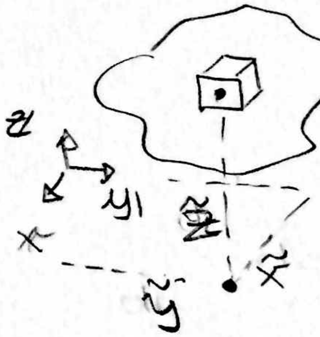
ENGR 14 - STATICS XVI Thu 03/03/16

FRICTION & CENTER WEEK

WHAT'S THE CENTER? (CHAPTERS 9.1 & 9.2)

9.1 CENTER ... OF GRAVITY, MASS, CENTROID



1D CENTER OF GRAVITY

$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int dW};\quad \bar{y} = \frac{\int \tilde{y}\,dW}{\int dW};\quad \bar{z} = \frac{\int \tilde{z}\,dW}{\int dW}$$

1D CENTER OF MASS

$dW = g\,dm$ (WEIGHT = g * MASS) CANCEL! $\frac{g}{g}$

$$\bar{x} = \frac{\int \tilde{x}\,dm}{\int dm};\quad \bar{y} = \frac{\int \tilde{y}\,dm}{\int dm};\quad \bar{z} = \frac{\int \tilde{z}\,dm}{\int dm}$$

1D CENTROID / GEOMETRIC CENTER

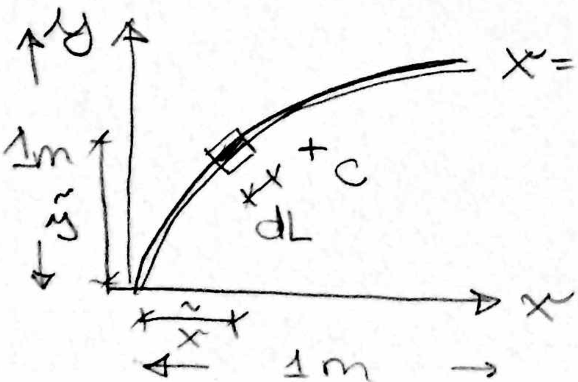
$dm = \rho\,dV$ (MASS = DENSITY * VOLUME) CANCEL $\frac{\rho}{\rho}$

$$\bar{x} = \frac{\int \tilde{x}\,dV}{\int dV};\quad \bar{y} = \frac{\int \tilde{y}\,dV}{\int dV};\quad \bar{z} = \frac{\int \tilde{z}\,dV}{\int dV}$$

⋮

CENTROID OF AN AREA / A LINE

EXAMPLE 9.1 CENTROID OF A ROD BENT TO AN ARCH?



PYTHAGORES

$$dL = \sqrt{(dx)^2 + (dy)^2}$$

$$= \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\; dy$$

ID WITH $x = y^2$ $\quad \frac{dx}{dy} = \frac{d(y^2)}{dy} = 2y$

$$dL = \sqrt{(2y)^2 + 1}\, dy$$

LINE ELEMENT WITH CENTROID @ $\tilde{x} = x$ AND $\tilde{y} = y$

ID INTO INTEGRAL EQUATION:

$$\bar{x} = \frac{\int_0^L \tilde{x}\, dL}{\int_0^L dL} = \frac{\int_0^{1m} x \sqrt{4y^2+1}\, dy}{\int_0^{1m} \sqrt{4y^2+1}\, dy} \quad (x = y^2)$$

$$= \frac{\int_0^{1m} y^2 \sqrt{4y^2+1}\, dy}{\int_0^{1m} \sqrt{4y^2+1}\, dy} \overset{\text{SEE APPENDIX}}{=} \frac{0.6063\,m}{0.1479}$$

$$\boxed{\bar{x} = 0.410\,m}$$

$$\bar{y} = \frac{\int_0^L \tilde{y}\, dL}{\int_0^L dL} = \frac{\int_0^{1m} y \sqrt{4y^2+1}\, dy}{\int_0^{1m} \sqrt{4y^2+1}\, dy} = \frac{0.8484\,m}{0.1479}$$

$$\boxed{\bar{y} = 0.574\,m}$$

9.2 COMPOSITE BODIES

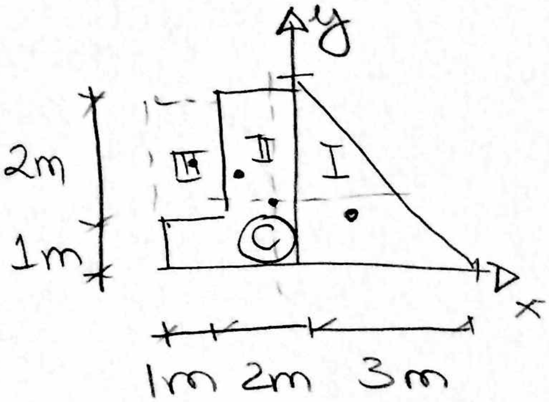
(WITH REGULAR STRUCTURE)

$$\boxed{\bar{x} = \frac{\sum \tilde{x} W}{\sum W} \; ; \; \bar{y} = \frac{\sum \tilde{y} W}{\sum W} \; ; \; \bar{z} = \frac{\sum \tilde{z} W}{\sum W}}$$

... DISCRETE, FINITE NUMBER OF WEIGHTS W WITH A KNOWN CENTER

HOLES ... HAVE A NEGATIVE WEIGHT & SIZE!

EXAMPLE 9.10: CENTROID OF A PLATE



▷ Hint: Triangle 1/3, 2/3

▷ assume: III is negative and II is a 3x3 square

TABLE

SEGMENT	AREA [m²]	$\tilde{x}$ [m]	$\tilde{y}$ [m]	$A\tilde{x}$ [m³]	$A\tilde{y}$ [m³]
I	$\frac{1}{2}\cdot 3\cdot 3 = 4.5$	+1	+1	+4.5	+4.5
II	$3\cdot 3 = 9$	−1.5	+1.5	−13.5	+13.5
III	$-1\cdot 2 = -2$	−2.5	+2	+5	−4
Σ	11.5	/	/	−4	+14

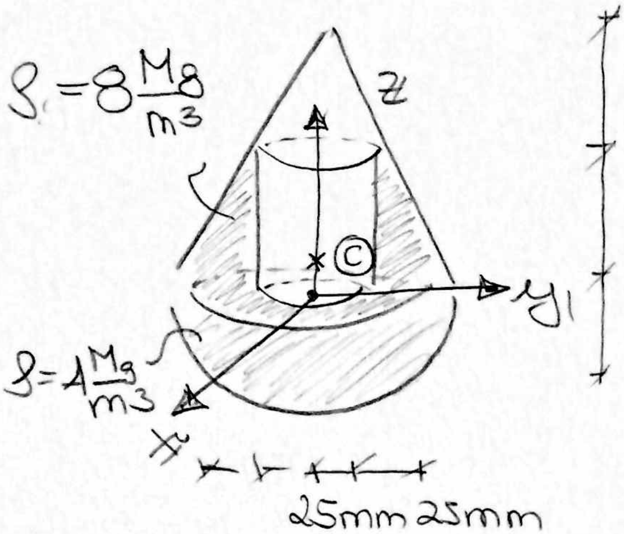
$$\bar{x} = \frac{\sum A\tilde{x}}{\sum_i A} = \frac{-4\,m^3}{11.5\,m^2} = -0.348\,m$$

$$\bar{y} = \frac{\sum A\tilde{y}}{\sum_i A} = \frac{+14}{11.5} = +1.22\,m$$

▷ CHECK in figure & comment! $C(\bar{x}, \bar{y})$

EXAMPLE 9.11 : CENTER OF MASS

$Mg = 1{,}000{,}000g = 1{,}000kg$



DECOMPOSE!

SYMMETRY: $\bar{x} = 0,\ \bar{y} = 0$

$\bar{C} = (0, 0, 14.6)$

TABLE

SEGMENT	mass m [kg]	$\tilde{z}$ [mm]	$\tilde{z}m$ [kg·mm]
I $V = \frac{1}{3}\pi \cdot r^2 \cdot h$, $\bar{z} = \frac{1}{4}h$	$\frac{1}{3} \cdot \pi \cdot 50^2 \cdot 200$ [+8] $\cdot 10^{-6}$ = 4.189	$\frac{1}{4} \cdot 200$ = +50	+ 209.440
II $V = \frac{2}{3}\pi r^3$, $\bar{z} = \frac{3}{8} \cdot h$	$\frac{2}{3} \cdot \pi \cdot 50^3$ [+4] $\cdot 10^{-6}$ = 1.047	$\frac{3}{8} \cdot [-50]$ = −18.75	− 19.635
III $V = \frac{1}{3}\pi r^2 \cdot h$, $\bar{z} = \frac{1}{4}h$	$\frac{1}{3} \cdot \pi \cdot 25^2 \cdot 100$ [−8] $\cdot 10^{-6}$ = −0.524	$100 + \frac{1}{4}100$ = 125	− 65.450
IV $V = \pi r^2 h$, $\bar{z} = h/2$	$\pi \cdot 25^2 \cdot 50$ [−8] $\cdot 10^{-6}$ = −1.571	$\frac{1}{2} \cdot 100$ = 50	− 78.540
Σ	3.142 kg	/	45.815 kg mm

$$\bar{z} = \frac{\Sigma \tilde{z} \cdot m}{\Sigma m} = \frac{45.815\,kg\,mm}{3.142\,kg} = \underline{14.6mm}$$ ⇝ COMMENT!