

ENGR 14 - STATICS VI THU, 01/21/16

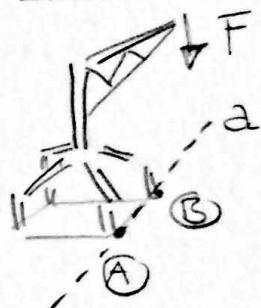
MOMENT WEEK

(CHAPTERS 4.5-4.7)

- WHAT'S A COUPLE?
- WHAT'S DISTRIBUTED loading?)

4.5 MOMENT ABOUT AN AXIS

- Problem: stability of a crane



moment about axis a / A-B

SCALAR FORMULATION

$$M_a = F d_a$$

F ... force

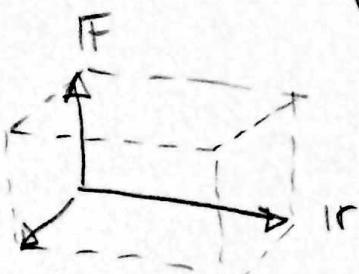
d_a ... distance from axis a
to line of action of force F

VECTOR FORMULATION

$$M_a = \mathbf{u}_{a\perp} \cdot (\mathbf{r} \times \mathbf{F}) = \det \begin{bmatrix} u_{ax} & r_x & F_x \\ u_{ay} & r_y & F_y \\ u_{az} & r_z & F_z \end{bmatrix}$$

... M_a is a scalar

volume of parallel epiped



u_{a\perp} ... unit vector along axis-a-

r ... position vector (to point

F ... force

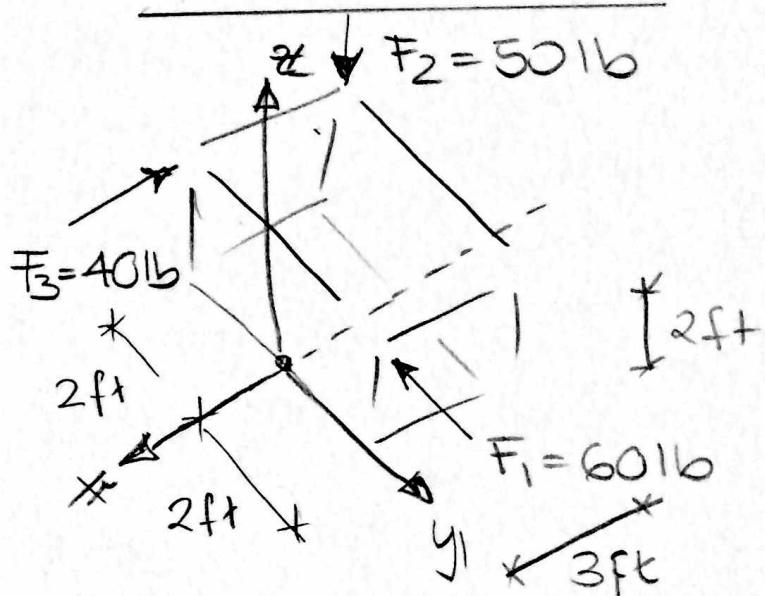
on line of
action)

u_{a\perp}

= READ... BUT FOCUS ON EXAMPLES! =

-II-

EXAMPLE 4.7



$$\text{clockwise } M_x = +F_1 \cdot d_1 + F_2 \cdot d_2 \quad \dots F_3 \text{ is parallel to } x$$

$$= +60 \text{ lb} \cdot 2 \text{ ft} + 50 \text{ lb} \cdot 2 \text{ ft} = [120 + 100] \text{ lb}$$

$$\boxed{M_x = 220 \text{ lb} \cdot \text{ft}}$$

$$\text{counter-clockwise } M_y = -F_2 \cdot d_2 - F_3 \cdot d_3 \quad \dots F_1 \text{ is parallel to } y$$

$$M_y = -50 \text{ lb} \cdot 3 \text{ ft} - 40 \text{ lb} \cdot 2 \text{ ft} = [-150 - 80] \text{ lb} \cdot \text{ft}$$

$$\boxed{M_y = -230 \text{ lb ft}}$$

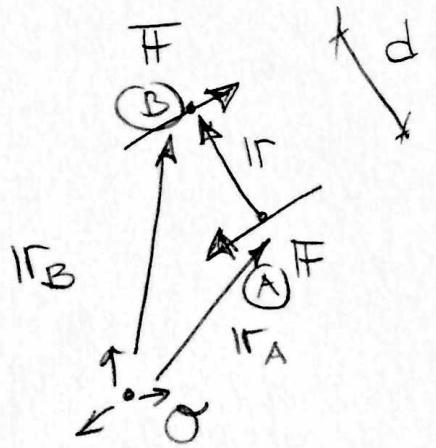
$$\text{upward } M_z = -F_3 \cdot d_3 \dots F_1 \text{ passes through } z \\ F_2 \text{ is parallel to } z$$

$$= -40 \text{ lb} \cdot 2 \text{ ft}$$

$$\boxed{M_z = -80 \text{ lb} \cdot \text{ft}}$$

4.6 MOMENTS OF A COUPLE

- "couple" .. two parallel forces with similar magnitude, opposite direction @ a finite distance



SCALAR FORMULATION

$$| M = F \cdot d | \dots \text{magnitude}$$

VECTOR FORMULATION

$$| \overrightarrow{M} = \overrightarrow{r} \times \overrightarrow{F} | \dots \text{magnitude \& direction}$$

$$\text{in 3d: } \overrightarrow{M} = \overrightarrow{r}_B \times \overrightarrow{F} + \overrightarrow{r}_A \times (-\overrightarrow{F})$$

$$= (\overrightarrow{r}_B - \overrightarrow{r}_A) \times \overrightarrow{F} = \overrightarrow{r} \times \overrightarrow{F}$$

Couple moment is a "free vector", it acts on any point, only depends on distance between forces \overrightarrow{r} , not on \overrightarrow{r}_A & \overrightarrow{r}_B , thus not on $\overrightarrow{0}$.

4.7 SIMPLIFICATION

resultant force & moment in 3d

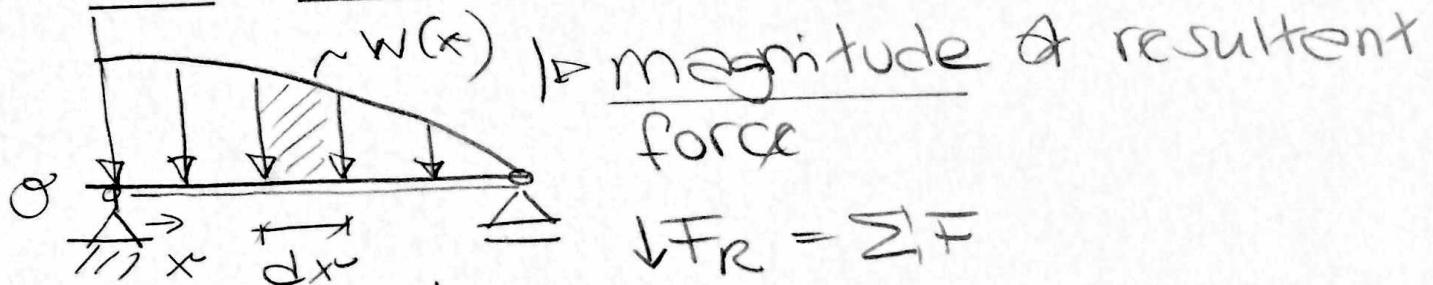
$$| \overrightarrow{F_R} = \sum \overrightarrow{F} \quad \text{and} \quad \overrightarrow{M_{R_O}} = \sum_{m_O} \overrightarrow{M} + \sum_{m_C} \overrightarrow{M} \} \begin{matrix} 3+3 \\ \text{moments} \quad \text{moments} \\ \text{of all forces} \quad \text{of all couples} \end{matrix} \} \text{eqns.}$$

and in 2d

$$| F_x = \sum F_x \quad F_y = \sum F_y \quad M_{R_O} = \sum M_O + \sum M \} \begin{matrix} 2+1 \\ \text{eqns.} \end{matrix}$$

4.8 SPECIAL FORCE SYSTEMS (read @ home)

4.9 DISTRIBUTED LOADING



↗ magnitude & resultant force

$$\downarrow F_R = \sum F$$

$$F_R = \int_L w(x) dx \quad \begin{matrix} w(x) \\ \text{AREA UNDER THE CURVE} \end{matrix}$$

↗ Location of resultant force

$$G M_{R.O.} = \sum M_O$$

$$G \quad \begin{matrix} + \\ - \bar{x} F_R = - \int_L x \cdot w(x) dx \end{matrix} \dots \text{substitute } F_R \\ \begin{matrix} + \\ \text{Position Force} \end{matrix}$$

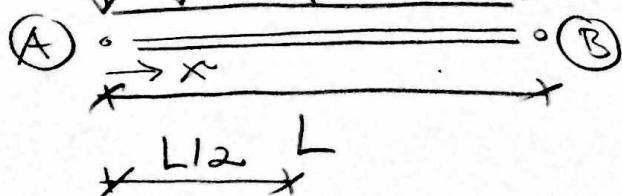
$$\bar{x} = \frac{\int_L x \cdot w(x) dx}{\int_L w(x) dx}$$

geometric center of $w(x)$ area

for simple geometries: check table
of last page of your book!

- EXAMPLE □

$$\downarrow \downarrow \downarrow \downarrow \downarrow \rightarrow w(x) = w = \text{const}$$



-V-

1) Magnitude / area

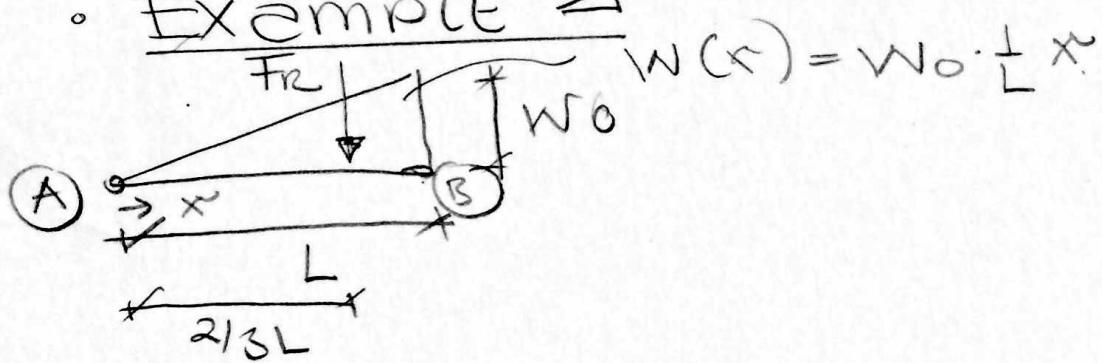
$$F_R = \int_0^L w(x) dx = \int_0^L w dx = w \cdot x \Big|_{x=0}^L = wL - wo = \underline{\underline{wL}}$$

1) LOCATION / CENTER OF GRAVITY (intuitive)

$$\bar{x} = \frac{\int_0^L x w dx}{\int_0^L w dx} = \frac{\frac{1}{2} x^2 w \Big|_{x=0}^L}{w x \Big|_{x=0}^L} = \frac{\frac{1}{2} L^2 w}{w L} = \underline{\underline{\frac{1}{2} L}}$$

(draw into figure!)

• EXAMPLE



1) Magnitude / AREA

$$F_R = \int_0^L w(x) dx = \int_0^L \frac{1}{2} w_0 x dx = \frac{1}{2} \left[\frac{1}{2} w_0 x^2 \right]_{x=0}^L = \underline{\underline{\frac{1}{2} wL}}$$

1) LOCATION / CENTER OF GRAVITY

$$\begin{aligned} \bar{x} &= \frac{\int_0^L x w(x) dx}{\int_0^L w(x) dx} = \frac{\int_0^L \frac{1}{2} w_0 x^2 dx}{\int_0^L \frac{1}{2} w_0 x dx} \\ &= \frac{\left[\frac{1}{2} \frac{w_0}{3} x^3 \right]_{x=0}^L}{\left[\frac{1}{2} w_0 \frac{1}{2} x^2 \right]_{x=0}^L} = \frac{\frac{1}{2} w_0 L^2}{\frac{1}{2} w_0 L} = \underline{\underline{\frac{2}{3} L}} \end{aligned}$$

→ IMPORTANT!