mon/wed/fri, 12:50-2:05pm, 370-370

from friction to heat

- the heat generated by the abrasive action of friction can be noticed when using this drinder to sharpen a metal blade
- friction is a force that resists the movement of two contacting surfaces relative to one another
- friction always acts tangent to the surface and is directed opposite to a possible motion

8.1 characteristics of dry friction
8.2 problems involving dry friction

pushing on the uniform crate of weight \( W \) sitting on a rough surface. If the magnitude \( P \) is small, the crate will remain in equilibrium and not move (left FBD). As \( P \) increases, the crate will either be on the verge of slipping, \( F = \mu_s \cdot M \), or, if the surface is very rough with large \( \mu_s \), the resultant force moves towards the corner and beyond, \( x > b/2 \), and the crate will tip over. Tipping also depends on the height \( h \) of the force \( P \).

example 8.1

The uniform crate shown in Fig. 8–7a has a mass of 20 kg. If a force \( P = 80 \text{ N} \) is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is \( \mu_s = 0.3 \).

example 8.3

The uniform 10-kg ladder in Fig. 8–9a rests against the smooth wall at \( B \), and the end \( A \) rests on the rough horizontal plane for which the coefficient of static friction is \( \mu_s = 0.3 \). Determine the angle of inclination \( \theta \) of the ladder and the normal reaction at \( B \) if the ladder is on the verge of slipping.

SOLUTION

Free-Body Diagram. As shown in Fig. 8–7b, the resultant normal force \( N_C \) must act a distance \( x \) from the crate’s center line in order to counteract the tipping effect caused by \( P \). There are three unknowns, \( F \), \( N_C \), and \( x \), which can be determined strictly from the three equations of equilibrium.

Equations of Equilibrium.

\[
\begin{align*}
\Sigma F_x &= 0; \\
\Sigma F_y &= 0; \\
\Sigma M_0 &= 0;
\end{align*}
\]

Solving.

\[
\begin{align*}
F &= 69.3 \text{ N} \\
N_C &= 236 \text{ N} \\
x &= -0.0908 \text{ m} = -9.08 \text{ mm}
\end{align*}
\]

Since \( x \) is negative it indicates the resultant normal force acts (slightly) to the left of the crate’s center line. No tipping will occur since \( x < 0.4 \text{ m} \). Also, the maximum frictional force which can be developed at the surface of contact is \( F_{\text{max}} = \mu_s N_C = 0.3(236 \text{ N}) = 70.8 \text{ N} \). Since \( F = 69.3 \text{ N} < 70.8 \text{ N} \), the crate will not slip, although it is very close to doing so.
example 8.3

Free-Body Diagram. As shown on the free-body diagram, Fig. 8-9b, the frictional force \( F_A \) must act to the right since impending motion at \( A \) is to the left.

Equations of Equilibrium and Friction. Since the ladder is on the verge of slipping, then \( F_A = \mu N_A = 0.3 N_A \). By inspection, \( N_A \) can be obtained directly.

\[ + \sum F_y = 0; \quad N_A - 10(9.81) N = 0 \quad N_A = 98.1 N \]

Using this result, \( F_A = 0.3(98.1) N = 29.43 N \). Now \( N_B \) can be found.

\[ - \sum F_x = 0; \quad 29.43 N - N_B = 0 \quad N_B = 29.43 N = 29.4 N \quad Ans. \]

Finally, the angle \( \theta \) can be determined by summing moments about point \( A \).

\[ \sum M_A = 0; \quad (29.43 N)(4 m) \sin \theta - [10(9.81) N](2 m) \cos \theta = 0 \]

\[ \frac{\sin \theta}{\cos \theta} = \tan \theta = 1.6667 \]

\[ \theta = 59.04^\circ = 59.0^\circ \quad Ans. \]

8.2 problems involving dry friction

center of a collection of particles

The location of the center of gravity from sum of all forces and moments of resultant

\[ \bar{x} = \frac{\sum x dW}{\sum dW} \quad \bar{y} = \frac{\sum y dW}{\sum dW} \quad \bar{z} = \frac{\sum z dW}{\sum dW} \]

9.1 center of gravity/mass and centroid

9.2 composite bodies

today's objectives

- to discuss the concept of the center of gravity, center of mass, and centroid
- to show how to determine the center of gravity and centroid for a system of particles
- to show how to determine the center of gravity and centroid for composite bodies

9. center of gravity and centroid

divide the structure!

the center of gravity of this water tank can be determined by dividing it into composite parts

\[ \bar{x} = \frac{\sum x W}{\sum W} \]
\[ \bar{y} = \frac{\sum y W}{\sum W} \]
\[ \bar{z} = \frac{\sum z W}{\sum W} \]
example 9.10

Locate the centroid of the plate area shown in Fig. 9-17a.

SOLUTION

Composite Parts. The plate is divided into three segments as shown in Fig. 9-17b. Here the area of the small rectangle ② is considered “negative” since it must be subtracted from the larger one ①.

Moment Arms. The centroid of each segment is located as indicated in the figure. Note that the \( x \) coordinates of ① and ② are negative.

9.2 composite bodies

example 9.11

Locate the center of mass of the assembly shown in Fig. 9-18a. The conical frustum has a density of \( \rho_c = 8 \text{ Mg/m}^3 \), and the hemisphere has a density of \( \rho_h = 4 \text{ Mg/m}^3 \). There is a 25-mm-radius cylindrical hole in the center of the frustum.

SOLUTION

Composite Parts. The assembly can be thought of as consisting of four segments as shown in Fig. 9-18b. For the calculations, ③ and ④ must be considered “negative” segments in order that the four segments, when added together, yield the total composite shape shown in Fig. 9-18a.

Moment Arm. Using the table on the inside back cover, the computations for the centroid \( \overline{z} \) of each piece are shown in the figure.

Summations. Because of symmetry, note that

\[ \overline{x} = \overline{y} = 0 \]

Thus, \( \overline{z} = \frac{\Sigma zm}{\Sigma m} = 45.815 \) mm. An answer.

9.2 composite bodies