mon/wed/fri, 12:50-2:05pm, 370-370

• **regular final**
  mon, 06/06/11, 8:30-10:30am, cubaud
  regular final (w/extra time and extra room)
  mon, 06/05/11, 8:30-11:30pm, durand 203, 180 min

• **makeup final**
  sun, 05/05/11, 3:00-5:00pm, 530-127, 120 min
  makeup final (w/extra time)
  sun, 05/05/11, 3:00-6:00pm, 530-127, 180 min

• closed book, closed notes, one page cheat sheet
• bring your **calculators!**
• **last minute office hours**
  sun, 05/05/11, 3:00-6:00, @ (but not in) 530-127

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**ENGR14 - Applied Mechanics - Statics**

**Final Exam - Spring 2011**

**June 06, 2011**

**Instructions**

A. This is an closed note / closed book exam.
B. You are allowed to bring a one page hand written cheat sheet and a calculator.
C. Always start with a Free Body Diagram. Demonstrate the derivation of your results.
D. Each of the six problems is worth 20 points.
E. Only five out of six problems will be graded for 100 points total.
F. You have to identify five out of six problems, which you would like to be graded.
G. You have 120 minutes to complete the exam.
**problem 01**

- to develop equations of equilibrium for a rigid body
- to introduce the concept of a free body diagram for a rigid body
- to show how to solve rigid body equilibrium problems

**example 5.15**

The homogeneous plate shown in Fig. 5-28a has a mass of 100 kg and is subjected to a force and couple moment along its edges. If it is supported in the horizontal plane by a roller at A, a ball-and-socket joint at B, and a cord at C, determine the components of reaction at these supports.

**SOLUTION (SCALAR ANALYSIS)**

**Free-Body Diagram.** There are five unknown reactions acting on the plate, as shown in Fig. 5-28b. Each of these reactions is assumed to act in a positive coordinate direction.

**Equations of Equilibrium.** Since the three-dimensional geometry is rather simple, a scalar analysis provides a direct solution to this problem. A force summation along each axis yields

\[ \sum F_x = 0; \quad B_x = 0 \quad \text{(Ans.)} \]
\[ \sum F_y = 0; \quad B_y = 0 \quad \text{(Ans.)} \]
\[ \sum F_z = 0; \quad A_z + B_z + T_c - 300 \text{ N} - 981 \text{ N} = 0 \quad (1) \]

Recall that the moment of a force about an axis is equal to the product of the force magnitude and the perpendicular distance (moment arm) from the line of action of the force to the axis. Also, forces that are parallel to an axis or pass through it create no moment about the axis. Hence, summing moments about the positive x and y axes, we have

\[ \sum M_x = 0; \quad T_c(2 \text{ m}) - 981 \text{ N}(1 \text{ m}) + B_x(2 \text{ m}) = 0 \quad (2) \]
\[ \sum M_y = 0; \quad 300 \text{ N}(1.5 \text{ m}) + 981 \text{ N}(1.5 \text{ m}) - B_y(3 \text{ m}) - A_y(3 \text{ m}) - 200 \text{ N} \cdot \text{m} = 0 \quad (3) \]

The components of the force at B can be eliminated if moments are summed about the x’ and y’ axes. We obtain

\[ \sum M_x = 0; \quad 981 \text{ N}(1 \text{ m}) + 300 \text{ N}(2 \text{ m}) - A_z(2 \text{ m}) = 0 \quad (4) \]
\[ \sum M_y = 0; \quad -300 \text{ N}(1.5 \text{ m}) - 981 \text{ N}(1.5 \text{ m}) - 200 \text{ N} \cdot \text{m} + T_c(3 \text{ m}) = 0 \quad (5) \]

Solving Eqs. 1 through 3 or the more convenient Eqs. 1, 4, and 5 yields

\[ A_z = 790 \text{ N} \quad B_x = -217 \text{ N} \quad T_c = 707 \text{ N} \quad \text{(Ans.)} \]

The negative sign indicates that B_x acts downward.

**NOTE:** The solution of this problem does not require a summation of moments about the z axis. The plate is partially constrained since the supports cannot prevent it from turning about the z axis if a force is applied to it in the x-y plane.

**example 5.16**

Determine the components of reaction that the ball-and-socket joint at A, the smooth journal bearing at B, and the roller support at C exert on the rod assembly in Fig. 5-29a.

**SOLUTION**

**Free-Body Diagram.** As shown on the free-body diagram, Fig. 5-29b, the reactive forces of the supports will prevent the assembly from rotating about each coordinate axis, and so the journal bearing at B only exerts reactive forces on the member.
**example 5.16**

*Equations of Equilibrium.* A direct solution for \( A_y \) can be obtained by summing forces along the \( y \) axis.

\[ \sum F_y = 0; \quad A_y = 0 \text{ Ans.} \]

The force \( F_c \) can be determined directly by summing moments about the \( y \) axis.

\[ \sum M_y = 0; \quad F_c (0.6 \text{ m}) - 900 \text{ N}(0.4 \text{ m}) = 0 \]

\[ F_c = 600 \text{ N} \text{ Ans.} \]

Using this result, \( B_z \) can be determined by summing moments about the \( x \) axis.

\[ \sum M_x = 0; \quad B_z (0.8 \text{ m}) + 600 \text{ N}(1.2 \text{ m}) - 900 \text{ N}(0.4 \text{ m}) = 0 \]

\[ B_z = -450 \text{ N} \text{ Ans.} \]

The negative sign indicates that \( B_z \) acts downward. The force \( B_z \) can be found by summing moments about the \( z \) axis.

\[ \sum M_z = 0; \quad -B_z (0.8 \text{ m}) - 900 \text{ N} = 0 \]

Thus,

\[ \sum F_z = 0; \quad A_z + 0 = 0 \quad A_z = 0 \text{ Ans.} \]

Finally, using the results of \( B_z \) and \( F_c \).

\[ \sum F_z = 0; \quad A_z + (-450 \text{ N}) + 600 \text{ N} - 900 \text{ N} = 0 \]

\[ A_z = 750 \text{ N} \text{ Ans.} \]

---

**problem 02**

- to show how to determine the forces in the members of a truss using the methods of joints
- to analyze the forces acting on the members of frames and machines composed of pin-connected members

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**6 truss structures**

**example 6.1**

Determine the force in each member of the truss shown in Fig. 6–8a and indicate whether the members are in tension or compression.

**SOLUTION**

Since we should have no more than two unknown forces at the joint and at least one known force acting there, we will begin our analysis at joint \( B \).

**Joint B.** The free-body diagram of the joint at \( B \) is shown in Fig. 6–8b. Applying the equations of equilibrium, we have

\[ \downarrow \sum F_x = 0; \quad 500 \text{ N} - F_{BC} \sin 45^\circ = 0 \quad F_{BC} = 707.1 \text{ N} \text{ Ans.} \]

\[ + \uparrow \sum F_y = 0; \quad F_{BC} \cos 45^\circ - F_{BA} = 0 \quad F_{BA} = 500 \text{ N} \text{ (T)} \text{ Ans.} \]

Since the force in member \( BC \) has been calculated, we can proceed to analyze joint \( C \) to determine the force in member \( CA \) and the support reaction at the rocker.

**Joint C.** From the free-body diagram of joint \( C \), Fig. 6–8c, we have

\[ \downarrow \sum F_x = 0; \quad -F_{CA} + 707.1 \cos 45^\circ \text{ N} = 0 \quad F_{CA} = 500 \text{ N} \text{ (T)} \text{ Ans.} \]

\[ + \uparrow \sum F_y = 0; \quad C_y - 707.1 \sin 45^\circ \text{ N} = 0 \quad C_y = 500 \text{ N} \text{ Ans.} \]

**Joint A.** Although it is not necessary, we can determine the components of the support reactions at joint \( A \) using the results of \( F_{CA} \) and \( F_{BA} \). From the free-body diagram, Fig. 6–8d, we have

\[ \downarrow \sum F_x = 0; \quad 500 \text{ N} - A_x = 0 \quad A_x = 500 \text{ N} \]

\[ + \uparrow \sum F_y = 0; \quad 500 \text{ N} - A_y = 0 \quad A_y = 500 \text{ N} \]

**NOTE:** The results of the analysis are summarized in Fig. 6–8e. Note that the free-body diagram of each joint (or pin) shows the effects of all the connected members and external forces applied to the joint, whereas the free-body diagram of each member shows only the effects of the end joints on the member.
problem 6.5

To show how to determine the forces in the members of a truss using the methods of joints.

To analyze the forces acting on the members of frames and machines composed of pin-connected members.

problem 03

6 truss structures

example 6.9

For the frame shown in Fig. 6-21a, draw the free-body diagram of (a) each member, (b) the pin at B, and (c) the two members connected together.

example 6.9

SOLUTION

Part (a): By inspection, members BA and BC are not two-force members. Instead, as shown on the free-body diagrams, Fig. 6-21b, BC is subjected to a force from the pins at B and C and the external force P. Likewise, AB is subjected to a force from the pins at A and B and the external couple moment M. The pin forces are represented by their x and y components.

Part (b): The pin at B is subjected to only two forces, i.e., the force of member BC and the force of member AB. For equilibrium, these forces or their respective components must be equal but opposite, Fig. 6-21c. Realize that Newton’s third law is applied between the pin and its connected members, i.e., the effect of the pin on the two members, Fig. 6-21b, and the equal but opposite effect of the two members on the pin, Fig. 6-21c.

Part (c): The free-body diagram of both members connected together, yet removed from the supporting pins at A and C, is shown in Fig. 6-21d. The force components B_x and B_y are not shown on this diagram since they are internal forces (Fig. 6-21b) and therefore cancel out. Also, to be consistent when later applying the equilibrium equations, the unknown force components at A and C must act in the same sense as those shown in Fig. 6-21b.
example 6.9

6 frame structures

problem 6.74

• to show how to use the method of sections to determine the internal loadings in a member

• to generalize this procedure by formulating equations that can be plotted so that they describe the internal shear and moment throughout a member

6 frame structures

example 6.9

7 shear & moment diagrams
example - simply supported beam

Shear and Moment Diagrams. When Eqs. 1 through 4 are plotted within the regions in which they are valid, the shear and moment diagrams shown in Fig. 7–11d are obtained. The shear diagram indicates that the internal shear force is always 2.5 kN (positive) within segment AB. Just to the right of point B, the shear force changes sign and remains at a constant value of 1.5 kN for segment BC. The moment diagram starts at zero, increases linearly to point B at \( x = 2 \text{ m} \), where \( M_{\text{max}} = -2.5 \text{ kN} \cdot \text{m} \), and thereafter decreases back to zero.

NOTE: It is seen in Fig. 7–11d that the graphs of the shear and moment diagrams are discontinuous where the concentrated force acts, i.e., at points A, B, and C. For this reason, as stated earlier, it is necessary to express both the shear and moment functions separately for regions between concentrated loads. It should be realized, however, that all loading discontinuities are mathematical, arising from the idealization of a concentrated force and couple moment. Physically, loads are always applied over a finite area, and if the actual load variation could be accounted for, the shear and moment diagrams would then be continuous over the shaft’s entire length.

example 7.6

SOLUTION

Support Reactions. The support reactions are shown on the shaft’s free-body diagram, Fig. 7–11a.

Shear and Moment Functions. The shaft is sectioned at an arbitrary distance \( x \) from point A, extending within the region AB, and the free-body diagram of the left segment is shown in Fig. 7–11b. The unknowns \( V \) and \( M \) are assumed to act in the positive sense on the right-hand face of the segment according to the established sign convention. Applying the equilibrium equations yields:

\[
\begin{align*}
V + \sum F_y &= 0; & V &= 2.5 \text{ kN} \\
\sum M &= 0; & M &= 2.5x \text{ kN} \cdot \text{m} \\
\end{align*}
\]

A free-body diagram for a left segment of the shaft extending a distance \( x \) within the region BC is shown in Fig. 7–11c. As always, \( V \) and \( M \) are shown acting in the positive sense. Hence,

\[
\begin{align*}
V + \sum F_y &= 0; & \quad V &= 2.5 \text{ kN} - 5 \text{ kN} - V = 0 \\
V &= -2.5 \text{ kN} \\
\sum M &= 0; & \quad M + 5 \text{ kN} (x - 2 \text{ m}) - 2.5 \text{ kN} (x) &= 0 \\
M &= (10 - 2.5x) \text{ kN} \cdot \text{m} \\
\end{align*}
\]

7 shear & moment diagrams

7 shear & moment diagrams

statics of the hanging problem

idealized free body diagram
cantiliver beam

\[
\begin{align*}
V &= W \\
M &= W \cdot 1 \\
V &= W \cdot \text{const.} \\
M &= (10 - 2.5x) \text{ kN} \cdot \text{m} \\
M_{\text{min}} &= -W \cdot [1-x] \\
\end{align*}
\]
friction
• the heat generated by the abrasive action of friction can be noticed when using this drinder to sharpen a metal blade
• friction is a force that resists the movement of two contacting surfaces relative to one another
• friction always acts tangent to the surface and is directed opposite to a possible motion

example 8.1
The uniform crate shown in Fig. 8–7a has a mass of 20 kg. If a force $P = 80$ N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$. 

slipping & tipping
pushing on the uniform crate of weight $W$ sitting on a rough surface. If the magnitude $P$ is small, the crate will remain in equilibrium and not move (left FBD). As $P$ increases, the crate will either be on the verge of slipping, $F = \mu_s \cdot M$, or, if the surface is very rough with large $\mu_s$, the resultant force moves towards the corner and beyond, $x>b/2$, and the crate will tip over. Tipping also depends on the height $h$ of the force $P$. 

8.2 problems involving dry friction
**example 8.1**

**Free-Body Diagram.** As shown in Fig. 8-7b, the resultant normal force \( N_C \) must act a distance \( x \) from the crate’s center line in order to counteract the tipping effect caused by \( P \). There are three unknowns, \( F \), \( N_C \), and \( x \), which can be determined directly from the three equations of equilibrium.

**Equations of Equilibrium.**

\[
\begin{align*}
\Sigma F_y &= 0; \\
80 \cos 30^\circ N - F &= 0 \\
\Sigma F_x &= 0; \\
-80 \sin 30^\circ N + N_C - 196.2 N &= 0 \\
\Sigma M_B &= 0; \\
80 \sin 30^\circ N (0.4 m) - 80 \cos 30^\circ N (0.2 m) + N_C (x) &= 0
\end{align*}
\]

Solving,

\[
\begin{align*}
F &= 69.3 N \\
N_C &= 236 N \\
x &= -0.0908 m = -9.08 mm
\end{align*}
\]

Since \( x \) is negative, it indicates the resultant normal force acts (slightly) to the left of the crate’s center line. No tipping will occur since \( x < 0.4 m \). Also, the minimum frictional force which can be developed at the surface of contact is \( F_{\text{min}} = \mu_s N_C = 0.3(236 N) = 70.8 N \). Since \( F = 69.3 N < 70.8 N \), the crate will not slip, although it is very close to doing so.

**example 8.3**

**Free-Body Diagram.** As shown on the free-body diagram, Fig. 8-9b, the frictional force \( F_A \) must act to the right since impending motion at \( A \) is to the left.

**Equations of Equilibrium and Friction.** Since the ladder is on the verge of slipping, then \( F_A = \mu_s N_A = 0.3 N_A \). By inspection, \( N_A \) can be obtained directly.

\[
\begin{align*}
\Sigma F_y &= 0; \\
N_A - 10(9.81) N &= 0 \\
N_A &= 98.1 N
\end{align*}
\]

Using this result, \( F_A = 0.3(98.1 N) = 29.43 N \). Now \( N_B \) can be found.

\[
\begin{align*}
\Sigma F_x &= 0; \\
29.43 N - N_B &= 0 \\
N_B &= 29.43 N = 29.4 N \\
\text{Ans.}
\end{align*}
\]

Finally, the angle \( \theta \) can be determined by summing moments about point \( A \).

\[
\begin{align*}
\Sigma M_A &= 0; \\
(29.43 N)(4 m) \sin \theta - [10(9.81) N](2 m) \cos \theta &= 0 \\
\frac{\sin \theta}{\cos \theta} &= \tan \theta = 1.6667 \\
\theta &= 59.04^\circ = 59.0^\circ \\
\text{Ans.}
\end{align*}
\]

**example 8.3**

The uniform 10-kg ladder in Fig. 8-9a rests against the smooth wall at \( B \), and the end \( A \) rests on the rough horizontal plane for which the coefficient of static friction is \( \mu_s = 0.3 \). Determine the angle of inclination \( \theta \) of the ladder and the normal reaction at \( B \) if the ladder is on the verge of slipping.
• to discuss the concept of the center of gravity, center of mass, and centroid
• to show how to determine the center of gravity and centroid for a system of particles
• to show how to determine the center of gravity and centroid for composite bodies
example 9.10

Locate the centroid of the plate area shown in Fig. 9-17a.

SOLUTION

Composite Parts. The plate is divided into three segments as shown in Fig. 9-17b. Here the area of the small rectangle ② is considered “negative” since it must be subtracted from the larger one ③.

Moment Arms. The centroid of each segment is located as indicated in the figure. Note that the x coordinates of ① and ② are negative.

9.2 composite bodies

mr equilibrium: isaac newton

powered by jacob

... and our all time hero is...

e14 - applied mechanics: statics

• regular final
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final exam