2D EQUILIBRIUM WEEK

WHAT'S FORCE & MOMENT EQUILIBRIUM IN 2D?

5.3 EQUATIONS OF EQUILIBRIUM (CHAPTER 5.3-5.4)

- Equilibrium in 2D
  \[ \sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0 \]

- Alternative set of equations
  \[ \sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0 \]
  such that \( \vec{r}_{AB} \times \vec{x} \) (A & B may not be perp to x)

- Or, alternatively
  \[ \sum M_A = 0 \quad \sum M_B = 0 \quad \sum M_C = 0 \]
  such that \( A, B, C \) do not lie on the same line

Remarks: Use one of the eqns you have not used before as a control!

\[ \sum M_0 = 0 \] first, select point \& such that two lines of action pass through it is direct solution & one face.

You may orient the x-y-system along your members!

If the source to a force is negative,
this implies that it acts opposite to the direction you have assumed!
**Example 5.5**

I. Information

II. Free Body Diagram (everybody, try themselves)

III. Solution

Which equation do we start with? Why?

\[ \sum M_A = 0 \]

\[ +B \cdot 0.75m - N \cdot 1.0m - M = 0 \]

\[ \rightarrow B = \frac{N \cdot 1.0m + M}{0.75m} \]

\[ = \frac{60 \text{ Nm} + 90 \text{ Nm}}{0.75m} = 200 \text{ N} \]

\[ \sum F_x = 0 \]

\[ A_x - B \cdot \sin 30^\circ = 0 \]

\[ \rightarrow A_x = B \cdot \sin 30^\circ = 200 \text{ N} \cdot 0.5 = 100 \text{ N} \]

\[ \sum F_y = 0 \]

\[ A_y - N - B \cdot \cos 30^\circ = 0 \]

\[ \rightarrow A_y = N + B \cdot \cos 30^\circ \]

\[ = 60 \text{ N} + 200 \text{ N} \cdot \frac{1}{2} \cdot \sqrt{3} = 233 \text{ N} \]

\[ \sum M_B = 0 \]

\[ -M - N[1.0m \cdot 0.75m \cdot \cos 30^\circ] + A_x \cdot 0.75m \cdot \sin 30^\circ + A_y \cdot 0.75m \cdot \cos 30^\circ \]

\[ \text{in project} = 0 \text{ Nm} \]
graphic control using vector addition

\[
\begin{align*}
Ax + Ay + B + N &= 0 \\
\text{establish a length scale!}
\end{align*}
\]

5.4 TWO- & THREE-FORCE MEMBERS

TWO FORCE MEMBERS
- pin-connected at both ends
- weightless
- no extra forces acting on it

\[
\begin{align*}
\Sigma F_x &= 0 & \Sigma F_y &= 0 \\
F_A &= F_B & F_A & F_B \text{ are equal & opposite} \\
\Sigma M &= 0 \\
F_A & F_B \text{ lie on the same line of action (\& do not produce a couple moment)}
\end{align*}
\]

Identifying two-force members is essential or you will not have enough equations to solve for the unknowns!

EXAMPLE 5.13!
THREE FORCE MEMBER

- Concurrent force system
- Parallel force system

\[ \Sigma M = 0 \]

requires that the three forces form a concurrent (meeting in \( \Theta \)) or parallel force system.

Special case of parallel force system: lines of action intersect at "infinity".

Example 5.13

I. Image projected
II. Free body diagram

Graphic solution

- \( C = 400 \text{ N} \)
- \( B \approx 1300 \text{ N} \)
- \( A \approx 1000 \text{ N} \)

Kinematics:

\[ \theta = ? \quad \tan \theta = \frac{0.5 \text{ m} + 0.2 \text{ m}}{0.4 \text{ m}} \quad \Rightarrow \theta = 60.3^\circ \]

This replaces \( \Sigma M = 0 \) equation, then use \( \Sigma F_x = 0 \) & \( \Sigma F_y = 0 \) to det. A & B.