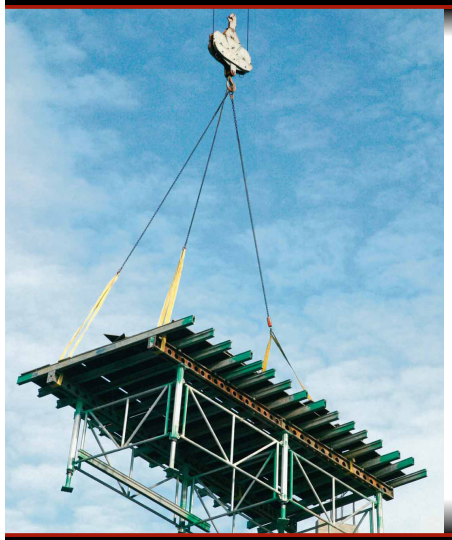


5.1 Determine the moment about point A of each of the three forces acting on the beam and determine the total moment of all three forces about A.

5.2 Determine the moment about point B of each of the three forces acting on the beam and determine the total moment of all three forces about B.

homework #02

what we've done so far...

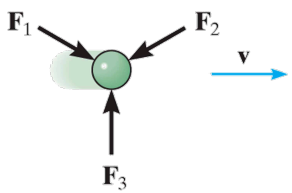


- to introduce the concept of the **free-body diagram** for a particle
- to show how to solve **particle equilibrium** problems using the equations of equilibrium
- when cables are used for hoisting loads, the must be selected so that they do not fail. today, we will show how to calculate cable forces for such cases

3. equilibrium of a particle

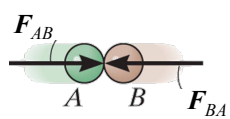
newton's three laws of motion

- first law
  equilibrium
  if $\sum \mathbf{F} = \mathbf{0}$ then $\mathbf{v}$ = const.



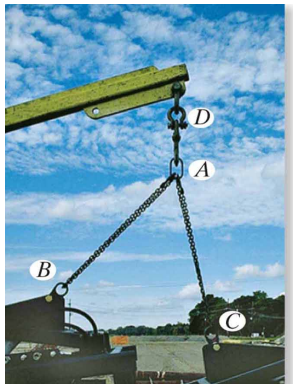
- second law
  accelerated motion
  $\mathbf{F} = \mathbf{m} \cdot \mathbf{a}$ ($\mathbf{a} = \mathbf{0}$)

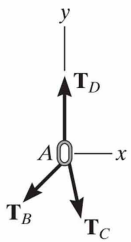
- third law
  actio = reactio
  $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$



3.1 equilibrium of a particle

procedure for drawing a FBD





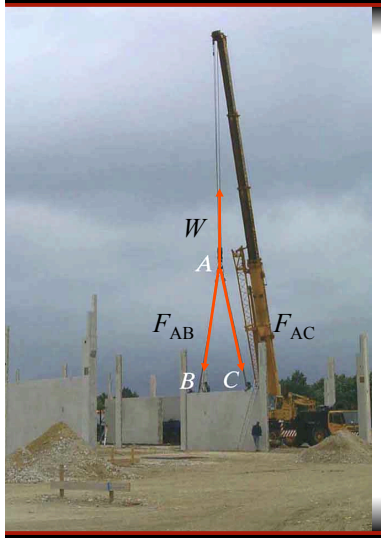
I. isolate the particle of interest - easy ;-)
here shown for particle A

II. show all forces - tricky!
3 cables, 3 tension forces
assume directions

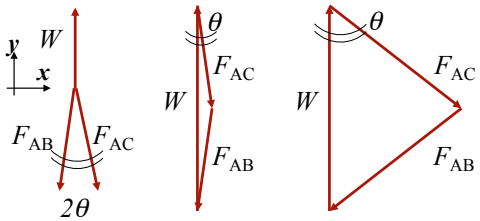
III. label each force - easy ;-)

3.2 free body diagram

conceptual problem 3-1



show that the longer the cables, the less the forces in each cable.



for longer cables, the angle θ becomes smaller, $\cos\theta$ becomes bigger, and F_{AB} and F_{AC} become smaller.

today's objectives

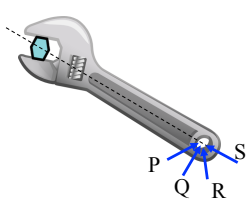
- to discuss the concept of a **moment of a force**
- to calculate the moment of a force in 2d and 3d using the scalar formulation
- to discuss the **cross product** as a tool to calculate moments
- to calculate the moment of a force in 2d and 3d using the vector formulation

concept quiz

1. a force of magnitude F can be applied in four different 2-d configurations (P,Q,R,S). select the cases resulting in the maximum and minimum torque values on the nut. (max, min).

A) (Q, P) B) (R, S)

C) (P, R) D) (Q, S)



2. if $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$, then what is the value of $\boldsymbol{M} \cdot \boldsymbol{r}$?

A) 0 B) 1

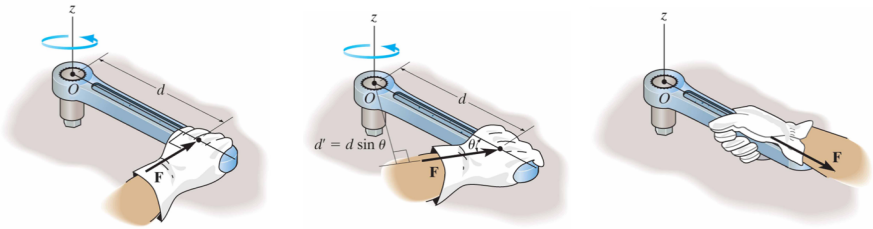
C) r^2 F D) none of the above.

engineering intuition

$M_0 = F \cdot d$ $[M] = \text{N}\cdot\text{m} = \text{lb}\cdot\text{ft}$

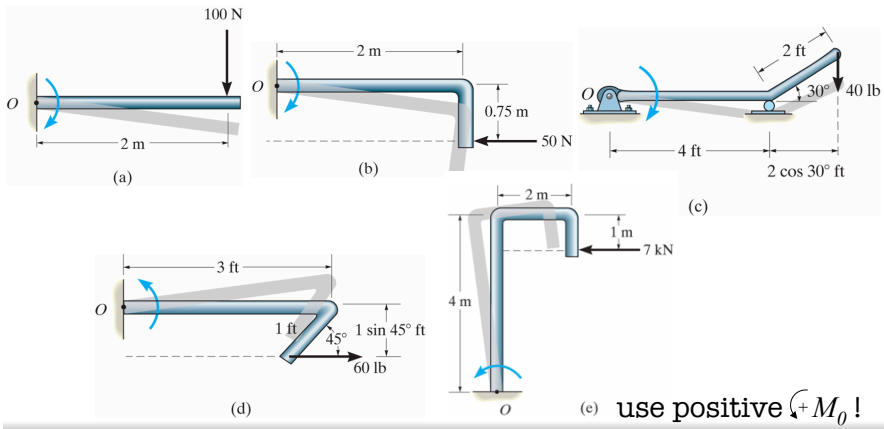
O ... fixed point
F ... force
d ... moment arm

largest moment smaller moment no moment



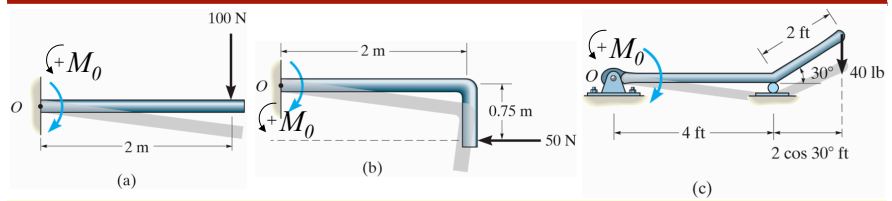
example 4.1

determine the moment M_O of the force F about point O



use positive $\circlearrowleft + M_O$!

example 4.1



For each case illustrated in Fig. 4–4, determine the moment of the force about point O.

use positive $\circlearrowleft + M_O$!

SOLUTION *(SCALAR ANALYSIS)*

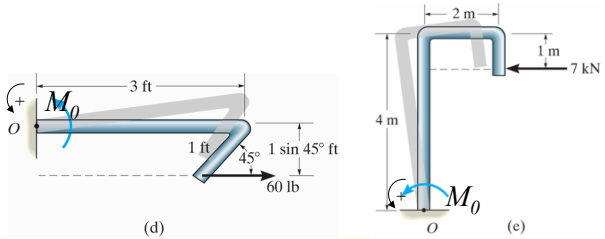
The line of action of each force is extended as a dashed line in order to establish the moment arm d. Also illustrated is the tendency of rotation of the member as caused by the force. Furthermore, the orbit of the force about O is shown as a colored curl. Thus,

Fig. 4–4*a* $\circlearrowleft + M_O = -(100\text{ N})(2\text{ m}) = -200\text{ N}\cdot\text{m}$ *Ans.*

Fig. 4–4*b* $\circlearrowleft + M_O = -(50\text{ N})(0.75\text{ m}) = -37.5\text{ N}\cdot\text{m}$ *Ans.*

Fig. 4–4*c* $\circlearrowleft + M_O = -(40\text{ lb})(4\text{ ft} + 2\cos 30°\text{ ft}) = -229\text{ lb}\cdot\text{ft}$ *Ans.*

example 4.1



For each case illustrated in Fig. 4–4, determine the moment of the force about point O.

use positive $\circlearrowleft + M_O$!

SOLUTION *(SCALAR ANALYSIS)*

The line of action of each force is extended as a dashed line in order to establish the moment arm d. Also illustrated is the tendency of rotation of the member as caused by the force. Furthermore, the orbit of the force about O is shown as a colored curl. Thus,

Fig. 4–4*d* $\circlearrowleft + M_O = (60\text{ lb})(1\sin 45°\text{ ft}) = 42.4\text{ lb}\cdot\text{ft}\ \circlearrowright$ *Ans.*

Fig. 4–4*e* $\circlearrowleft + M_O = (7\text{ kN})(4\text{ m} - 1\text{ m}) = 21.0\text{ kN}\cdot\text{m}\ \circlearrowright$ *Ans.*

example 4.2

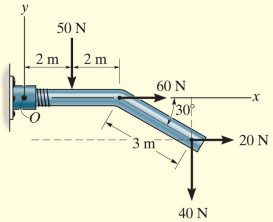


Fig. 4–5

Determine the resultant moment of the four forces acting on the rod shown in Fig. 4–5 about point O.

SOLUTION

Assuming that positive moments act in the $+\mathbf{k}$ direction, i.e., counterclockwise, we have

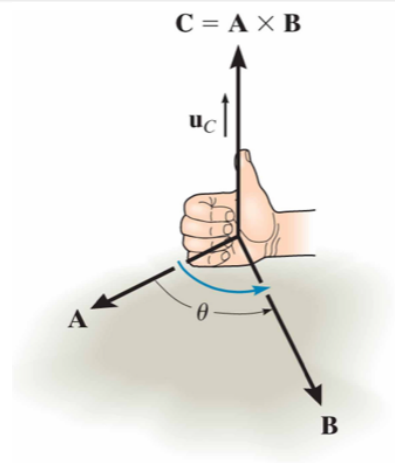
$$\circlearrowleft + (M_R)_O = \Sigma Fd;$$

$$(M_R)_O = -50\text{ N}(2\text{ m}) + 60\text{ N}(0) + 20\text{ N}(3\sin 30°\text{ m}) - 40\text{ N}(4\text{ m} + 3\cos 30°\text{ m})$$

$$(M_R)_O = -334\text{ N}\cdot\text{m} = 334\text{ N}\cdot\text{m}\ \circlearrowright$$ *Ans.*

For this calculation, note how the moment-arm distances for the 20-N and 40-N forces are established from the extended (dashed) lines of action of each of these forces.

right-handed system



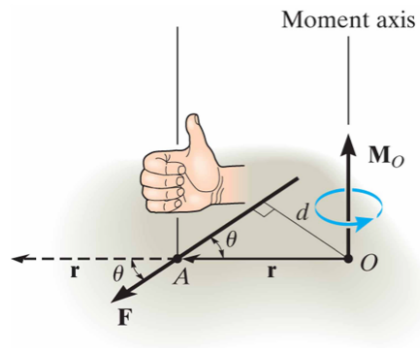
$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = A \cdot B \cdot \sin\theta \; \boldsymbol{u}_C$$

- $\boldsymbol{C}$ is a vector
- $\boldsymbol{C}$ is orthogonal to $\boldsymbol{A}$ and $\boldsymbol{B}$
- C is the area enclosed by $\boldsymbol{A}$ and $\boldsymbol{B}$, i.e., $A \cdot B \cdot \sin\theta$

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = \det \begin{bmatrix} A_x & B_x & \boldsymbol{x} \\ A_y & B_y & \boldsymbol{y} \\ A_z & B_z & \boldsymbol{z} \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} A_y B_z - A_z B_y \\ A_z B_x - A_x B_z \\ A_x B_y - A_y B_x \end{bmatrix} \quad \circlearrowleft +$$

right-handed system



$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F} = r \cdot F \cdot \sin\theta \; \boldsymbol{u}_M$$

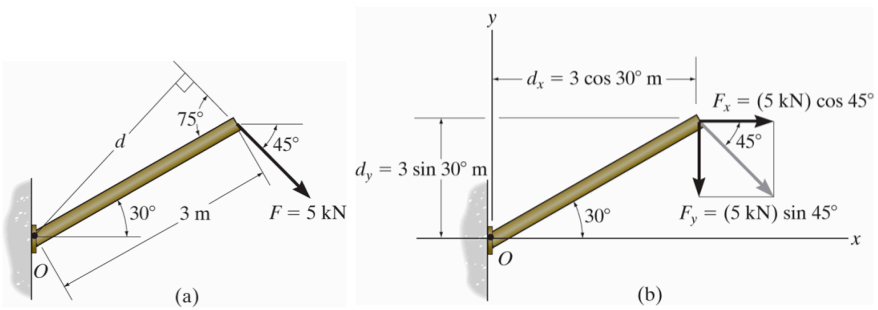
- $\boldsymbol{M}_O$ is a vector
- $\boldsymbol{M}_O$ is orthogonal to $\boldsymbol{r}$ and $\boldsymbol{F}$
- M_O is the area enclosed by $\boldsymbol{r}$ and $\boldsymbol{F}$, i.e., $r \cdot F \cdot \sin\theta$

$$\circlearrowleft + \; \boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F} = \det \begin{bmatrix} r_x & F_x & \boldsymbol{x} \\ r_y & F_y & \boldsymbol{y} \\ r_z & F_z & \boldsymbol{z} \end{bmatrix}$$

$$\circlearrowleft + \; \boldsymbol{M}_O = \begin{bmatrix} r_y F_z - r_z F_y \\ r_z F_x - r_x F_z \\ r_x F_y - r_y F_x \end{bmatrix}$$

example 4.5

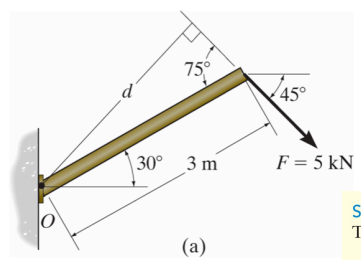
determine the moment $\circlearrowleft + M_0$ of the force F about point O



use two different approaches and compare the results!

example 4.5

determine the moment $\circlearrowleft + M_0$ of the force F about point O



SOLUTION I

The moment arm d in Fig. 4–18a can be found from trigonometry.

$$d = (3\text{ m}) \sin 75° = 2.898\text{ m}$$

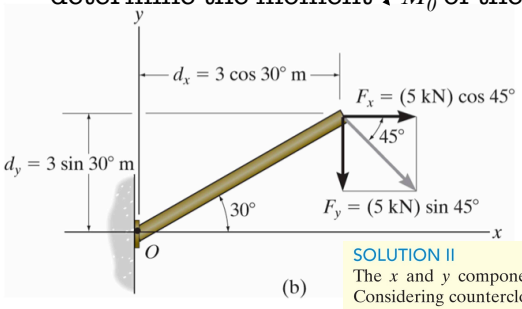
Thus,

$$M_O = Fd = (5\text{ kN})(2.898\text{ m}) = 14.5\text{ kN}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

Since the force tends to rotate or orbit clockwise about point O, the moment is directed into the page.

example 4.5

determine the moment $\circlearrowleft + M_0$ of the force F about point O



SOLUTION II

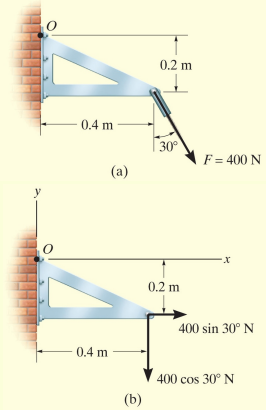
The x and y components of the force are indicated in Fig. 4–18b. Considering counterclockwise moments as positive, and applying the principle of moments, we have

$$\circlearrowleft + M_O = -F_x d_y - F_y d_x$$
$$= -(5 \cos 45° \text{ kN})(3 \sin 30° \text{ m}) - (5 \sin 45° \text{ kN})(3 \cos 30° \text{ m})$$
$$= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \circlearrowright$$

Ans.

example 4.6

Force **F** acts at the end of the angle bracket in Fig. 4–19a. Determine the moment of the force about point O.



SOLUTION I *(SCALAR ANALYSIS)*

The force is resolved into its x and y components, Fig. 4–19b, then

$$\circlearrowleft + M_O = 400 \sin 30° \text{ N}(0.2 \text{ m}) - 400 \cos 30° \text{ N}(0.4 \text{ m})$$
$$= -98.6 \text{ N} \cdot \text{m} = 98.6 \text{ N} \cdot \text{m} \circlearrowright$$

or

SOLUTION II *(VECTOR ANALYSIS)*

Using a Cartesian vector approach, the force and position vectors, Fig. 4–19c, are

$$\mathbf{r} = \{0.4\mathbf{i} - 0.2\mathbf{j}\} \text{ m}$$
$$\mathbf{F} = \{400 \sin 30° \mathbf{i} - 400 \cos 30° \mathbf{j}\} \text{ N}$$
$$= \{200.0\mathbf{i} - 346.4\mathbf{j}\} \text{ N}$$