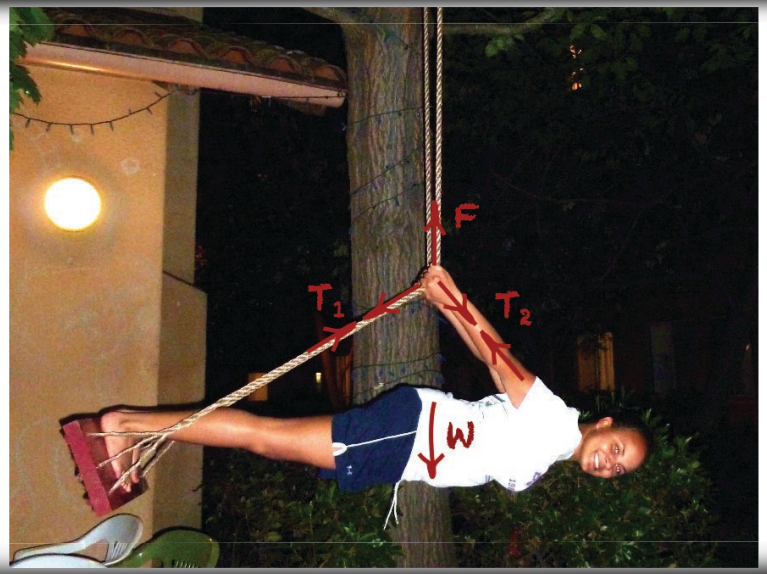
e14 - applied mechanics: statics



mon/wed/fri, 12:50-2:05pm, 370-370

e14 - applied mechanics: statics

	w06		3d equilibrium week	ch 5
mon	may	02	what's a free body diagram of a 3d system?	5.5
wed	may	04	what force and moment equilibrium in 3d?	5.6-5.7
fri	may	06	problem session 4	
	w07		structures week	ch 6
mon	may	09	what's truss structure?	6.1-6.3
wed	may	11	what's a frame?	6.6
fri	may	13	problem session 5	
	w08		internal force week	ch 7
mon	may	16	what's inside?	7.1
wed	may	18	midterm 2, in class, closed book, 1 cheat sheet	cubberley
fri	may	20	what's a shear and moment diagram?	7.2-7.3
	w09		friction and center week	ch 8-9
mon	may	23	~~what's friction?~~	8.1-8.2
~~wed~~	~~may~~	~~25~~	what's the center?	9.1-9.2
fri	may	27	~~problem session 6~~	
	~~w10~~		that's it week	
mon	may	30	memorial day / no class	
wed	jun	01	what's a was this all about?	
fri	jun	03	end of quarter / no class	
sat	jun	04	problems, problems, problems ...	
mon	jun	06	final, in class, closed book, 1 cheat sheet	

fifth homework due

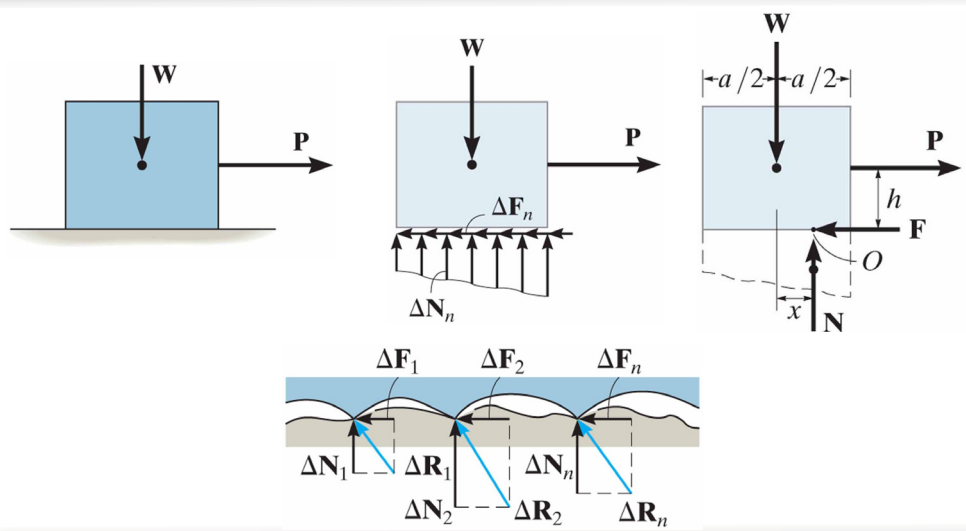
syllabus

from friction to heat

- the heat generated by the abrasive action of friction can be noticed when using this drinder to sharpen a metal blade
- friction is a force that risists the movement of two contacting surfaces relative to one another
- friction always acts **tangent to the surface** and is directed **opposite to a possible motion**

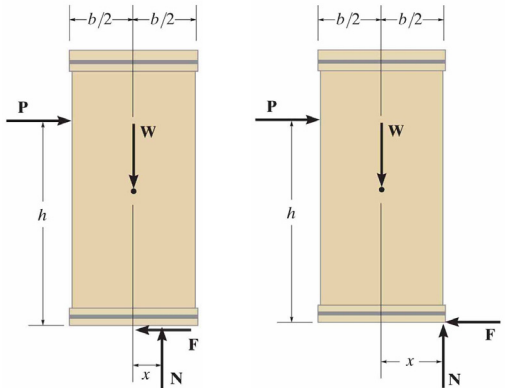
8.1 characteristics of dry friction

theory of dry friction



8.1 characteristics of dry friction

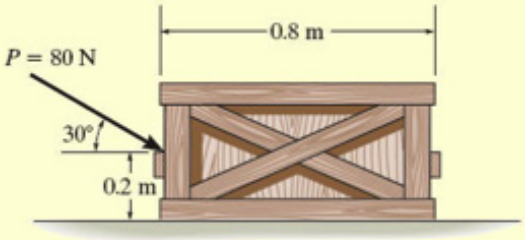
slipping & tipping



pushing on the uniform crate of weight W sitting on a rough surface. if the magnitude P is small, the crate will remain in equilibrium and not move (left FBD). as P increases, the crate will either be on the verge of slipping, $F = \mu_s \cdot M$, or, if the surface is very rough with large μ_s, the resultant force moves towards the corner and beyond, x>b/2, and the crate will tip over. tipping also depends on the height h of the force P.

example 8.1

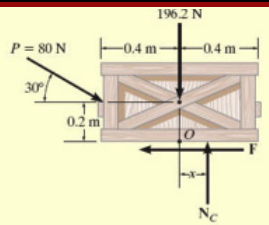
The uniform crate shown in Fig. 8–7*a* has a mass of 20 kg. If a force $P = 80$ N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$.



example 8.1

SOLUTION

Free-Body Diagram. As shown in Fig. 8–7*b*, the *resultant* normal force $\mathbf{N}_C$ must act a distance x from the crate's center line in order to counteract the tipping effect caused by **P**. There are *three unknowns*, F, N_C, and x, which can be determined strictly from the *three* equations of equilibrium.



(b)

Equations of Equilibrium.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 80 \cos 30^\circ \text{ N} - F = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad -80 \sin 30^\circ \text{ N} + N_C - 196.2 \text{ N} = 0$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad 80 \sin 30^\circ \text{ N}(0.4 \text{ m}) - 80 \cos 30^\circ \text{ N}(0.2 \text{ m}) + N_C(x) = 0$$

Solving,

$$F = 69.3 \text{ N}$$

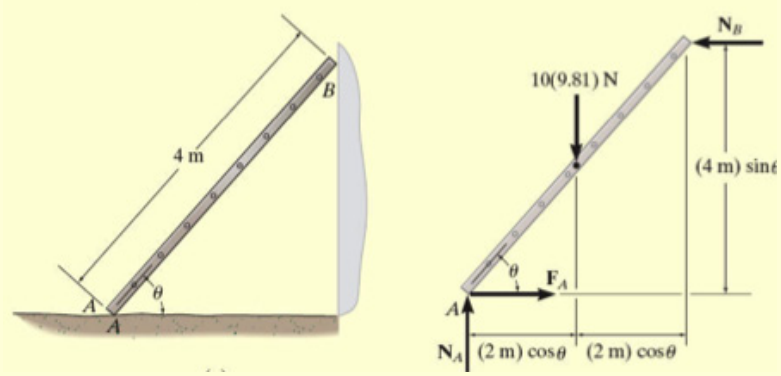
$$N_C = 236 \text{ N}$$

$$x = -0.00908 \text{ m} = -9.08 \text{ mm}$$

Since x is negative it indicates the *resultant* normal force acts (slightly) to the *left* of the crate's center line. No tipping will occur since $x < 0.4$ m. Also, the *maximum* frictional force which can be developed at the surface of contact is $F_{max} = \mu_s N_C = 0.3(236 \text{ N}) = 70.8$ N. Since $F = 69.3 \text{ N} < 70.8$ N, the crate will *not slip*, although it is very close to doing so.

example 8.3

The uniform 10-kg ladder in Fig. 8–9*a* rests against the smooth wall at B, and the end A rests on the rough horizontal plane for which the coefficient of static friction is $\mu_s = 0.3$. Determine the angle of inclination θ of the ladder and the normal reaction at B if the ladder is on the verge of slipping.



example 8.3

Free-Body Diagram. As shown on the free-body diagram, Fig. 8–9*b*, the frictional force $\mathbf{F}_A$ must act to the right since impending motion at A is to the left.

Equations of Equilibrium and Friction. Since the ladder is on the verge of slipping, then $F_A = \mu_s N_A = 0.3N_A$. By inspection, N_A can be obtained directly.

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 10(9.81)\text{ N} = 0 \qquad N_A = 98.1\text{ N}$$

Using this result, $F_A = 0.3(98.1\text{ N}) = 29.43\text{ N}$. Now N_B can be found.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 29.43\text{ N} - N_B = 0$$

$$N_B = 29.43\text{ N} = 29.4\text{ N} \qquad \textit{Ans.}$$

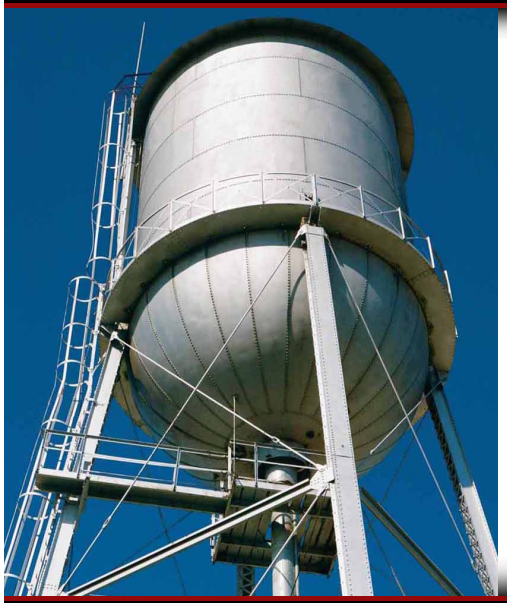
Finally, the angle θ can be determined by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad (29.43\text{ N})(4\text{ m})\sin\theta - [10(9.81)\text{ N}](2\text{ m})\cos\theta = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = 1.6667$$

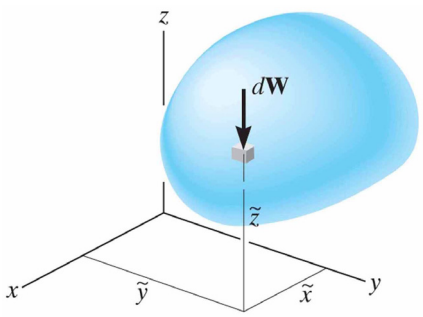
$$\theta = 59.04° = 59.0° \qquad \textit{Ans.}$$

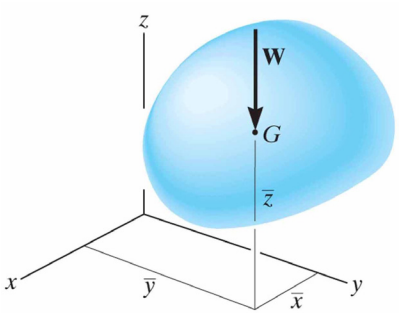
today's objectives



- to discuss the concept of the center of gravity, center of mass, and centroid
- to show how to determine the center of gravity and centroid for a **system of particles**
- to show how to determine the center of gravity and centroid for **composite bodies**

center of a collection of particles

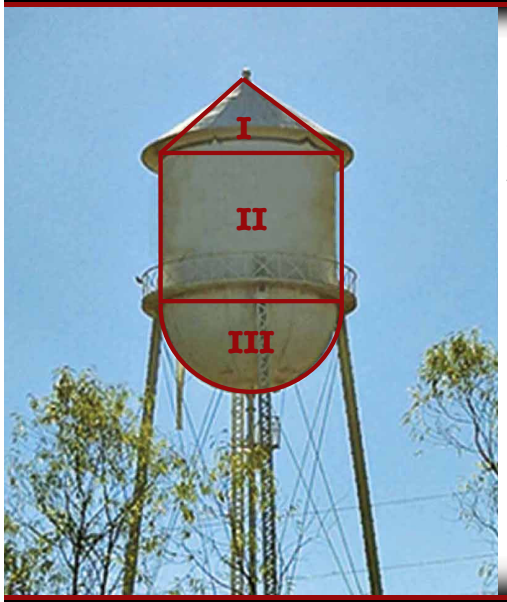




location of the center of gravity from sum of all forces and moments of resultant

$$\bar{x} = \frac{\square\, \tilde{x}\, dW}{\square\, dW} \qquad \bar{y} = \frac{\square\, \tilde{y}\, dW}{\square\, dW} \qquad \bar{z} = \frac{\square\, \tilde{z}\, dW}{\square\, dW}$$

divide the structure!



the center of gravity of this water tank can be determined by dividing it into composite parts

$$\bar{x} = \frac{\square\, \tilde{x}\, W}{\square\, W}$$

$$\bar{y} = \frac{\square\, \tilde{y}\, W}{\square\, W}$$

$$\bar{z} = \frac{\square\, \tilde{z}\, W}{\square\, W}$$

example 9.10

Locate the centroid of the plate area shown in Fig. 9–17*a*.

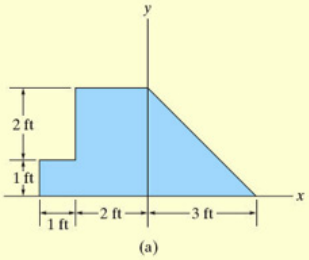
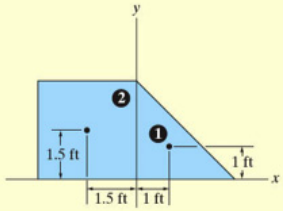


Fig. 9–17

SOLUTION

Composite Parts. The plate is divided into three segments as shown in Fig. 9–17*b*. Here the area of the small rectangle ③ is considered "negative" since it must be subtracted from the larger one ②.

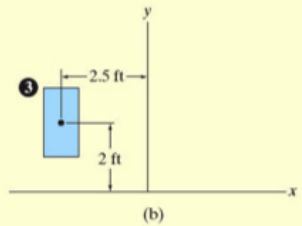
Moment Arms. The centroid of each segment is located as indicated in the figure. Note that the $\tilde{x}$ coordinates of ② and ③ are *negative*.



example 9.10

Summations. Taking the data from Fig. 9–17*b*, the calculations are tabulated as follows:

Segment	A (ft^2)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}A$ (ft^3)	$\tilde{y}A$ (ft^3)
1	$\frac{1}{2}(3)(3) = 4.5$	1	1	4.5	4.5
2	$(3)(3) = 9$	−1.5	1.5	−13.5	13.5
3	$-(2)(1) = -2$	−2.5	2	5	−4
	$\Sigma A = 11.5$			$\Sigma\tilde{x}A = -4$	$\Sigma\tilde{y}A = 14$



Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348 \text{ ft} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22 \text{ ft} \qquad \textit{Ans.}$$

NOTE: If these results are plotted in Fig. 9–17, the location of point *C* seems reasonable.

example 9.11

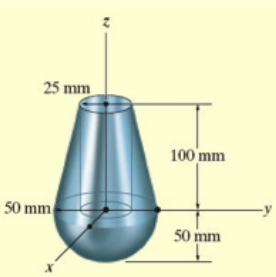


Fig. 9–18

Locate the center of mass of the assembly shown in Fig. 9–18*a*. The conical frustum has a density of $\rho_c = 8$ Mg/m^3, and the hemisphere has a density of $\rho_h = 4$ Mg/m^3. There is a 25-mm-radius cylindrical hole in the center of the frustum.

SOLUTION

Composite Parts. The assembly can be thought of as consisting of four segments as shown in Fig. 9–18*b*. For the calculations, ③ and ④ must be considered as "negative" segments in order that the four segments, when added together, yield the total composite shape shown in Fig. 9–18*a*.

Moment Arm. Using the table on the inside back cover, the computations for the centroid $\tilde{z}$ of each piece are shown in the figure.

Summations. Because of *symmetry*, note that

$$\bar{x} = \bar{y} = 0 \qquad \textit{Ans.}$$

Since $W = mg$, and g is constant, the third of Eqs. 9–6 becomes $\bar{z} = \Sigma\tilde{z}m/\Sigma m$. The mass of each piece can be computed from $m = \rho V$ and used for the calculations. Also, 1 Mg/m^3 = 10^{-6} kg/mm^3, so that

example 9.11

Segment	m (kg)	$\tilde{z}$(mm)	$\tilde{z}m$ (kg · mm)
1	$8(10^{-6})\left(\frac{1}{3}\right)\pi(50)^2(200) = 4.189$	50	209.440
2	$4(10^{-6})\left(\frac{2}{3}\right)\pi(50)^3 = 1.047$	−18.75	−19.635
3	$-8(10^{-6})\left(\frac{1}{3}\right)\pi(25)^2(100) = -0.524$	100 + 25 = 125	−65.450
4	$-8(10^{-6})\pi(25)^2(100) = -1.571$	50	−78.540
	$\Sigma m = 3.142$		$\Sigma\tilde{z}m = 45.815$

Thus, $$\bar{z} = \frac{\Sigma\tilde{z}m}{\Sigma m} = \frac{45.815}{3.142} = 14.6 \text{ mm} \qquad \textit{Ans.}$$