office hours: what e14 students ask vs. what they are really asking

**TUE/THU, 11:30-1:20PM, 550-200**

**Syllabus**

<table>
<thead>
<tr>
<th>Day</th>
<th>Date</th>
<th>Topic</th>
<th>Reading</th>
<th>HW due</th>
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<tr>
<td>W01</td>
<td></td>
<td><strong>Force Week</strong></td>
<td>Ch 1-2</td>
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<tr>
<td>Tue</td>
<td>01/05</td>
<td>What's statics?</td>
<td>1.1-1.5</td>
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<tr>
<td>Thu</td>
<td>01/07</td>
<td>What's a force?</td>
<td>2.1-2.9</td>
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<td>W02</td>
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<td><strong>Particle Week</strong></td>
<td>Ch 3</td>
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<tr>
<td>Tue</td>
<td>01/11</td>
<td>What's a free body diagram at a particle?</td>
<td>3.1-3.2</td>
<td>HW1</td>
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<tr>
<td>Thu</td>
<td>01/14</td>
<td>What's a force equilibrium at a particle?</td>
<td>3.3-3.4</td>
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<tr>
<td>W03</td>
<td></td>
<td><strong>Moment Week</strong></td>
<td>Ch 4</td>
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<tr>
<td>Tue</td>
<td>01/19</td>
<td>What's a moment?</td>
<td>4.1-4.4</td>
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<tr>
<td>Thu</td>
<td>01/21</td>
<td>What's a couple? What's distributed loading?</td>
<td>4.5-4.7</td>
<td>HW2</td>
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<td>W04</td>
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<td><strong>Practice Week</strong></td>
<td>Ch 1-4</td>
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<tr>
<td>Tue</td>
<td>01/26</td>
<td>Problems, problems, problems...</td>
<td>1.1-1.7</td>
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<tr>
<td>Thu</td>
<td>01/28</td>
<td><strong>Midterm 1</strong>, in class, closed book, 1 cheat sheet</td>
<td>1.1-1.7</td>
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<tr>
<td>W05</td>
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<td><strong>2d Equilibrium Week</strong></td>
<td>Ch 5</td>
<td></td>
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<tr>
<td>Tue</td>
<td>02/02</td>
<td>What's a free body diagram of a 2d system?</td>
<td>5.1-5.2</td>
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<tr>
<td>Thu</td>
<td>02/04</td>
<td>What's force and moment equilibrium in 2d?</td>
<td>5.3-5.4</td>
<td>HW3</td>
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</table>

**Textbook / e-textbook**


today’s objectives

• to show how to add 3d forces and resolve forces into components using the parallelogram law
• to express 3d force and position in Cartesian vector form and explain how to determine the vector’s magnitude and direction
• to introduce the 3d dot product in order to determine the angle between two vectors or the projection of one vector onto another

2. forces - wrap up, 3d

2.2 vector operations

vector addition $R = A + B = B + A$

vector subtraction $R = A - B = -B + A$

2.3 vector addition and forces

I. find the resultant force

$F_R = F_1 + F_2$

II. find the components of a force

$F_R = F_u + F_v$

2.3 vector addition and forces
2.3 vector addition and forces

Ia. find the resultant of several forces

\[ F_R = F_1 + F_2 + F_3 \]

double parallelogram

2.4 addition of forces (2d)

problem 2.35 - 12th edition

the contact point between the femur and tibia bones of the leg is at A. If a **vertical force of 175 lb** is applied at this point, determine the components along the x and y axes. Note that the y-component represents the normal force on the load bearing region of the bones. Both the x- and y-components of this force cause synovial fluid to be squeezed out of the bearing space.

\[ F_x = 175 \cdot \frac{5}{13} \text{ lb} = 67.3 \text{ lb} \]
\[ F_y = 175 \cdot \frac{12}{13} \text{ lb} = 161.5 \text{ lb} \]

bone density changes with force

Computational modeling of bone density profiles in response to gait: A subject-specific approach

Henry Pang², Abhishek P. Shiwalkar³, Chris M. Madormo⁴, Rebecca E. Taylor⁴, Thomas P. Andriacchi¹, Ellen Kuhl¹²⁴

class project me337

mechanics of growth

why this is important...

<table>
<thead>
<tr>
<th>trial</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>max knee force [N]</td>
<td>572</td>
<td>549</td>
<td>579</td>
<td>560</td>
</tr>
<tr>
<td>max knee force [% BW]</td>
<td>94.9</td>
<td>94.4</td>
<td>99.4</td>
<td>96.2</td>
</tr>
</tbody>
</table>
bone density changes with force

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why this is important...

force addition in cartesian coordinates

$\mathbf{F} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$

- we can easily add and subtract forces using a Cartesian coordinate system
  
  $F_R = \begin{bmatrix} F_{Rx} \\ F_{Ry} \\ F_{Rz} \end{bmatrix}$
  
  $F_{Rx} = \sum F_x$
  
  $F_{Ry} = \sum F_y$
  
  $F_{Rz} = \sum F_z$

- we can determine the magnitude of a force using Pythagoras (3d)
  
  $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2}$

- we can determine the direction of a force using trigonometry
  
  $\theta = \tan^{-1}[F_{Ry} / F_{Rx}]$

$F_{Rx} = |\mathbf{F}| \cos \theta$

$F_{Ry} = |\mathbf{F}| \sin \theta$

2.5 addition of forces (3d)

the resultant force acting on the bow of the ship can be determined by first representing each rope force as a cartesian vector and then summing the components $F_x, F_y, F_z$
**example 2.8**

express force as Cartesian vector
- calculate the angle \( \alpha \)

\[
\begin{align*}
F &= 200 \text{ N} \\
\alpha &= 45^\circ \quad \text{and} \quad 60^\circ
\end{align*}
\]

- calculate the components \( F_x, F_y, F_z \)
- control \( F \)

**2.5 addition of cartesian vectors (3d)**

**example**

express force as Cartesian vector
- calculate the angle \( \alpha \)

\[
\begin{align*}
cos^2 \alpha + cos^2 \beta + cos^2 \gamma &= 1 \\
cos^2 \alpha + cos^2 60^\circ + cos^2 45^\circ &= 1 \\
\cos \alpha &= \sqrt{[1 - 0.5^2 - 0.707^2]} = 0.5 \\
\alpha &= 60^\circ
\end{align*}
\]

- calculate the components \( F_x, F_y, F_z \)
- control \( F \)

\[
F = \sqrt{[100^2 + 100^2 + 141^2]} \text{ N} = 200 \text{ N}
\]

**2.6 addition of cartesian vectors (3d)**

**position vectors define the direction**

establish a cartesian coordinate system and determine the coordinates of points \( A \) and \( B \). determine the position vector \( r = r_B - r_A \) along the cable. its magnitude \( r \) determines the length of the cable, its unit vector \( u = r / r \) defines the direction.

**2.7 position vectors (3d)**

**force projection onto a given direction**

the force \( F \) along the chain can be represented as a cartesian vector by calculating the position vector \( r \) along the chain. then we determine the unit vector \( u = r / r \) and the magnitude \( F \), such that \( F = Fu \).

**2.8 forces along a line**
I. angle between two force vectors

We can determine the angle $\theta$ between the rope and the beam by formulating the unit vectors along the beam $u_b$ and the rope $u_r$ and then using the dot product $\cos \theta = u_b \cdot u_r$.

II. projection of force along a direction

We can determine the projection of the cable force along the beam $F_b$ by first finding the unit vector $u_b$ along the beam and then taking the dot product $F_b = F \cdot u_b$.

Example 2.15

The frame shown in Fig. 2.43a is subjected to a horizontal force $F = [0, 300, 0]$ N. Determine the magnitudes of the components of this force parallel and perpendicular to member $AB$.

Example 2.15

The magnitude of the component of $F$ along $AB$ is equal to the dot product of $F$ and the unit vector $u_b$, which defines the direction of $AB$, Fig. 2.43b. Since

$$u_b = \frac{2i + 6j + 3k}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = 0.2861i + 0.857j + 0.429k$$

then

$$F_{AB} = F \cos \theta = F \cdot u_b = (300i)(0.2861) + (300)(0.857j) + (0)(0.429k)$$

$$= (85.86i) + (257.1N)$$

Ans.
3. equilibrium of a particle

- to introduce the concept of the free-body diagram for a particle
- to show how to solve particle equilibrium problems using the equations of equilibrium
- when cables are used for hoisting loads, they must be selected so that they do not fail. today, we will show how to calculate cable forces for such cases.

3.1 equilibrium condition of a particle

- Newton’s three laws of motion
- equilibrium if $\sum \mathbf{F} = 0$ then $v = \text{const.}$
- third law: $F_{\text{act}} = -F_{\text{react}}$

3.2 free body diagram

- procedure for drawing a FBD
  1. isolate the particle of interest - easy ;)
  2. show all forces - tricky!
  3. label each force - easy ;)

- assumptions - cables and pulleys
  - cables, unless otherwise stated, have a negligible weight and cannot stretch. they can only support tension along their axis.
  - pulleys, for now, we assume that pulleys are frictionless, i.e., the tension force of a cable that passes over a pulley may change its direction but not its magnitude.
  - unless otherwise stated, we will assume that all cables have a negligible weight and cannot stretch. they can only support tension along their axis.

- today’s objectives
  - to introduce the concept of the free-body diagram for a particle
### example 3.1

**draw the free body diagrams of the sphere E, the cord CE, and the knot C.**

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**example 3.1**

- **Cord CE.** When the cord CE is isolated from its surroundings, its free-body diagram shows only two forces acting on it, namely, the force of the sphere and the force of the knot, Fig. 3–3c. Notice that $F_{CE}$ shown here is equal but opposite to that shown in Fig. 3–3b, a consequence of Newton's third law of action–reaction. Also, $F_{CE}$ and $F_{CA}$ pull on the cord and keep it in tension so that it doesn't collapse. For equilibrium, $F_{CE} = F_{CA}$.

- **Knot.** The knob at C is subjected to three forces, Figs. 3–3d. They are caused by the cords $CE$ and $CD$ and the spring $CD$. As required, the free-body diagram shows all these forces labeled with their magnitudes and directions. It is important to recognize that the weight of the sphere does not directly act on the knot. Instead, the cord CE subjects the knot to this force.

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**procedure for drawing a FBD**

1. *isolate the particle of interest - easy ;-)*
2. *show all forces - tricky!*
3. *label each force - easy ;-)*

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**example 3.1**

The sphere in Fig. 3–3a has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord CE, and the knot at C.

**SOLUTION**

**Sphere.** Once the supports are removed, we can see that there are four forces acting on the sphere, namely, its weight, 6 kg ($9.81 \text{ m/s}^2 = 58.9 \text{ N}$), the force of cord $CE$, and the two normal forces caused by the smooth inclined planes. The free-body diagram is shown in Fig. 3–3b.
3.2 free body diagram

procedure for drawing a FBD

... try this @home

ΣF_y = 0: S_1 = S_2 = 1/sinα F/2

equilibrium sponsored by eichhof

3.2 free body diagram

don't try this @home

W = 0.8kN, α=20°, S = 12kN

W = 12kN, α=90°, S = 12kN

3.2 free body diagram