mon/wed/fri, 12:50-2:05pm, 370-370

Homework II - Chapters 3 and 4
due Friday, 04/15/11, 12:50pm, 370-370

For late homework, you are responsible to arrange drop off with our grader Kaushik Mani, kmani@stanford.edu. Once you have used up your three late days, you will no longer receive points for your homework. Here are our office hours and emails.

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<thead>
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<th>who</th>
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<tbody>
<tr>
<td>Tuesdays</td>
<td>06:00 - 07:30pm</td>
<td>Durand 247</td>
<td>Charbel</td>
<td><a href="mailto:ceid@stanford.edu">ceid@stanford.edu</a></td>
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<tr>
<td>Wednesdays</td>
<td>02:30 - 04:00pm</td>
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<td>Ellen</td>
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<tr>
<td>Wednesdays</td>
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<td>10:00 - 11:30am</td>
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<tr>
<td>Thursdays</td>
<td>01:00 - 02:30pm</td>
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<td>Estevan</td>
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For this homework, you need to be familiar with chapters 3 and 4 of your book! All solutions for problems 1 - 4 must include a free body diagram!
newton’s three laws of motion

- first law
  equilibrium
  if \( \sum \mathbf{F} = \mathbf{0} \) then \( \mathbf{v} = \text{const.} \)

- second law
  accelerated motion
  \( \mathbf{F} = m \cdot \mathbf{a}^{\neq} = 0 \)

- third law
  actio = reactio
  \( \mathbf{F}_{AB} = - \mathbf{F}_{BA} \)

3. equilibrium of a particle

• to introduce the concept of the free-body diagram for a particle
• to show how to solve particle equilibrium problems using the equations of equilibrium
• when cables are used for hoisting loads, the must be selected so that they do not fail. today, we will show how to calculate cable forces for such cases

3.1 equilibrium condition of a particle

3.2 free body diagram

I. isolate the particle of interest - easy ;-) here shown for particle A

II. show all forces - tricky!
  3 cables, 3 tension forces assume directions

III. label each force - easy ;-)
conceptual problem 3-1

Show that the longer the cables, the less the forces in each cable.

\[ W \]

\[ F_{AB} \]

\[ F_{AC} \]

\[ 2\theta \]

For longer cables, the angle \( \theta \) becomes smaller, \( \cos \theta \) becomes bigger, and \( F_{AB} \) and \( F_{AC} \) become smaller.

3.3 coplanar force systems

engineering intuition

\[ M_0 = F \cdot d \]

\[ [M] = N \cdot m = lb \cdot ft \]

\( O \) ... fixed point
\( F \) ... force
\( d \) ... moment arm

largest moment  smaller moment  no moment

determine the moment \( M_0 \) of the force \( F \) about point \( O \)

today’s objectives

- To discuss the concept of a moment of a force
- To calculate the moment of a force in 2d and 3d using the scalar formulation
- To discuss the cross product as a tool to calculate moments
- To calculate the moment of a force in 2d and 3d using the vector formulation

4. force system resultants

example 4.1

4.1 moment - scalar formulation

4.1 moment - scalar formulation
**Example 4.2**

**SOLUTION**
Assuming that positive moments act in the +k direction, i.e., counterclockwise, we have

\[
\Sigma M_{R_0} = \Sigma Fd;
\]

\[
M_{R_0} = -50 \text{N}(2 \text{m}) + 60 \text{N}(0) + 20 \text{N}(3 \sin 30^\circ \text{m})
\]

\[
-40 \text{N}(4 \text{m} + 3 \cos 30^\circ \text{m})
\]

\[
M_{R_0} = -334 \text{N}\cdot\text{m} = 334 \text{N}\cdot\text{m} \uparrow
\]

For this calculation, note how the moment-arm distances for the 20-N and 40-N forces are established from the extended (dashed) lines of action of each of these forces.

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**4.1 moment - scalar formulation**

**right-handed system**

\[
C = A \times B = A \cdot B \cdot \sin \theta \ u_C
\]

- C is a vector
- C is orthogonal to A and B
- C is the area enclosed by A and B, i.e., \(A \cdot B \cdot \sin \theta\)

\[
C = A \times B = \begin{vmatrix} A_x & B_x & x \\ A_y & B_y & y \\ A_z & B_z & z \end{vmatrix}
\]

\[
C = \begin{bmatrix} A_y B_z - A_z B_y \\ A_x B_z - A_z B_x \\ A_x B_y - A_y B_x \end{bmatrix}
\]

... if your thumb is the x axis...

... and your middle finger is the z axis...

... then your index finger is...

... pointing right at your abs...
right-handed system

\[ M_O = r \times F = r \cdot F \cdot \sin \theta \ u_M \]

- \( M_O \) is a vector
- \( M_O \) is orthogonal to \( r \) and \( F \)
- \( M_O \) is the area enclosed by \( r \) and \( F \), i.e., \( r \cdot F \cdot \sin \theta \)

\[ \begin{vmatrix} r_x & F_x & x \\ r_y & F_y & y \\ r_z & F_z & z \end{vmatrix} \]

\[ \begin{vmatrix} r_y F_z - r_z F_y \\ r_z F_x - r_x F_z \\ r_x F_y - r_y F_x \end{vmatrix} \]

\( M_O \) is a vector
\( M_O \) is orthogonal to \( r \) and \( F \)
\( M_O \) is the area enclosed by \( r \) and \( F \), i.e., \( r \cdot F \cdot \sin \theta \)

**Example 4.5**

Determine the moment \( M_O \) of the force \( F \) about point \( O \)

**Solution I**

The moment arm \( d \) in Fig. 3-18a can be found from trigonometry.

\[ d = (3 \text{ m}) \sin 75° = 2.898 \text{ m} \]

Thus,

\[ M_O = Fd = (5 \text{ kN})(2.898 \text{ m}) = 14.5 \text{ kN} \cdot \text{m} \quad \text{Ans.} \]

Since the force tends to rotate or orbit clockwise about point \( O \), the moment is directed into the page.

**Solution II**

The \( x \) and \( y \) components of the force are indicated in Fig. 3-18b.

Considering counterclockwise moments as positive, and applying the principle of moments, we have

\[ \begin{align*}
\downarrow + M_O &= -F_x d_y - F_y d_x \\
&= -(5 \text{ kN})(3 \sin 30° \text{ m}) - (5 \sin 45° \text{ kN})(3 \cos 30° \text{ m}) \\
&= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \\
&\quad \text{Ans.}
\end{align*} \]